Majorana Neutrinos, Neutrino Oscillations, $\mu \rightarrow e + \gamma$ Decay and Beyond

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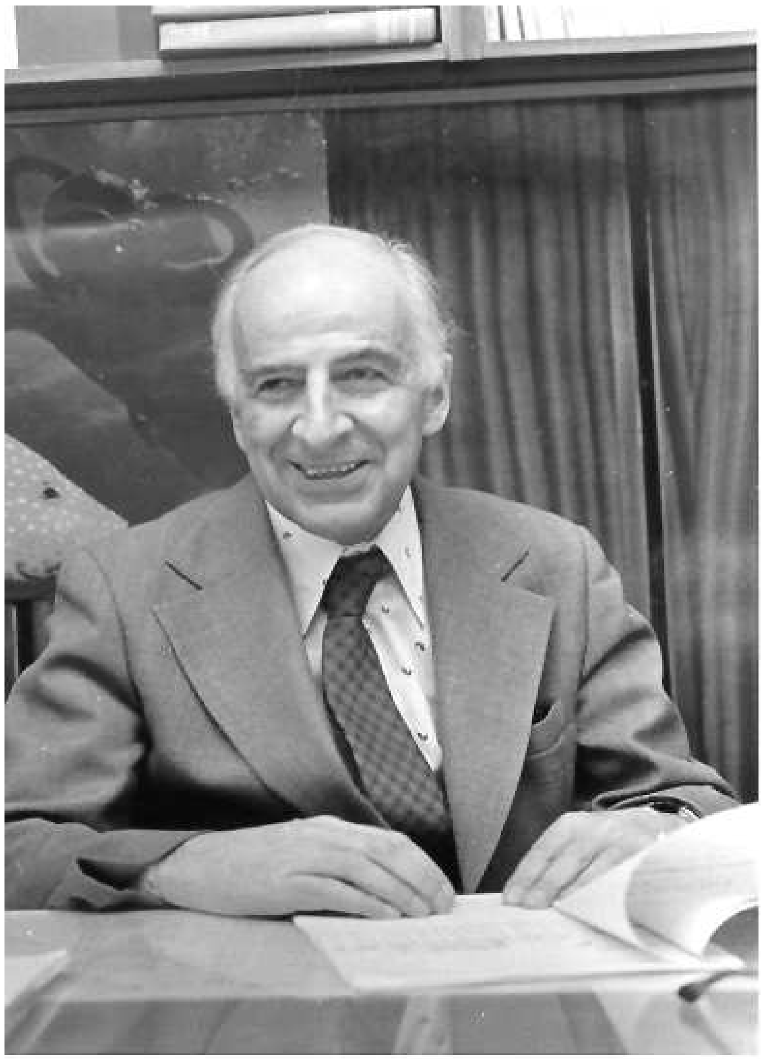
JINR, Dubna
February 18, 2011



Main Topics of Research in Neutrino Physics:

- Understanding the Properies of Majorana Neutrinos.
- The Problem of Establishing the Nature - Dirac or Majorana, of Massive Neutrinos.
- Theory of Neutrino Oscillations.
- Lepton Flavour Violating Processes ($\mu \rightarrow e + \gamma$, etc.)
- Understanding the Origin of the Emerging Patterns of Neutrino Masses and Mixing (Symmetries).
- The Possible Connection between the Generation of Neutrino Masses and of the Baryon Asymmetry of the Universe.

Compelling Evidences for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

V. Gribov, B. Pontecorvo, 1969

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

		n	2	3	4
mixing angles:		$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:					
• ν_j- Dirac:		$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:		$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980;

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.65 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.039$ (0.053) 2σ (3σ).

T. Schwetz *et al.*, arXiv:0808.2016

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987
  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};

    S.M. Bilenky, S. Pascoli, S.T.P.,2000;
    S. Pascoli, S.T.P., L. Wolfenstein, 2002
    S. Pascoli, S.T.P., 2002
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

    S.T.P., T. Shindou, Y. Takanishi, 2007
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

    S. Pascoli, S.T.P., A. Riotto, 2006;
    E. Molinaro, S.T.P., 2008

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$

$$\mu^- + \ (\mathrm{A,Z}) \rightarrow \mu^+ + \ (\mathrm{A,Z-2})$$

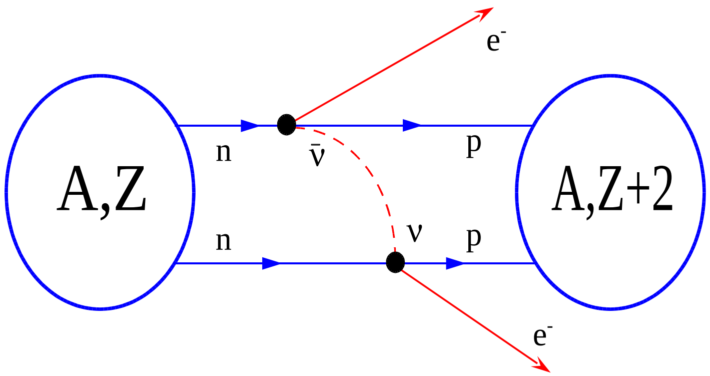
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\mathrm{A,Z}) \rightarrow (\mathrm{A,Z+2}) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

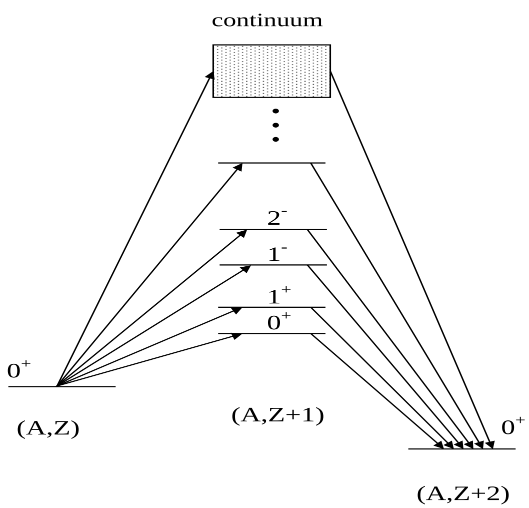
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \rightarrow uue^- e^- (\overline{\nu}_e \overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$\left|<m>\right| \cong \left|\sqrt{\Delta m^2_{\odot}}\,\sin^2\theta_{12} e^{i\alpha_{21}} + \sqrt{\Delta m^2_{31}}\,\sin^2\theta_{13} e^{i\alpha_{31}}\right|\ ,\ m_1 \ll m_2 \ll m_3\ \text{(NH)},$$

$$\left|<m>\right| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha_{21}}\ \sin^2\theta_{12}\right|\ ,\ m_3 < (\ll) m_1 < m_2\ \text{(IH)},$$

$$\left|<m>\right| \cong m\left|\cos^2\theta_{12} + e^{i\alpha_{21}}\ \sin^2\theta_{12}\right|\ ,\ m_{1,2,3} \cong m \gtrsim 0.10\ \text{eV}\ \ \text{(QD)},$$

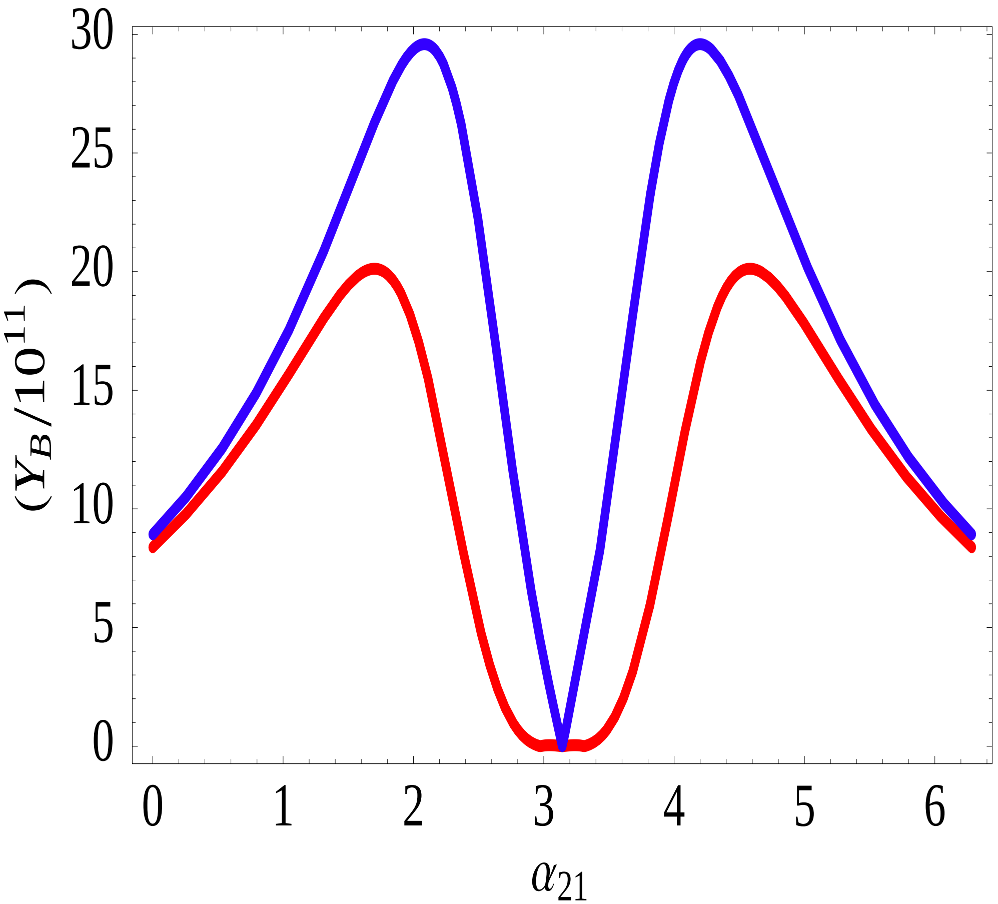
CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi$;

$$\left|<m>\right| \lesssim 5 \times 10^{-3}\ \text{eV},\ \text{NH};$$

$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013\ \text{eV} \lesssim \left|<m>\right| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055\ \text{eV},\quad \text{IH};$$

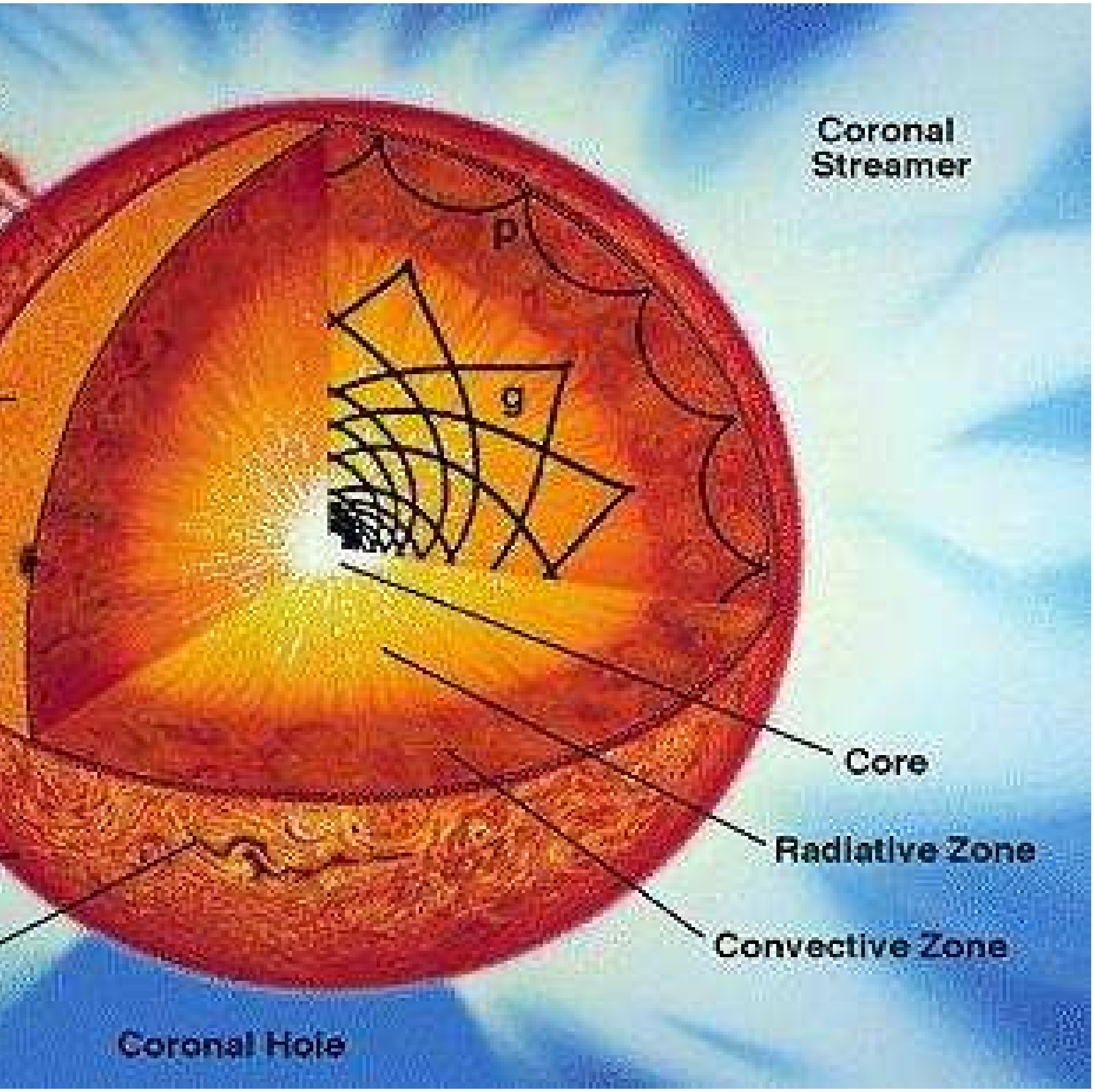
$$m\cos 2\theta_{12} \lesssim \left|<m>\right| \lesssim m,\ m \gtrsim 0.10\ \text{eV},\ \ \text{QD}\ .$$

S. Pascoli, S.T.P., 2002



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



Coronal
Streamer
p
g
Core
Radiative Zone
Convective Zone
Coronal Hole

MSW Transitions of Solar Neutrinos in the Sun (and the Hydrogen Atom)

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \quad (1)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) = N_e(t_0)\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1 R_\odot$, $R_\odot = 6.96 \times 10^5$km

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t,t_0) = (Z/Z_0)^{c-a}\ e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'}\ A'_e(t,t_0),$$

$A'_e(t,t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{\ Z\frac{d^2}{dZ^2} + (c - Z)\ \frac{d}{dZ} - a\right\} A'_e(t,t_0) = 0,$$

where

$$a = 1 + ir_0\ \frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\ \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$\Psi(\overrightarrow{r}) = \frac{1}{r}\psi_{kl}(r) Y_{lm}(\theta', \phi')$,

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$\psi'_{kl}(Z) = Z^{-c/2}\ e^{Z/2}\ \psi_{kl}(r)$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2\ \frac{r}{a_0}\sqrt{-E_{kl}/E_I},\ \ a \equiv a_{kl} = l + 1 - \sqrt{-E_I/E_{kl}},\ \ c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\ \Phi(a-c+1, 2-c; Z)$; $\Phi(a',c'; Z=0) = 1$, $a', c' \neq 0, -1, -2, \ldots$.

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2}\ \sin 2\theta\ \left\{ \Phi(a-c, 2-c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}}\ \Phi(a-1, c; Z_0)\ \right\}.$$

Sun: $N_e(x) \cong N_e(x_0) e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1 R_\odot$, $R_\odot \cong 7 \times 10^5$ km

The region of $\nu_\odot$ production:

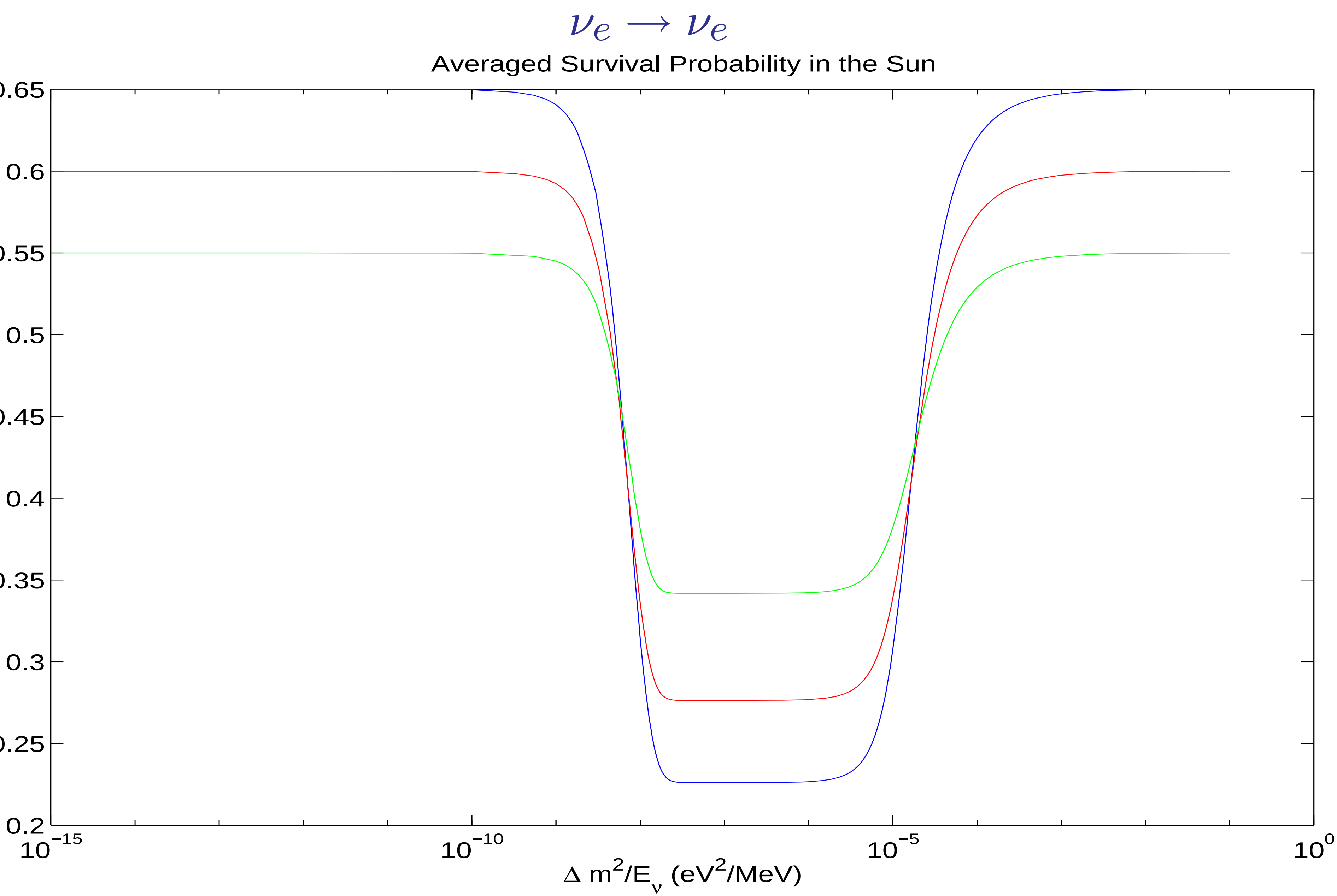
$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$: $|Z_0| > 500$ (!)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

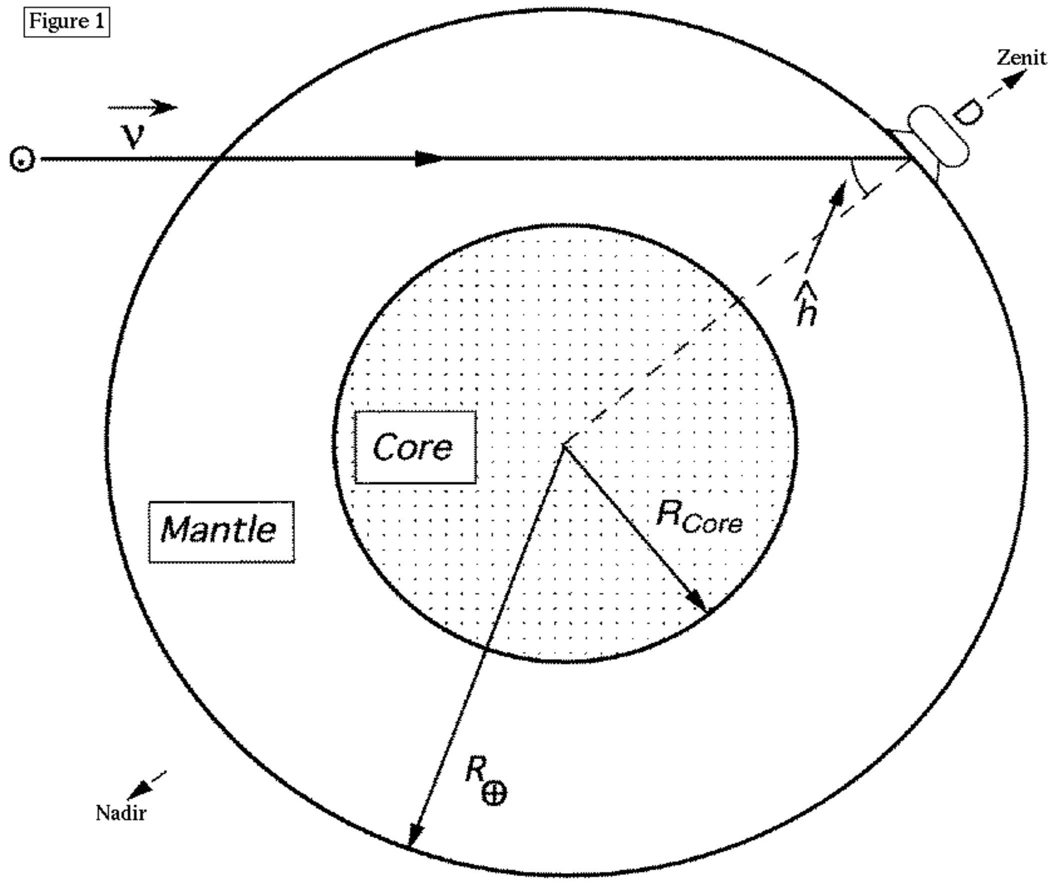
$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988



$\nu_e \longrightarrow \nu_e$
Averaged Survival Probability in the Sun
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
10^{-15}
10^{-10}
10^{-5}
10^{0}
$\Delta m^2/E_\nu$ (eV^2/MeV)

The Earth



Earth: $R_{core} = 3446$ km, $R_{mant} = 2885$ km

Earth: $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

The Earth

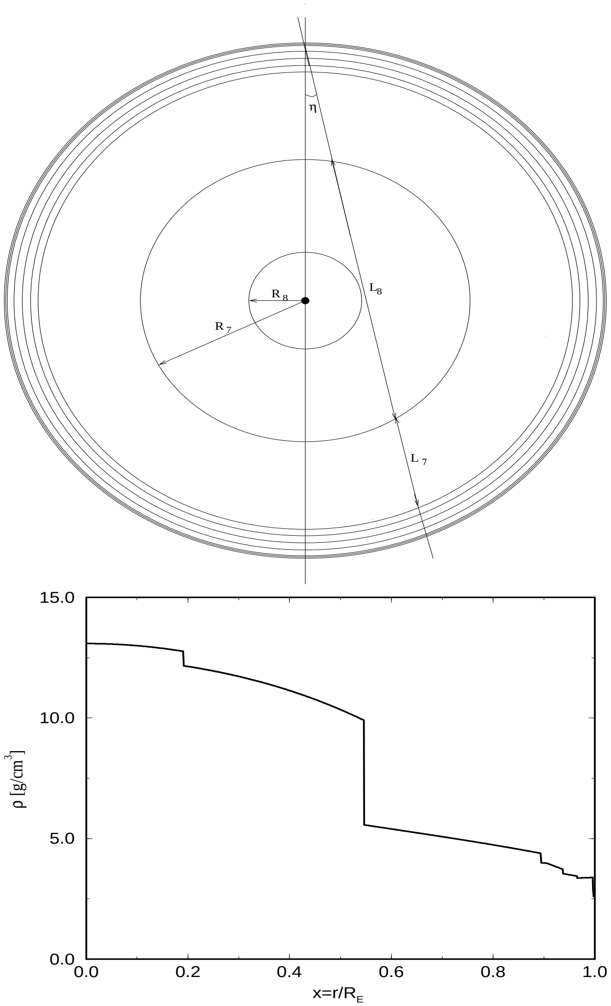
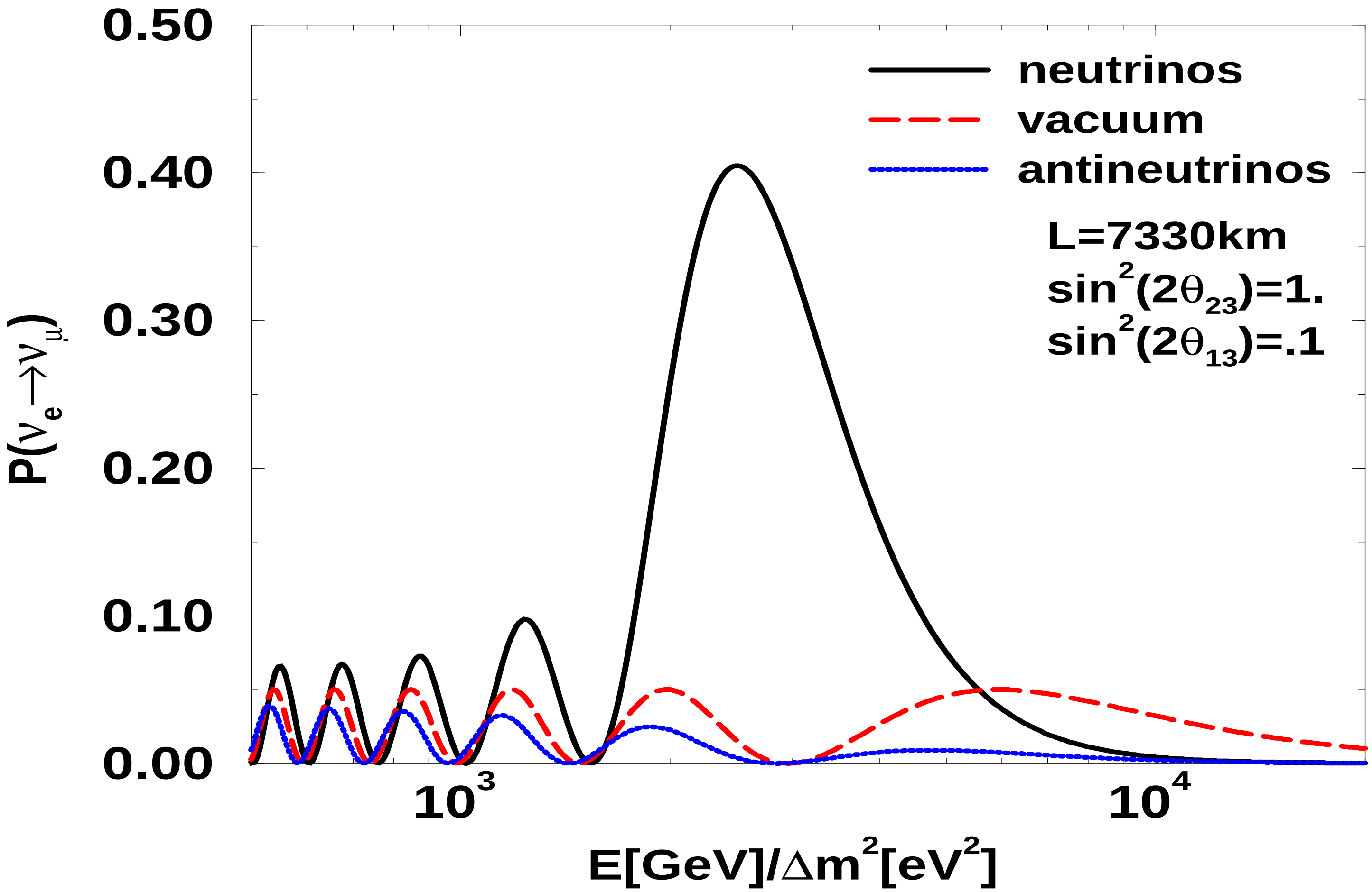


FIG. 1. Density profile of the Earth.

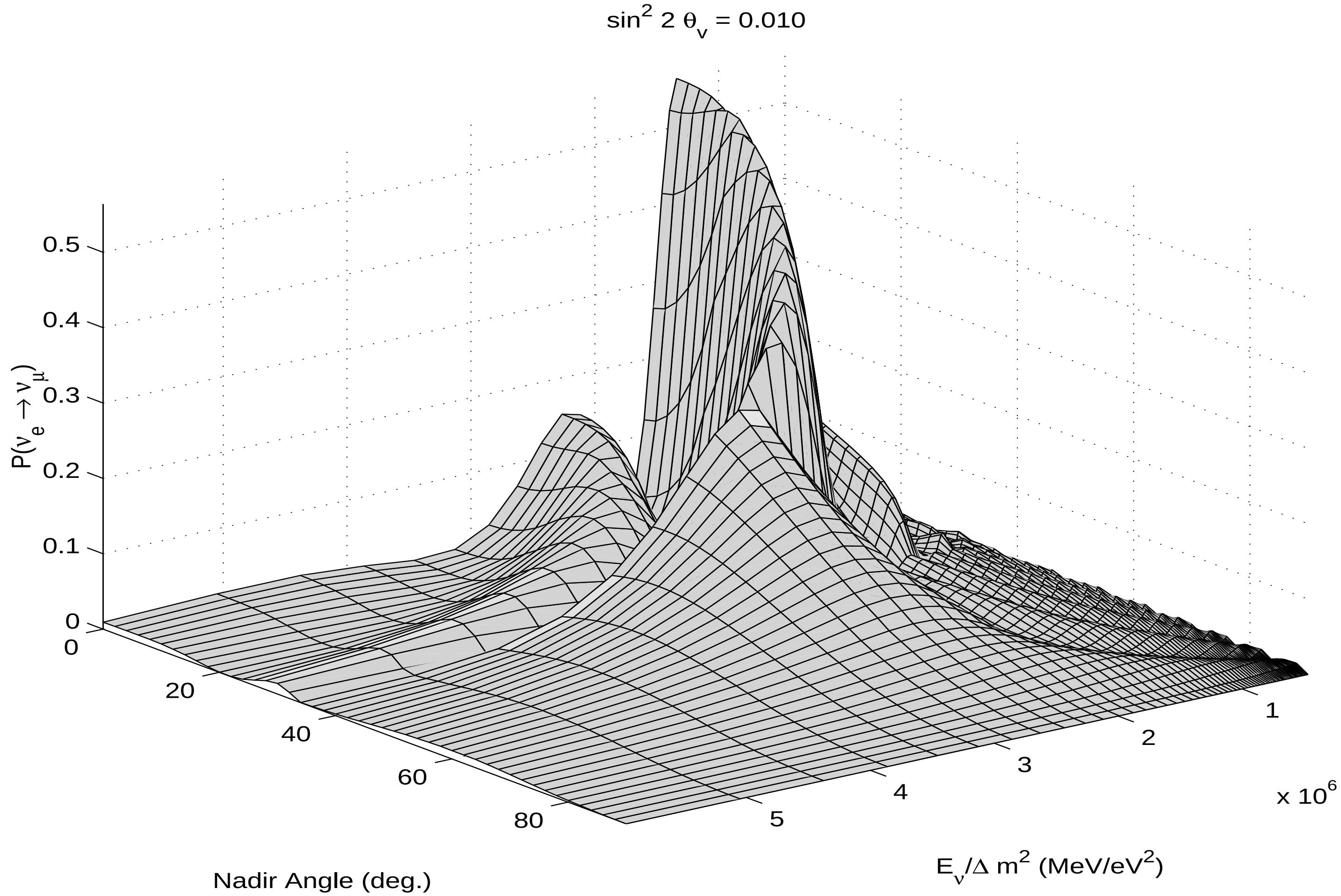
$R_c = 3446$ km, $R_m = 2885$ km; $\bar{N}_e^{mant} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 5.7\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



I. Mocioiu, R. Shrock, 2000

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)

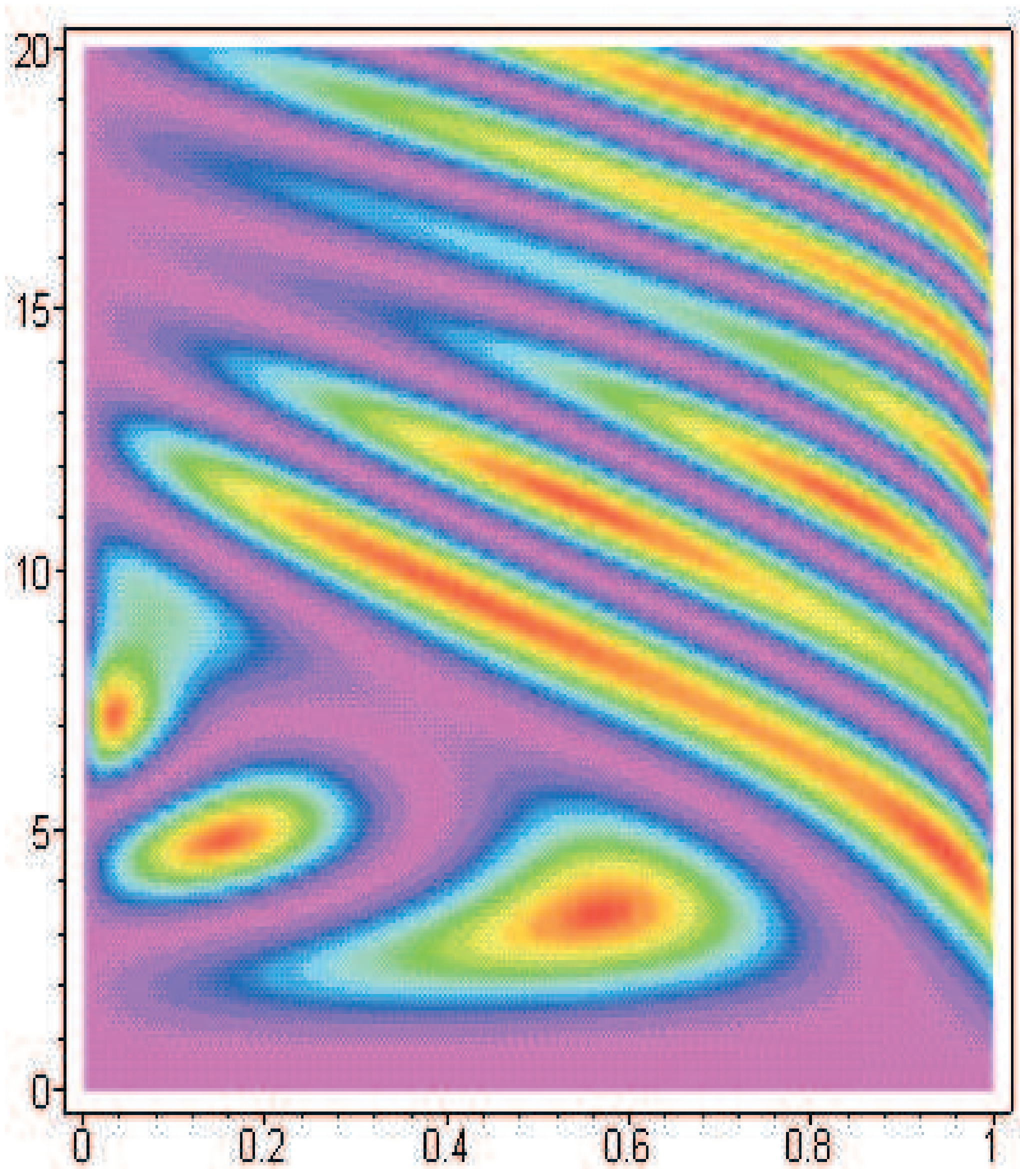


S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_{\rm v} \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{\rm atm}$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2}P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E$ $[10^{-7}eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

• **For Earth center crossing ν's ($\theta_n = 0$) and, e.g.** $\sin^2 2\theta_{13} = 0.01$, **NOLR occurs at $E \cong 4$ GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

S.T.P., hep-ph/9805262

• **For the Earth core crossing ν's:** $P_{2\nu} = 1$ **due to NOLR when**

$$\tan \Phi^{\rm man}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}}\,,$$

$$\tan \Phi^{\rm core}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta'_m}{-\cos(2\theta''_m)\cos(2\theta''_m - 4\theta'_m)}}$$

$\Phi^{\rm man}$ ($\Phi^{\rm core}$) - **phase accumulated in the Earth mantle (core),**
θ'_m (θ''_m) - **the mixing angle in the Earth mantle (core).**

$P_{2\nu} = 1$ **due to NOLR for $\theta_n = 0$ (Earth center crossing ν's) at,**
e.g. $\sin^2 2\theta_{13} = 0.034;\ 0.154$, $E \cong 3.5;\ 5.2$ **GeV ($\Delta m^2(atm) = 2.5 \times 10^{-3}$ eV^2).**

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

$\nu_\odot$, $\Delta m^2_{\rm atm}$, CHOOZ Data:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{6}\ (\frac{\pi}{5.4})$, $\quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}$, $\quad \theta_{13} < \frac{\pi}{12}$

$$U_{\rm PMNS} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1 + \lambda)$, $\quad \sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1 - \lambda)$,

- $\lambda \cong (0.20 - 0.25)$: $\quad \theta_\odot + \theta_c = \pi/4$?

Natural Possibility:

$$U = U_{\text{lep}}^{\dagger}(\lambda)\ U_{\text{bim}}$$

with

$$U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- $U_{\text{lep}}^{\dagger}(\lambda)$ - from diagonalization of the l^- mass matrix,

- $U_{\text{bim(tri)}}$ - from diagonalization of the $\nu-$mass matrix

Further, $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$.

- U_{bim} can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

This symmetry cannot be exact.

For $\sin^{\ell}\theta_{ij} \equiv \lambda_{ij}$ "small", $\lambda_{12} \gg \lambda_{13}$ (natural),

$\sin^2\theta_{12} = \frac{1}{2} - \sin\theta_{13}\cos\delta$, U_{bim} ,

δ is the Dirac CPV phase,

P. Frampton, S.T.P., W. Rodejohann, 2004

Can be tested experimentally.

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\tan\theta_{23} = M_{e\tau}/M_{e\mu}$,

$m_3 = 0$ - spectrum with IH, $m_1 = m_2$, $\chi_{1,2}$ - equivalent to one Dirac ν, Ψ .

Adding L'-breaking term, e.g. M_{ee}, $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, leads to $m_1 \neq m_2$ compatible with $\Delta m_\odot^2$.

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}^{\nu}_{M}(x) = -\,\frac{1}{2}\,\overline{\nu^c_{l'R}}(x)\ M_{l'l}\ \nu_{lL}(x) + h.c.\ ,\ \ \nu^c_{l'R} \equiv C\ \left(\overline{\nu_{l'L}}(x)\right)^{\mathsf{T}}$$

$\mathcal{L}^{\nu}_{M}(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- Dirac ν, Ψ, is equivalent to two Majorana ν's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are "maximally mixed":

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,,\quad m_1 = m_2 = m_D > 0\,, \eta_{jCP} = i\rho_j\,, \rho_1 = -\rho_2\ \ (C\ (\overline{\chi_j})^{\mathsf{T}} = \rho_j \chi_j)$$

Example ZKM ν : $\nu_{eL}(x) = \Psi_L = \dfrac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}\,,\ \nu_{\mu L}(x) = \Psi^C_L = \dfrac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$

- Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}^{\nu}_{M}(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to "leading order" $m_1 = m_2$, but due to "higher order" corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- Suppose: $m_1 = m_2$, $\rho_1 = -\rho_2$, but $\chi_{1,2}$ are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L} \cos\phi + \chi_{2L} \sin\phi = \Psi_L \cos\phi' + \Psi^C_L \sin\phi'$$

All Majorana effects are $\sim m_D \cos\phi' \sin\phi'$

Pontecorvo, 1957, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}\,, \quad m_1 \neq m_2 > 0\,, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - Majorana, maximal mixing .

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos\theta_C + \Psi_{2L} \sin\theta_C\,,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin\theta_C + \Psi_{2L} \cos\theta_C\,,$$

$\Psi_{1,2}$ - Dirac (composite), θ_C- the Cabbibo angle .

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

Instead of Conclusions

We are at the beginning of the Road...

Still a lot of work to be done...