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FUNCTIONAL INTEGRAL APPROACH IN THE THEORY OF COLOR SUPERCONDUCTIVITY*

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*Lectures at the VI International School in Theoretical Physics, Vung Tau, 27 December 1999 – 8 January 2000.

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The functional integral method for studying the superconducting pairing of quarks with the formation of the diquarks as well as the quark-antiquark pairing in dense QCD is presented. The dynamical equations for the superconducting order parameters are the nonlinear integral equations for the composite quantum fields describing the quark-quark or quark-antiquark systems. These composite fields are the bilocal fields if the pairing is generated by the gluon exchange, while for the instanton induced pairing interactions they are the local ones. The expressions of the free energy densities are derived. The binding of three quarks is also discussed.

Представлен метод функционального интегрирования в рамках КХД для изучения явления сверхпроводящего спаривания кварков в плотной среде с образованием дикварков и кварк-антикварковых пар. Динамическими уравнениями для параметров порядка сверхпроводимости являются нелинейные интегральные уравнения для составных квантовых полей, описывающих кварк-кварковые или кварк-антикварковые системы. Данные составные поля являются билокальными в случае, если спаривание генерируется глюонным обменом, и локальными, если парное взаимодействие вызывается инстантоном. Получены выражения для плотностей свободной энергии. Также обсуждается система трех кварков.

INTRODUCTION

The superconducting pairing of quarks due to the gluon exchange in QCD with the formation of the diquark condensate was proposed by Barrois [1] and Frautschi [2] since more than two decades and then studied by Bailin and Love [3], Donoglue and Sateesh [4], Iwasaki and Iwado [5]. Recently, in a series of papers [6–21] there was aroused a new interest to the existence of the diquark Bose condensate in the QCD dense matter — the color superconductivity. The connection between the color superconductivity and the chiral phase transition in QCD was studied by Berges and Rajagopal [22], Harada and Shibata [23]. There exists also the spontaneous parity violation, as it was shown by Pisarski and Rischke [24].

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For the study of many-body systems of relativistic particles with the internal degrees of freedom as well as with the virtual creation and annihilation of the particle-antiparticle pairs the functional integral technique is a powerful mathematical tool. This method was applied to the study of the color superconductivity as well as the quark-antiquark pairing [25–28].

In this series of lectures we present the basics of the functional integral method for the study of the color superconductivity in QCD at finite temperature and density (the so-called dense QCD or thermal QCD). As the physical origin of the superconducting pairing of quarks we consider two different commonly discussed mechanisms: the direct local four-fermion interactions of the quark field and the quark-quark nonlocal interaction due to the gluon exchange. The direct four-quark interaction might be induced by the instantons [11, 12].

We follow the general method of the functional integral approach in the theory of superconductivity [29, 30]. We start from the expressions of the partition functions of the systems of interacting quarks and antiquarks with some local or nonlocal quark-quark interactions. Then we introduce the local or bilocal composite fields describing the diquarks or quark-antiquark pairs, establish the effective action of these composite fields and derive their field equations. The order parameters of the ground state of the diquark or quark-antiquark condensate are the expectation values of the composite fields in the corresponding state of the system. Due to the translational invariance of the ground state these expectation values are coordinate-independent (for the local fields) or depend only on the difference of the coordinates (for the bilocal ones). The equations for the order parameters are the special cases of the field equations. The expressions of the free energy density in the corresponding phases are also derived.

We work in the imaginary time formalism and write briefly

$$x = (\mathbf{x}, \tau)\,, \qquad \int dx = \int\limits_0^\beta d\tau \int d\mathbf{x}, \qquad \beta = \frac{1}{kT},$$

k is the Boltzmann constant and T is the temperature. We denote ψ_A, $\overline{\psi}^A$ the quark field and its Dirac conjugate, where $A = (\alpha, a, i)$ is the set consisting of the Dirac spinor index $\alpha = 1, 2, 3, 4$, the flavor index $i = 1, 2, 3, ..., N_f$ and the color one $a = 1, 2, 3, ..., N_c$. The internal symmetry groups are assumed to be $SU\left(N_f\right)_f$ and $SU\left(N_c\right)_c$. The partition function of the system of free quarks and antiquarks with the chemical potential μ and at the temperature T can be expressed in the form of the functional integral

$$Z_0 = \int \left[D\psi\right]\left[D\overline{\psi}\right] \exp\left\{-\int dx \overline{\psi}^A\left(x\right) L_A^B \psi_B\left(x\right)\right\}, \tag{1}$$

where

$$L_A^B = \delta_a^b \delta_j^j \left[\gamma_4 \left(\frac{\partial}{\partial \tau} - \mu \right) + \boldsymbol{\gamma} \boldsymbol{\nabla} + M \right]_\alpha^\beta , \tag{2}$$

and M is the bare quark mass.

Introduce the generating functional

$$\begin{aligned} Z_0 [\eta, \overline{\eta}] &= \int [D\psi] \left[D\overline{\psi} \right] \exp \left\{ - \int dx \overline{\psi}^A (x) L_A^B \psi_B (x) \right\} \times \\ &\times \exp \left\{ - \int dx \left[\overline{\eta}^A (x) \psi_A (x) + \overline{\psi}^A (x) \eta_A (x) \right] \right\} \end{aligned} \tag{3}$$

with anticommuting parameters $\eta_A (x)$ and $\overline{\eta}^A (x)$. The $2n$-point Green functions are expressed in terms of the functional derivatives of $Z_0 [\eta, \overline{\eta}]$ at the special value $\eta_A (x) = \overline{\eta}^A (x) = 0$:

$$\begin{aligned} & G_{A_1,...,A_n}^{B_1,...,B_n} (x_1, x_2, ..., x_n; y_1, y_2, ..., y_n) = \\ = & \left\langle T \left\{ \psi_{A_1} (x_1) \psi_{A_2} (x_2) ... \psi_{A_n} (x_n) \overline{\psi}^{B_n} (y_n) ... \overline{\psi}^{B_2} (y_2) \overline{\psi}^{B_1} (y_1) \right\} \right\rangle = \\ = & (-1)^n \frac{1}{Z_0} \left. \frac{\delta^{2n} Z_0 [\eta, \overline{\eta}]}{\delta \overline{\eta}^{A_1} (x_1) ... \delta \overline{\eta}^{A_n} (x_n) \delta \eta_{B_n} (y_n) ... \delta \eta_{B_1} (y_1)} \right|_{\eta = \overline{\eta} = 0} . \end{aligned} \tag{4}$$

In particular

$$\begin{aligned} G_A^B (x; y) &= G_A^B (x - y) = \left\langle T \left\{ \psi_A (x) \overline{\psi}^B (y) \right\} \right\rangle = \\ &= - \frac{1}{Z_0} \left. \frac{\delta^2 Z_0 [\eta, \overline{\eta}]}{\delta \overline{\eta}^A (x) \delta \eta_B (y)} \right|_{\eta = \overline{\eta} = 0} . \end{aligned} \tag{5}$$

Denote

$$\begin{gathered} S_A^B (x - y) = \delta_a^b \delta_i^j S_\alpha^\beta (x - y) = \delta_a^b \delta_i^j S_\alpha^\beta (\mathbf{x} - \mathbf{y}, \tau - \sigma) , \\ S_\alpha^\beta (\mathbf{x} - \mathbf{y}, \tau - \sigma) = S_{\alpha\beta} (\mathbf{x} - \mathbf{y}, \tau - \sigma) \end{gathered} \tag{6}$$

the solution of the equation

$$\begin{gathered} L_A^B S_B^C (x - y) = \delta_A^C \delta (x - y) , \\ x = (\mathbf{x}, \tau) , \quad y = (\mathbf{y}, \ \sigma) , \quad \delta (x - y) = \delta (\mathbf{x} - \mathbf{y}) \delta (\tau - \sigma) , \end{gathered} \tag{7}$$

$$\begin{aligned} & \left[\gamma_4 \left(\frac{\partial}{\partial \tau} - \mu \right) + \boldsymbol{\gamma} \boldsymbol{\nabla} + M \right]_{\alpha\beta} S_{\beta\gamma} (\mathbf{x} - \mathbf{y}, \tau - \sigma) = \\ & \qquad = \delta_{\alpha\gamma} \delta (\mathbf{x} - \mathbf{y}) \delta (\tau - \sigma) . \end{aligned} \tag{8}$$

Shifting the functional integration variables

$$\psi_B(x) \rightarrow \psi_B(x) + \int dy S_B^D(x-y)\,\eta_D(y),$$
$$\overline{\psi}^A(x) \rightarrow \overline{\psi}^A(x) + \int dz \overline{\eta}^C(z)\,S_C^A(z-x)$$

in the r.h.s. of the formula (1), we derive the explicit expression of the generating functional (3)

$$Z_0[\eta,\overline{\eta}] = Z_0 \exp\left\{\int dx \int dy \overline{\eta}^A(x)\,S_A^B(x-y)\,\eta_B(y)\right\}. \tag{9}$$

Substituting this expression into the r.h.s. of the formula (5), we obtain the two-point Green function

$$G_A^B(x-y) = \left\langle T\left\{\psi_A(x)\,\overline{\psi}^B(y)\right\}\right\rangle = S_A^B(x-y). \tag{10}$$

Similarly, from the formulae (4) and (9) it follows the Wick theorem for the $2n$-point Green functions of the free fermionic fields.

Introduce the Fourier transform $\widetilde{S}_{\alpha\beta}(p) = \widetilde{S}_{\alpha\beta}(\mathbf{p},\varepsilon_m)$ of $S_{\alpha\beta}(x) = S_{\alpha\beta}(\mathbf{x},\tau)$:

$$S_{\alpha\beta}(\mathbf{x},\tau) = \frac{1}{\beta}\sum_m \mathrm{e}^{i\varepsilon_m t}\frac{1}{(2\pi)^3}\int d\mathbf{p}\;\mathrm{e}^{i\mathbf{px}}\;\widetilde{S}_{\alpha\beta}(\mathbf{p},\varepsilon_m),$$
$$\varepsilon_m = (2m+1)\,\frac{\pi}{2}, \tag{11}$$

m being the integers $m = 0, \pm 1, \pm 2, \ldots$ From Eq. (8) it follows that

$$\widetilde{S}_{\alpha\beta}(p) = \left(\frac{1}{i\widehat{p}+M}\right)_{\alpha\beta} = \frac{(-i\widehat{p}+M)_{\alpha\beta}}{p^2+M} \tag{12}$$

with the notations

$$\widehat{p} = (\varepsilon_m + i\mu)\,\gamma_4 + \boldsymbol{\gamma}\mathbf{p}, \qquad p^2 = (\varepsilon_m + i\mu)^2 + \mathbf{p}^2. \tag{13}$$

In the calculations we shall use also the expression $\widetilde{S}_{\alpha\beta}(-p) = \widetilde{S}_{\alpha\beta}(-\mathbf{p}, -\varepsilon_m)$. For the convenience we write it in the form

$$\widetilde{S}_{\alpha\beta}(-p) = \left(\frac{1}{-i\widehat{p}'+M}\right)_{\alpha\beta} = \frac{(i\widehat{p}'+M)_{\alpha\beta}}{p^2+M}, \tag{14}$$

where

$$\widehat{p}' = (\varepsilon_m - i\mu)\,\gamma_4 + \gamma\mathbf{p}, \qquad p'^{\,2} = (\varepsilon_m - i\mu)^2 + \mathbf{p}^2. \tag{15}$$

To study the color superconductivity we consider the quark-quark pairing in QCD with the formation of the diquark condensate. Then we investigate the quark-antiquark pairing. The corresponding composite fields are the meson ones. We discuss also the possibility of extending our reasonings to the study of the binding of three quarks. The composite particles in this case are the baryons.

1. QUARK-QUARK PAIRING

For the simplicity in writing the formulae we begin our study by considering the superconducting quark-quark pairing due to some direct four-fermion coupling of quarks with the interaction Lagrangian

$$L_{\text{int}} = \frac{1}{2}\overline{\psi}^{A}(x)\overline{\psi}^{C}(x)V_{CA}^{BD}\psi_D(x)\psi_B(x), \qquad V_{AC}^{BD} = V_{CA}^{DB} = -V_{CA}^{BD} = -V_{AC}^{DB}. \tag{16}$$

The partition function of the system equals

$$\begin{aligned} Z &= \int [D\psi][D\overline{\psi}] \exp\left\{-\int dx\overline{\psi}^{A}(x)L_A^B\psi_B(x)\right\} \times \\ &\times \exp\left\{\frac{1}{2}\int dx\overline{\psi}^{A}(x)\overline{\psi}^{C}(x)V_{CA}^{BD}\psi_D(x)\psi_B(x)\right\}. \end{aligned} \tag{17}$$

Introduce the antisymmetric bispinor local fields $\Phi_{CA}(x)$, $\overline{\Phi}^{AC}(x)$

$$\overline{\Phi}^{CA}(x) = -\overline{\Phi}^{AC}(x), \quad \Phi_{AC}(x) = -\Phi_{CA}(x), \tag{18}$$

and the functional integral

$$Z_0^{\Phi} = \int [D\Phi]\left[D\overline{\Phi}\right] \exp\left\{-\frac{1}{2}\int dx\overline{\Phi}^{AC}(x)\, V_{CA}^{BD}(x-y)\,\Phi_{DB}(x)\right\}. \tag{19}$$

By shifting the functional integration variables

$$\begin{aligned} \Phi_{DB}(x) &\to \Phi_{DB}(x) + \psi_D(x)\,\psi_B(x), \\ \overline{\Phi}^{AC}(x) &\to \overline{\Phi}^{AC}(x) + \overline{\psi}^{A}(x)\,\overline{\psi}^{C}(x), \end{aligned}$$

we establish the Hubbard–Stratonovich transformation

$$\begin{aligned} &\exp\left\{\frac{1}{2}\int dx\overline{\psi}^{A}(x)\,\overline{\psi}^{C}(x)\,V_{CA}^{BD}\psi_D(x)\,\psi_B(x)\right\} = \\ &= \frac{1}{Z_0^{\Phi}}\int [D\Phi]\left[D\overline{\Phi}\right]\exp\left\{-\frac{1}{2}\int dx\overline{\Phi}^{AC}(x)\,V_{CA}^{BD}\Phi_{DB}(x)\right\} \times \\ &\times \exp\left\{-\frac{1}{2}\int dx\left[\overline{\Delta}^{BD}(x)\,\psi_D(x)\,\psi_B(x) + \overline{\psi}^{A}(x)\,\overline{\psi}^{C}(x)\,\Delta_{CA}(x)\right]\right\}, \end{aligned} \tag{20}$$

where

$$\overline{\Delta}^{BD}(x) = \overline{\Phi}^{AC}(x)\,V_{CA}^{BD}, \qquad \Delta_{CA}(x) = V_{CA}^{BD}\Phi_{DB}(x), \tag{21}$$

$$\overline{\Delta}^{DB}(x) = -\overline{\Delta}^{BD}(x), \qquad \Delta_{AC}(x) = -\Delta_{CA}(x), \tag{22}$$

and rewrite the partition function (17) in the form

$$Z = \frac{1}{Z_0^{\Phi}} \int [D\Phi] \left[D\overline{\Phi}\right] \exp\left\{S_{\text{eff}}\left[\Phi, \overline{\Phi}\right]\right\} \tag{23}$$

with the effective action

$$S_{\text{eff}}\left[\Phi, \overline{\Phi}\right] = -\frac{1}{2} \int dx \overline{\Phi}^{BD}(x) V_{DB}^{AC} \Phi_{CA}(x) + W\left[\Delta, \overline{\Delta}\right], \tag{24}$$

$$\begin{aligned} \exp\left\{W\left[\Delta, \overline{\Delta}\right]\right\} = 1 + \sum_{n=1}^{\infty} \Gamma^{(2n)}\left[\Delta, \overline{\Delta}\right] = \\ = \left\langle \mathbf{T}\left[\exp\left\{-\frac{1}{2}\int dx \left[\overline{\psi}^{A}(x)\,\overline{\psi}^{C}(x) \times \right.\right.\right.\right. \\ \left.\left.\left.\left. \times \Delta_{CA}(x) + \overline{\Delta}^{BD}(x)\,\psi_D(x)\,\psi_B(x)\right]\right\}\right]\right\rangle. \end{aligned} \tag{25}$$

Calculations give

$$\begin{aligned} \Gamma^{(2)}\left[\Delta, \overline{\Delta}\right] = W^{(2)}\left[\Delta, \overline{\Delta}\right] = \frac{1}{2}\int dx_1 \int dx_2 \overline{\Delta}^{A_1C_1}(x_1)\, S_{C_1}^{C_2}(x_1 - x_2) \times \\ \times \Delta_{C_2A_2}(x_2)\, S_{A_1}^{\mathrm{T}A_2}(x_2 - x_1), \end{aligned} \tag{26}$$

$$\Gamma^{(4)}\left[\Delta, \overline{\Delta}\right] = \frac{1}{2!}\left(W^{(2)}\left[\Delta, \overline{\Delta}\right]\right)^2 + W^{(4)}\left[\Delta, \overline{\Delta}\right], \tag{27}$$

$$\begin{aligned} W^{(4)}\left[\Delta, \overline{\Delta}\right] = -\frac{1}{4}\int dx_1 ... \int dx_4 \overline{\Delta}^{A_1C_1}(x_1)\, S_{C_1}^{C_2}(x_1 - x_2)\, \Delta_{C_2A_2}(x_2) \times \\ \times S_{A_3}^{\mathrm{T}A_2}(x_2 - x_3)\, \overline{\Delta}^{A_3C_3}(x_3)\, S_{C_3}^{C_4}(x_3 - x_4)\, \Delta_{C_4A_4}(x_4)\, S_{A_1}^{\mathrm{T}A_4}(x_4 - x_1), \end{aligned} \tag{28}$$

$$\begin{aligned} \Gamma^{(6)}\left[\Delta, \overline{\Delta}\right] = \frac{1}{3!}\left(W^{(2)}\left[\Delta, \overline{\Delta}\right]\right)^3 + \\ + W^{(2)}\left[\Delta, \overline{\Delta}\right] W^{(4)}\left[\Delta, \overline{\Delta}\right] + W^{(6)}\left[\Delta, \overline{\Delta}\right], \end{aligned} \tag{29}$$

$$\begin{aligned} W^{(6)}\left[\Delta, \overline{\Delta}\right] = \frac{1}{6}\int dx_1 ... \int dx_6 \overline{\Delta}^{A_1C_1}(x_1)\, S_{C_1}^{C_2}(x_1 - x_2)\, \Delta_{C_2A_2}(x_2) \times \\ \times S_{A_3}^{\mathrm{T}A_2}(x_2 - x_3) ... \overline{\Delta}^{A_5C_5}(x_5)\, S_{C_5}^{C_6}(x_5 - x_6)\, \Delta_{C_6A_6}(x_6)\, S_{A_1}^{\mathrm{T}A_6}(x_6 - x_1), \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{30}$$

with

$$S_B^{\mathrm{T}A}(x_1 - x_2) = S_B^A(x_2 - x_1).$$

We have then

$$W\left[\Delta, \overline{\Delta}\right] = \sum_{n=1}^{\infty} W^{(2n)}\left[\Delta, \overline{\Delta}\right]. \tag{31}$$

From the variational principle for the effective action we derive the field equation

$$\frac{1}{2}\Delta_{CA}(x) = V_{CA}^{BD} \sum_{n=1}^{\infty} \frac{\delta W^{(2n)}\left[\Delta, \overline{\Delta}\right]}{\delta \overline{\Delta}^{BD}(x)}. \tag{32}$$

Using the explicit expressions of $W^{(2n)}\left[\Delta, \overline{\Delta}\right]$, we obtain

$$\begin{aligned}
\Delta_{CA}(x) = V_{CA}^{BD}\Big\{ &\int dx_2 S_D^{C_2}(x - x_2)\,\Delta_{C_2A_2}(x_2)\,S_B^{\mathrm{T}A_2}(x_2 - x) - \\
&- \int dx_2 ... \int dx_4 S_D^{C_2}(x - x_2)\,\Delta_{C_2A_2}(x_2)\,S_{A_3}^{\mathrm{T}A_2}(x_2 - x_3) \times \\
&\times \overline{\Delta}^{A_3C_3}(x_3)\,S_{C_3}^{C_4}(x_3 - x_4)\,\Delta_{C_4A_4}(x_4)\,S_B^{\mathrm{T}A_4}(x_4 - x) + \\
&+ \int dx_2 ... \int dx_6 S_D^{C_2}(x - x_2)\,\Delta_{C_2A_2}(x_2)\,S_{A_3}^{\mathrm{T}A_2}(x_2 - x_3) \times \\
&\times \overline{\Delta}^{A_3C_3}(x_3)\,S_{C_3}^{C_4}(x_3 - x_4) ... \Delta_{C_6A_6}(x_6)\,S_B^{\mathrm{T}A_6}(x_6 - x) - ... \Big\}.
\end{aligned} \tag{33}$$

In the special class of the constant solutions

$$\Delta_{CA}(x) = \Delta_{CA} = V_{CA}^{BD}\Phi_{DB}, \qquad \overline{\Delta}^{BD}(x) = \overline{\Delta}^{BD} = \overline{\Phi}^{AC} V_{CA}^{BD}, \tag{34}$$

we have the extended BCS gap equation

$$\begin{aligned}
\Delta_{CA} = V_{CA}^{BD} \frac{1}{\beta} \sum_m \frac{1}{(2\pi)^3} \int d\mathbf{p} \times \\
\times \left[\widetilde{S}(\mathbf{p}, \varepsilon_m)\,\Delta \widetilde{S}^{\mathrm{T}}(-\mathbf{p}, -\varepsilon_m) \frac{1}{1 + \overline{\Delta}\,\widetilde{S}(\mathbf{p}, \varepsilon_m)\,\Delta\,\widetilde{S}^{\mathrm{T}}(-\mathbf{p}, -\varepsilon_m)} \right]_{DB},
\end{aligned} \tag{35}$$

where $\widetilde{S}(\mathbf{p}, \varepsilon_m)$, $\widetilde{S}^{\mathrm{T}}(-\mathbf{p}, -\varepsilon_m)$, Δ and $\overline{\Delta}$ are the matrices with the elements $\widetilde{S}_A^B(\mathbf{p}, \varepsilon_m)$, $\widetilde{S}_A^{\mathrm{T}B}(-\mathbf{p}, -\varepsilon_m)$, Δ_{CA}, and $\overline{\Delta}^{AC}$.

At the values of the superconducting order parameters Δ_{CA} and $\overline{\Delta}^{BD}$ satisfying the extended BCS gap equation the effective action equals

$$S_{\text{eff}}\left[\Phi,\overline{\Phi}\right]=\left(\frac{1}{2}-\frac{1}{4}\right)\int dx_1 ... \int dx_4 \,\mathrm{Tr}\left[\overline{\Delta} S\left(x_1-x_2\right)\Delta S^{\mathrm{T}}\left(x_2-x_3\right)\times\right.$$
$$\left.\times\,\overline{\Delta}\,S\left(x_3-x_4\right)\Delta\,S^{\mathrm{T}}\left(x_4-x_1\right)\right]-$$
$$-\left(\frac{1}{2}-\frac{1}{6}\right)\int dx_1 ... \int dx_6 \,\mathrm{Tr}\left[\overline{\Delta}\,S\left(x_1-x_2\right)\Delta\,S^{\mathrm{T}}\left(x_2-x_3\right)\times\right.$$
$$\left.\times\, ...\,\overline{\Delta}\,S\left(x_5-x_6\right)\Delta\,S^{\mathrm{T}}\left(x_6-x_1\right)\right]+ ... . \quad (36)$$

Denote $F\left[\mathbf{x};\Delta\right]$ the free energy density of the condensate. The effective action is expressed in terms of this free energy density in the following manner

$$S_{\text{eff}}\left[\Phi,\overline{\Phi}\right]=-\beta\int d\mathbf{x}F\left[\mathbf{x};\Delta\right]. \quad (37)$$

Comparing (36) with (37), we obtain

$$F\left[\mathbf{x};\,\Delta\right]=F\left[\Delta\right]=-\frac{1}{\beta}\sum_m\frac{1}{\left(2\pi\right)^3}\int d\mathbf{p}\,\times$$
$$\times\,\mathrm{Tr}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)\left[\overline{\Delta}\,\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\,\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^2-\right.$$
$$-\left(\frac{1}{2}-\frac{1}{6}\right)\left[\overline{\Delta}\,\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\,\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^3+$$
$$\left.+\left(\frac{1}{2}-\frac{1}{8}\right)\left[\overline{\Delta}\,\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\,\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^4- ...\right\}. \quad (38)$$

Summing up the infinite series, we write the r. h. s of the formula (38) in the compact form

$$F\left[\mathbf{x};\,\Delta\right]=\frac{1}{\beta}\sum_m\frac{1}{\left(2\pi\right)^3}\int d\mathbf{p}\,\frac{1}{2}\,\mathrm{Tr}\left[\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}\,\times\right.$$
$$\times\left\{\frac{1}{1+\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}}-\right.$$
$$\left.\left.-\int_0^1 d\alpha\frac{1}{1+\alpha\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}}\right\}\right]. \quad (39)$$

Let us discuss the general form of the superconducting order parameters. Consider first the constants Δ_{AC}. We have

$$\Delta_{AC} = \Delta_{(ai\alpha)(ck\gamma)} = (\gamma_5 C)_{\alpha\gamma}\, \Delta^S_{(ai)(ck)} + (C)_{\alpha\gamma}\, \Delta^P_{(ai)(ck)}, \tag{40}$$

where $\Delta^S_{(ai)(ck)}$ are the scalar constants while the $\Delta^P_{(ai)(ck)}$ are the pseudoscalar ones. If the parity is conserved, then all pseudoscalar constants $\Delta^P_{(ai)(ck)}$ must be zero. The existence of nonvanishing pseudoscalar constants $\Delta^P_{(ai)(ck)}$ would signify the spontaneous breaking of the parity conservation. Because of the condition (22) the constants $\Delta^S_{(ai)(ck)}$ and $\Delta^P_{(ai)(ck)}$ must have the property

$$\Delta^S_{(ck)(ai)} = \Delta^S_{(ai)(ck)}, \qquad \Delta^P_{(ck)(ai)} = \Delta^P_{(ai)(ck)}. \tag{41}$$

This means, in particular, that if they are symmetric (antisymmetric) with respect to the flavor indices i and j, they must be also symmetric (antisymmetric) with respect to the color ones a and b.

For the study of the quark-quark pairing due to the gluon exchange we start from the partition function in the form

$$Z = \int [D\psi][D\overline{\psi}] \exp\left\{ -\int dx \overline{\psi}^A(x) L^B_A \psi_B(x) \right\} \times$$
$$\times \exp\left\{ \frac{1}{2} \int dx \int dy \overline{\psi}^A(x) \overline{\psi}^C(y) V^{BD}_{CA}\,(x-y)\, \psi_D(y) \psi_B(x) \right\}, \tag{42}$$

where

$$V^{BD}_{CA}\,(x-y) = -\frac{g^2}{2\pi^2} \sum_I (\gamma_\mu \otimes \lambda_I)^B_A\, (\gamma_\mu \otimes \lambda_I)^D_C\, \frac{1}{(x-y)^2},$$
$$(\gamma_\mu \otimes \lambda_I)^B_A = (\gamma_\mu)^\beta_\alpha\, (\lambda_I)^b_a\, \delta^j_i,\ \sum_I (\lambda_I)^b_a\, (\lambda_I)^d_c = \frac{1}{2}\left[\delta^b_c \delta^d_a - \frac{1}{N_c} \delta^b_a \delta^d_c \right], \tag{43}$$

λ_I are the Gell-Mann matrices of the color symmetry group. In order to describe the diquark systems we introduce some composite bilocal bispinor fields $\Phi_{BD}\,(x,y)\,, \overline{\Phi}^{AC}\,(x,y)$ obeying the Fermi–Dirac statistics

$$\Phi_{DB}\,(y,x) = -\Phi_{BD}\,(x,y)\,, \quad \overline{\Phi}^{CA}\,(y,x) = -\overline{\Phi}^{AC}\,(x,y)\,, \tag{44}$$

and the functional integral over these bosonic fields

$$Z^\Phi_0 = \int [D\Phi]\, [D\overline{\Phi}] \exp\left\{ -\frac{1}{2} \int dx \int dy \overline{\Phi}^{AC}\,(x,y)\, V^{BD}_{CA}\,(x-y)\, \Phi_{DB}\,(y,x) \right\}. \tag{45}$$

By means of the shift of the functional integration variables

$$\begin{aligned}\Phi_{DB}(y,x) &\rightarrow \Phi_{DB}(y,x)+\psi_D(y)\,\psi_B(x)\,,\\ \overline{\Phi}^{AC}(x,y) &\rightarrow \overline{\Phi}^{AC}(x,y)+\overline{\psi}^{A}(x)\,\overline{\psi}^{C}(y)\,,\end{aligned}$$

we can establish the Hubbard–Stratonovich transformation

$$\begin{aligned}&\exp\left\{\frac{1}{2}\int dx\int dy\overline{\psi}^{A}(x)\,\overline{\psi}^{C}(y)\,V_{CA}^{BD}(x-y)\,\psi_D(y)\,\psi_B(x)\right\}=\\ &=\frac{1}{Z_0^{\Phi}}\int[D\Phi]\left[D\overline{\Phi}\right]\exp\left\{-\frac{1}{2}\int dx\int dy\overline{\Phi}^{AC}(x,y)V_{CA}^{BD}(x-y)\Phi_{DB}(y,x)\right\}\times\\ &\qquad\times\exp\left\{-\frac{1}{2}\int dx\int dy\left[\overline{\Delta}^{BD}(x,y)\psi_D(y)\psi_B(x)+\right.\right.\\ &\qquad\qquad\left.\left.+\ \overline{\psi}^{A}(x)\overline{\psi}^{C}(y)\Delta_{CA}(y,x)\right]\right\},\end{aligned}\tag{46}$$

$$\begin{aligned}\Delta_{CA}(y,x)&=V_{CA}^{BD}(x-y)\,\Phi_{DB}(y,x)\,,\\ \overline{\Delta}^{BD}(x,y)&=\overline{\Phi}^{AC}(x,y)\,V_{CA}^{BD}(x-y)\,,\end{aligned}\tag{47}$$

$$\Delta_{AC}(x,y)=-\Delta_{CA}(y,x)\,,\qquad \overline{\Delta}^{DB}(y,x)=-\overline{\Delta}^{BD}(x,y)\,,\tag{48}$$

and rewrite the partition function

$$\begin{aligned}Z=&\frac{1}{Z_0^{\Phi}}\int[D\Phi]\left[D\overline{\Phi}\right]\exp\left\{-\frac{1}{2}\int dx\int dy\overline{\Phi}^{AC}(x,y)\,V_{CA}^{BD}(x-y)\times\right.\\ &\left.\times\,\Phi_{DB}(y,x)\right\}\int[D\psi]\left[D\overline{\psi}\right]\exp\left\{-\int dx\overline{\psi}^{A}(x)\,L_A^B\psi_B(x)\right\}\times\\ &\times\exp\left\{-\frac{1}{2}\int dx\int dy\left[\overline{\Delta}^{BD}(x,y)\,\psi_D(y)\,\psi_B(x)+\right.\right.\\ &\left.\left.+\,\overline{\psi}^{A}(x)\,\overline{\psi}^{C}(y)\,\Delta_{CA}(y,x)\right]\right\}\end{aligned}\tag{49}$$

in the form (23) with the effective action

$$S_{\text{eff}}\left[\Phi,\,\overline{\Phi}\right]=-\frac{1}{2}\int dx\int dy\overline{\Phi}^{AC}(x,y)\,V_{CA}^{BD}(x-y)\,\Phi_{DB}(y,x)+W\left[\Delta,\,\overline{\Delta}\right],\tag{50}$$

$$\exp\left\{W\left[\Delta,\,\overline{\Delta}\right]\right\} = 1 + \sum_{n=1}^{\infty}\Gamma^{(2n)}\left[\Delta,\,\overline{\Delta}\right] =$$
$$= \left\langle \mathbf{T}\left[\exp\left\{-\frac{1}{2}\int dx\int dy\left[\overline{\Delta}^{BD}(x,y)\psi_D(y)\psi_B(x) + \right.\right.\right.\right.$$
$$\left.\left.\left.\left. + \overline{\psi}^{A}(x)\overline{\psi}^{C}(y)\Delta_{CA}(y,x)\right]\right\}\right]\right\rangle. \quad (51)$$

Calculations give

$$\Gamma^{(2)}\left[\Delta,\,\overline{\Delta}\right] = W^{(2)}\left[\Delta,\,\overline{\Delta}\right] = \frac{1}{2}\int dx_1\int dy_1\int dx_2\int dy_2 \times$$
$$\times\,\overline{\Delta}^{A_1C_1}\left(x_1,y_1\right)S_{C_1}^{C_2}\left(y_1-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_1}^{\mathrm{T}A_2}\left(x_2-x_1\right), \quad (52)$$

$$S_{A_1}^{\mathrm{T}A_2}\left(x_2-x_1\right) = S_{A_1}^{A_2}\left(x_1-x_2\right), \quad (53)$$

$$\Gamma^{(4)}\left[\Delta,\,\overline{\Delta}\right] = \frac{1}{2}\left(W^{(2)}\left[\Delta,\,\overline{\Delta}\right]\right)^2 + W^{(4)}\left[\Delta,\,\overline{\Delta}\right], \quad (54)$$

$$W^{(4)}\left[\Delta,\,\overline{\Delta}\right] = -\frac{1}{4}\int dx_1\int dy_1\int dx_2\int dy_2\int dx_3\int dy_3\int dx_4\int dy_4 \times$$
$$\times\,\overline{\Delta}^{A_1C_1}\left(x_1,y_1\right)S_{C_1}^{C_2}\left(y_1-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\times\,\overline{\Delta}^{A_3C_3}\left(x_3,y_3\right)S_{C_3}^{C_4}\left(y_3-y_4\right)\Delta_{C_4A_4}\left(y_4,x_4\right)S_{A_1}^{\mathrm{T}A_4}\left(x_4-x_1\right), \quad (55)$$

$$\Gamma^{(6)}\left[\Delta,\,\overline{\Delta}\right] = \frac{1}{3!}\left(W^{(2)}\left[\Delta,\,\overline{\Delta}\right]\right)^3 +$$
$$+ W^{(2)}\left[\Delta,\,\overline{\Delta}\right]W^{(4)}\left[\Delta,\,\overline{\Delta}\right] + W^{(6)}\left[\Delta,\,\overline{\Delta}\right], \quad (56)$$

$$W^{(6)}\left[\Delta,\,\overline{\Delta}\right] = \frac{1}{6}\int dx_1\int dy_1\int dx_2\int dy_2 \ldots \int dx_5\int dy_5\int dx_6\int dy_6 \times$$
$$\times\,\overline{\Delta}^{A_1C_1}\left(x_1,y_1\right)S_{C_1}^{C_2}\left(y_1-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\times\,\overline{\Delta}^{A_3C_3}\left(x_3,y_3\right)S_{C_3}^{C_4}\left(y_3-y_4\right)\Delta_{C_4A_4}\left(y_4,x_4\right)S_{A_5}^{\mathrm{T}A_4}\left(x_4-x_5\right)\times$$
$$\times\,\overline{\Delta}^{A_5C_5}\left(x_5,y_5\right)S_{C_5}^{C_6}\left(y_5-y_6\right)\Delta_{C_6A_6}\left(y_6,x_6\right)S_{A_1}^{\mathrm{T}A_6}\left(x_6-x_1\right), \quad (57)$$

$$\Gamma^{(8)}\left[\Delta,\,\overline{\Delta}\right] = \frac{1}{4!}\left(W^{(2)}\left[\Delta,\,\overline{\Delta}\right]\right)^4 + \frac{1}{2}\left(W^{(2)}\left[\Delta,\,\overline{\Delta}\right]\right)^2 W^{(4)}\left[\Delta,\,\overline{\Delta}\right] +$$
$$+ \frac{1}{2}\left(W^{(4)}\left[\Delta,\,\overline{\Delta}\right]\right)^2 + W^{(2)}\left[\Delta,\,\overline{\Delta}\right]W^{(6)}\left[\Delta,\,\overline{\Delta}\right] + W^{(8)}\left[\Delta,\,\overline{\Delta}\right], \quad (58)$$

$$W^{(8)}\left[\Delta,\overline{\Delta}\right]=-\frac{1}{8}\int dx_1\int dy_1\int dx_2\int dy_2...\int dx_7\int dy_7\int dx_8\int dy_8\times$$
$$\times\overline{\Delta}^{A_1C_1}\left(x_1,y_1\right)S_{C_1}^{C_2}\left(y_1-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\times\overline{\Delta}^{A_3C_3}\left(x_3,y_3\right)S_{C_3}^{C_4}\left(y_3-y_4\right)\Delta_{C_4A_4}\left(y_4,x_4\right)S_{A_5}^{\mathrm{T}A_4}\left(x_4-x_5\right)\times$$
$$\times\overline{\Delta}^{A_5C_5}\left(x_5,y_5\right)S_{C_5}^{C_6}\left(y_5-y_6\right)\Delta_{C_6A_6}\left(y_6,x_6\right)S_{A_7}^{\mathrm{T}A_6}\left(x_6-x_7\right)\times$$
$$\times\overline{\Delta}^{A_7C_7}\left(x_7,y_7\right)S_{C_7}^{C_8}\left(y_7-y_8\right)\Delta_{C_8A_8}\left(y_8,x_8\right)S_{A_1}^{\mathrm{T}A_8}\left(x_8-x_1\right),$$
$$.............................\qquad(59)$$

It is easy to verify that

$$W\left[\Delta,\overline{\Delta}\right]=\sum_{n=1}^{\infty}W^{(2n)}\left[\Delta,\overline{\Delta}\right].\qquad(60)$$

From the variational principle for the effective action we derive the field equation

$$\frac{1}{2}\Delta_{CA}\left(y,x\right)=V_{CA}^{BD}\left(x-y\right)\sum_{n=1}^{\infty}\frac{\delta W^{(2n)}\left[\Delta,\overline{\Delta}\right]}{\delta\overline{\Delta}^{BD}\left(x,y\right)}.\qquad(61)$$

It has the explicit form

$$\Delta_{CA}\left(y,x\right)=$$
$$=V_{CA}^{BD}\left(x-y\right)\left\{\int dx_2\int dy_2S_D^{C_2}\left(y-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_B^{\mathrm{T}A_2}\left(x_2-x\right)-\right.$$
$$-\int dx_2\int dy_2...\int dx_4\int dy_4S_D^{C_2}\left(y-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\times\overline{\Delta}^{A_3C_3}\left(x_3,y_3\right)S_{C_3}^{C_4}\left(y_3-y_4\right)\Delta_{C_4A_4}\left(y_4,x_4\right)S_B^{\mathrm{T}A_4}\left(x_4-x\right)+$$
$$+\int dx_2\int dy_2...\int dx_6\int dy_6S_D^{C_2}\left(y-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\times\overline{\Delta}^{A_3C_3}\left(x_3,y_3\right)S_{C_3}^{C_4}\left(y_3-y_4\right)\Delta_{C_4A_4}\left(y_4,x_4\right)S_{A_5}^{\mathrm{T}A_4}\left(x_4-x_5\right)\times$$
$$\times\overline{\Delta}^{A_5C_5}\left(x_5,y_5\right)S_{C_5}^{C_6}\left(y_5-y_6\right)\Delta_{C_6A_6}\left(y_6,x_6\right)S_B^{\mathrm{T}A_6}\left(x_6-x\right)-$$
$$-\int dx_2\int dy_2...\int dx_8\int dy_8S_D^{C_2}\left(y-y_2\right)\Delta_{C_2A_2}\left(y_2,x_2\right)S_{A_3}^{\mathrm{T}A_2}\left(x_2-x_3\right)\times$$
$$\left.\times\,...\,\overline{\Delta}^{A_7C_7}(x_7,y_7)S_{C_7}^{C_8}(y_7-y_8)\Delta_{C_8A_8}(y_8,x_8)S_B^{\mathrm{T}A_8}x_8-x)+...\right\}.\qquad(62)$$

Considering the solutions of this equation in the special class of functions depending only on the difference of the coordinates

$$\Delta_{CA}\left(y,x\right)=\Delta_{CA}\left(y-x\right), \qquad \overline{\Delta}^{BD}\left(x,y\right)=\overline{\Delta}^{BD}\left(x-y\right), \tag{63}$$

performing the Fourier transformations

$$\Delta_{CA}\left(\mathbf{y}-\mathbf{x},\sigma-\tau\right)=\frac{1}{\beta}\sum_{m}\mathrm{e}^{i\varepsilon_m(\sigma-\tau)}\frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{e}^{i\mathbf{p}(\mathbf{y}-\mathbf{x})}\widetilde{\Delta}_{CA}\left(\mathbf{p},\varepsilon_m\right),$$

$$\overline{\Delta}^{BD}(\mathbf{x}-\mathbf{y},\tau-\sigma)=\frac{1}{\beta}\sum_{m}\mathrm{e}^{i\varepsilon_m(\tau-\sigma)}\frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{e}^{i\mathbf{p}(\mathbf{x}-\mathbf{y})}\widetilde{\overline{\Delta}}^{BD}\left(\mathbf{p},\varepsilon_m\right), \tag{64}$$

$$V_{CA}^{DB}(\mathbf{x}-\mathbf{y},\tau-\sigma)=\frac{1}{\beta}\sum_{m}\mathrm{e}^{i\omega_m(\sigma-\tau)}\frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{e}^{i\mathbf{p}(\mathbf{y}-\mathbf{x})}\widetilde{V}_{CA}^{DB}(\mathbf{p},\omega_m),$$

$$\varepsilon=(2m+1)\,\frac{\pi}{\beta}, \qquad \omega_m=2m\frac{\pi}{\beta},$$

and introducing matrices $\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{\overline{\Delta}}\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)$ with the elements $\widetilde{\Delta}_{CA}\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{\overline{\Delta}}^{BD}\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{S}_A^B\left(\mathbf{p},\varepsilon_m\right)$, $\widetilde{S}_B^{\mathrm{T}A}\left(-\mathbf{p},-\varepsilon_m\right)$, we derive the extended BCS gap equation for the superconducting quark-quark pairing in QCD

$$\begin{aligned}\widetilde{\Delta}_{CA}(\mathbf{p},\varepsilon_m)=\frac{1}{\beta}\sum_{n}\frac{1}{(2\pi)^3}\int d\mathbf{q}\widetilde{V}_{CA}^{BD}\left(\mathbf{p}-\mathbf{q},\varepsilon_m-\varepsilon_n\right)\times&\\ \times\Bigg[\widetilde{S}(\mathbf{q},\varepsilon_n)\widetilde{\Delta}(\mathbf{q},\varepsilon_n)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{q},-\varepsilon_n\right)\times&\\ \times\frac{1}{1+\widetilde{\overline{\Delta}}\left(\mathbf{q},\varepsilon_n\right)\widetilde{S}\left(\mathbf{q},\varepsilon_n\right)\widetilde{\Delta}\left(\mathbf{q},\varepsilon_n\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{q},-\varepsilon_n\right)}\Bigg]_{DB}.&\end{aligned} \tag{65}$$

At the values of the fields $\Delta_{CA}\left(y,x\right)$ and $\overline{\Delta}^{BD}\left(x,y\right)$ satisfying the equation (62) the effective action (50) equals

$$S_{\mathrm{eff}}\left[\Phi,\overline{\Phi}\right]=W\left[\Delta,\,\overline{\Delta}\right]-\int dx\int dy\overline{\Delta}^{AC}\left(x,y\right)\frac{\delta W\left[\Delta,\,\overline{\Delta}\right]}{\delta\overline{\Delta}^{AC}\left(x,y\right)}=$$

$$=-\left(\frac{1}{4}-\frac{1}{2}\right)\int dx_1\int dy_1...\int dx_4\int dy_4\,\mathrm{Tr}\left[\overline{\Delta}\left(x_1,y_1\right)S\left(y_1-y_2\right)\Delta\left(y_2,x_2\right)\times\right.$$
$$\left.\times\,S^{\mathrm{T}}\left(x_2-x_3\right)\overline{\Delta}\left(x_3,y_3\right)S\left(y_3-y_4\right)\Delta\left(y_4,x_4\right)S^{\mathrm{T}}\left(x_4-x_1\right)\right]+$$
$$+\left(\frac{1}{6}-\frac{1}{2}\right)\int dx_1\int dy_1...\int dx_6\int dy_6\,\mathrm{Tr}\left[\overline{\Delta}\left(x_1,y_1\right)S\left(y_1-y_2\right)\Delta\left(y_2,x_2\right)\times\right.$$
$$\left.\times\,S^{\mathrm{T}}\left(x_2-x_3\right)...\,\overline{\Delta}\left(x_5,y_5\right)S\left(y_5-y_6\right)\Delta\left(y_6,x_6\right)S^{\mathrm{T}}\left(x_6-x_1\right)\right]-$$
$$-\left(\frac{1}{8}-\frac{1}{2}\right)\int dx_1\int dy_1...\int dx_8\int dy_8\,\mathrm{Tr}\left[\overline{\Delta}\left(x_1,y_1\right)S\left(y_1-y_2\right)\Delta\left(y_2,x_2\right)\times\right.$$
$$\left.\times\,S^{\mathrm{T}}\left(x_2-x_3\right)...\,\overline{\Delta}\left(x_7,y_7\right)S\left(y_7-y_8\right)\Delta\left(y_8,x_8\right)S^{\mathrm{T}}\left(x_8-x_1\right)\right]+\,...\,,\qquad(66)$$

where $\Delta\left(y,x\right),\ \overline{\Delta}\left(y,x\right),\ S\left(x-y\right),\ S^{\mathrm{T}}\left(x-y\right)$ denote the matrices with the elements $\Delta_{CA}\left(y,x\right)$, $\overline{\Delta}^{AC}\left(x,y\right)$, $S_A^B\left(x-y\right)$, $S_B^{\mathrm{T}A}(x-y)$. In the case of the special class (63) of the solutions $\Delta_{CA}\left(y-x\right)$, $\overline{\Delta}^{AC}\left(x-y\right)$ the effective action is expressed in terms of the free energy by means of the formula (37). Comparing the expressions (37) and (66) and performing the Fourier transformation, we obtain

$$F\left[\mathbf{x},\Delta\right]=F\left[\Delta\right]=-\frac{1}{\beta}\sum_m\frac{1}{\left(2\pi\right)^3}\int d\mathbf{p}\,\times$$
$$\times\,\mathrm{Tr}\left\{\left(\frac{1}{2}-\frac{1}{4}\right)\left[\widetilde{\overline{\Delta}}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^2-\right.$$
$$-\left(\frac{1}{2}-\frac{1}{6}\right)\left[\widetilde{\overline{\Delta}}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^3+$$
$$\left.+\left(\frac{1}{2}-\frac{1}{8}\right)\left[\widetilde{\overline{\Delta}}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\right]^4-...\right\}.\qquad(67)$$

This series can be also written in the compact form of the integral representation

$$F[\Delta]=\frac{1}{\beta}\sum_m\frac{1}{\left(2\pi\right)^3}\int d\mathbf{p}\frac{1}{2}\,\mathrm{Tr}\left[\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}\left(\mathbf{p},\varepsilon_m\right)\times\right.$$
$$\times\left\{\frac{1}{1+\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}\left(\mathbf{p},\varepsilon_m\right)}-\right.$$
$$\left.\left.-\int_0^1 d\alpha\frac{1}{1+\alpha\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\Delta\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}^{\mathrm{T}}\left(-\mathbf{p},-\varepsilon_m\right)\overline{\Delta}\left(\mathbf{p},\varepsilon_m\right)}\right\}\right].\qquad(68)$$

In order to simplify the expression for $\Delta_{AC}\left(\mathbf{p},\varepsilon_m\right)$ we introduce 4-vector p_μ with the fourth component $p_4=\varepsilon_m$ and denote $\Delta_{AC}\left(\mathbf{p},\varepsilon_m\right)$by $\Delta_{AC}\left(p\right)$. The condition (48) is now written in the form

$$\Delta_{BA}\left(-p\right)=-\Delta_{AB}\left(p\right). \tag{69}$$

The bispinors $\Delta_{AB}\left(p\right)$ are expressed in terms of the Dirac bispinors

$$\begin{aligned}\Delta_{AB}\left(p\right)=\Delta_{(ai\alpha)(bj\beta)}\left(p\right)=\left(\gamma_5 C\right)_{\alpha\beta}\Delta^S_{(ai)(bj)}\left(p\right)+\left(C\right)_{\alpha\beta}\Delta^P_{(ai)(bj)}\left(p\right)+\\+\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\Delta^V_{\mu\,(ai)(bj)}\left(p\right)+\left(\gamma_\mu C\right)_{\alpha\beta}\Delta^A_{\mu\,(ai)(bj)}\left(p\right)+\\+\left(\sigma_{\mu\nu}\gamma_5 C\right)_{\alpha\beta}\Delta^t_{\mu\nu\,(ai)(bj)}\left(p\right).\end{aligned} \tag{70}$$

From the conditions (69) it follows that under the interchange $(ai)\leftrightarrow(bj)$ and the inversion $p\rightarrow -p$ the scalar, pseudoscalar and vector functions are invariant while the pseudovector and tensor functions change their sign

$$\begin{aligned}\Delta^{S,P}_{(bj)(ai)}(-p)=\Delta^{S,P}_{(ai)(bj)}(p),\ \Delta^V_{\mu\,(bj)(ai)}(-p)=\Delta^V_{\mu\,(ai)(bj)}(p),\\ \Delta^A_{\mu\,(bj)(ai)}(-p)=-\Delta^A_{\mu\,(ai)(bj)}(p),\ \Delta^t_{\mu\nu\,(bj)(ai)}(-p)=-\Delta^t_{\mu\nu\,(ai)(bj)}(p).\end{aligned} \tag{71}$$

In general, the existence of nonvanishing superconducting order parameters which are not the singlets of the color and/or flavor groups and lower, the free energy would mean the spontaneous breaking of the color and/or flavor symmetries. For the systems with the isomorphic color and flavor groups $SU(N)_c$ and $SU(N)_f$, there may exist the superconducting order parameters which are the irreducible spinor representations of the groups $SU(N)_c$ and $SU(N)_f$ but the singlet of the «diagonal» $SU(N)$ subgroup of the direct product $SU(N)_c\otimes SU(N)_f$. In this case we have the «color-flavor locking».

2. QUARK-ANTIQUARK PAIRING

The direct four-fermion coupling of the quark fields with the interaction Lagrangian (16) or the nonlocal interaction of the quark fields with the effective action given in the r.h.s of the formula (42) are also the origins of the quark-antiquark pairing. In the case of the quark-antiquark pairing due to the direct four-fermion coupling of the quark field we use the interaction Lagrangian in the form

$$\begin{aligned}L_{\rm int}=\frac{1}{2}\overline{\psi}^A\left(x\right)\psi_B\left(x\right)U^{BD}_{AC}\overline{\psi}^C\left(x\right)\psi_D\left(x\right),\\ U^{DB}_{CA}=-U^{BD}_{AC}=-U^{DB}_{CA}=U^{DB}_{AC},\end{aligned} \tag{72}$$

where instead of the constants V_{CA}^{BD} in (16) we use the new notations

$$U_{AC}^{BD} = V_{CA}^{BD}. \tag{73}$$

The partition function of the system equals

$$Z = \int [D\psi] \left[D\overline{\psi}\right] \exp\left\{-\int dx \overline{\psi}^{A}(x) L_{A}^{B} \psi_{B}(x)\right\} \times \\ \times \exp\left\{\frac{1}{2}\int dx \overline{\psi}^{A}(x)\psi_{B}(x) U_{AC}^{BD} \overline{\psi}^{C}(x)\psi_{D}(x)\right\}. \tag{74}$$

Introducing the local hermitian fields $\Phi_{A}^{B}(x)$ and the functional integral

$$Z_{0}^{\Phi} = \int [D\Phi] \exp\left\{-\frac{1}{2}\int dx \Phi_{B}^{A}(x) U_{AC}^{BD} \Phi_{D}^{C}(x)\right\}, \tag{75}$$

we establish the Hubbard–Stratonovich transformation

$$\exp\left\{\frac{1}{2}\int dx \overline{\psi}^{A}(x)\psi_{B}(x) U_{AC}^{BD} \overline{\psi}^{C}(x)\psi_{D}(x)\right\} = \\ = \frac{1}{Z_{0}^{\Phi}} \int [D\Phi] \exp\left\{-\frac{1}{2}\int dx \Phi_{B}^{A}(x) U_{AC}^{BD} \Phi_{D}^{C}(x)\right\} \times \\ \times \exp\left\{-\int dx \overline{\psi}^{A}(x)\psi_{B}(x) \Delta_{A}^{B}(x)\right\}, \tag{76}$$

$$\Delta_{A}^{B}(x) = U_{AC}^{BD} \Phi_{D}^{C}(x), \tag{77}$$

and rewrite the partition function in the form

$$Z = \frac{Z_{0}}{Z_{0}^{\Phi}} \int [D\Phi] \exp\left\{S_{\text{eff}}[\Phi]\right\} \tag{78}$$

with the effective action

$$S_{\text{eff}}[\Phi] = -\frac{1}{2}\int dx \Phi_{B}^{A}(x) U_{AC}^{BD} \Phi_{D}^{C}(x) + W[\Delta], \tag{79}$$

$$\exp\{W[\Delta]\} = 1 + \sum_{n=1}^{\infty} \Gamma^{(n)}[\Delta] = \\ = \left\langle \mathbf{T}\left[\exp\left\{-\int dx \Delta_{A}^{B}(x) \overline{\psi}^{A}(x)\psi_{B}(x)\right\}\right]\right\rangle. \tag{80}$$

Calculations give

$$W\left[\Delta\right]=\sum_{n=1}^{\infty}W^{(n)}\left[\Delta\right], \tag{81}$$

$$W^{(1)}\left[\Delta\right]=\int dx\Delta_A^B\left(x\right)S_B^A\left(0\right), \tag{82}$$

$$W^{(2)}\left[\Delta\right]=-\frac{1}{2}\int dx_1\int dx_2\,\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A_2}\left(x_1-x_2\right)\times \\ \times\,\Delta_{A_2}^{B_2}\left(x_2\right)S_{B_2}^{A_1}\left(x_2-x_1\right), \tag{83}$$

$$W^{(3)}\left[\Delta\right]=\frac{1}{3}\int dx_1...\int dx_3\,\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A_2}\left(x_1-x_2\right)\Delta_{A_2}^{B_2}\left(x_2\right)\times \\ \times\,S_{B_2}^{A_3}\left(x_2-x_3\right)\Delta_{A_3}^{B_3}\left(x_3\right)S_{B_3}^{A_1}\left(x_3-x_1\right), \tag{84}$$

....

$$W^{(n)}\left[\Delta\right]=\frac{\left(-1\right)^{n+1}}{n}\int dx_1...\int dx_n\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A_2}\left(x_1-x_2\right)\Delta_{A_2}^{B_2}\left(x_2\right)\times \\ \times\,S_{B_2}^{A_3}\left(x_2-x_3\right)...\Delta_{A_n}^{B_n}\left(x_n\right)S_{B_n}^{A_1}\left(x_n-x_1\right). \tag{85}$$

From the variational principle

$$\frac{\delta\,S_{\text{eff}}\left(\Phi\right)}{\delta\,\Phi_D^C\left(x\right)}=0 \tag{86}$$

we derive the field equation

$$\Delta_C^D\left(x\right)=U_{CA}^{DB}\frac{\delta W\left[\Delta\right]}{\delta\Delta_A^B\left(x\right)}=U_{CA}^{DB}\mathbf{S}_B^A\left(x,x\right), \tag{87}$$

where $\mathbf{S}_B^A\left(y,x\right)$ is the two-point Green function of the quark field in the presence of the pairing interaction

$$\mathbf{S}_B^A\left(y,x\right)=S_B^A\left(y-x\right)-\int dx_1S_B^{A_1}\left(y-x_1\right)\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A}\left(x_1-x\right)+ \\ +\int dx_1\int dx_2S_B^{A_1}\left(y-x_1\right)\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A_2}\left(x_1-x_2\right)\Delta_{A_2}^{B_2}\left(x_2\right)S_{B_2}^{A}\left(x_2-x\right)- \\ -\int dx_1...\int dx_3S_B^{A_1}\left(y-x_1\right)\Delta_{A_1}^{B_1}\left(x_1\right)S_{B_1}^{A_2}\left(x_1-x_2\right)\Delta_{A_2}^{B_2}\left(x_2\right)\times \\ \times\,S_{B_2}^{A_3}\left(x_2-x_3\right)\Delta_{A_3}^{B_3}\left(x_3\right)S_{B_3}^{A}\left(x_3-x\right)+... \tag{88}$$

It satisfies the Schwinger–Dyson equation

$$\mathbf{S}_B^A(y,x) = S_B^A(y-x) - \int dz S_B^C(y-z)\,\Delta_C^D(z)\,\mathbf{S}_D^A(z,x). \tag{89}$$

In the special class of the constant solutions of the field equation (87)

$$\Delta_B^A(x) = \Delta_B^A = \text{const} \tag{90}$$

$\mathbf{S}_B^A(y,x)$ depends only on the coordinate difference

$$\mathbf{S}_B^A(y,x) = \mathbf{S}_B^A(y-x). \tag{91}$$

For its Fourier transform we have then an algebraic equation. Denote Δ the matrix with the elements Δ_A^B. From the equation (89) it follows that

$$\widetilde{\mathbf{S}}(\mathbf{p},\varepsilon_m) = \widetilde{S}(\mathbf{p},\varepsilon_m) - \widetilde{S}(\mathbf{p},\varepsilon_m)\,\Delta\widetilde{\mathbf{S}}(\mathbf{p},\varepsilon_m) \tag{92}$$

and

$$\frac{1}{\widetilde{\mathbf{S}}(\mathbf{p},\varepsilon_m)} = \frac{1}{\widetilde{S}(\mathbf{p},\varepsilon_m)} + \Delta. \tag{93}$$

With the field satisfying the equation (87) the effective action equals

$$\begin{aligned}
S_{\text{eff}}[\Phi] = W[\Delta] - \frac{1}{2}\int dx \Delta_A^B(x)\,\frac{\delta W[\Delta]}{\delta\Delta_A^B(x)} = \left(1-\frac{1}{2}\right)\int dx \Delta_A^B(x)\,S_B^A(0) + \\
+\left(\frac{1}{3}-\frac{1}{2}\right)\int dx_1 \int dx_2 \int dx_3\,\Delta_{A_1}^{B_1}(x_1)\,S_{B_1}^{A_2}(x_1-x_2)\,\Delta_{A_2}^{B_2}(x_2)\times \\
\times S_{B_2}^{A_3}(x_2-x_3)\,\Delta_{A_3}^{B_3}(x_3)\,S_{B_3}^{A_1}(x_3-x_1) - \\
-\left(\frac{1}{4}-\frac{1}{2}\right)\int dx_1 ... \int dx_4\,\Delta_{A_1}^{B_1}(x_1)\,S_{B_1}^{A_2}(x_1-x_2)\,\Delta_{A_2}^{B_2}(x_2)\times \\
\times S_{B_2}^{A_3}(x_2-x_3)\,...\,\Delta_{A_4}^{B_4}(x_4)\,S_{B_4}^{A_1}(x_4-x_1) + ... = \\
= \text{Tr}\left\{\left(1-\frac{1}{2}\right)\int dx\Delta(x)\,S(0) + \right. \\
+\left(\frac{1}{3}-\frac{1}{2}\right)\int dx_1 \int dx_2 \int dx_3\,\Delta(x_1)\,S(x_1-x_2)\times \\
\times\Delta(x_2)\,S(x_2-x_3)\,\Delta(x_3)\,S(x_3-x_1) - \\
-\left(\frac{1}{4}-\frac{1}{2}\right)\int dx_1 ... \int dx_4\,\Delta(x_1)\,S(x_1-x_2)\,\Delta(x_2)\times \\
\left.\times S(x_2-x_3)\,...\,\Delta(x_4)\,S(x_4-x_1) + ...\right\},
\end{aligned} \tag{94}$$

where $\Delta(x)$ is the matrix with elements $\Delta_A^B(x)$. It follows that in the case of the constant solutions (90) of the field equations (87) we have the following formula determining the free energy density $F[\mathbf{x};\Delta]$:

$$
\begin{aligned}
F[\mathbf{x};\Delta] = F[\Delta] = &\left(1-\frac{1}{2}\right)\Delta_A^B S_B^A(0) + \\
&+\left(\frac{1}{3}-\frac{1}{2}\right)\int dy\int dz \Delta_A^B S_B^C(x-y)\,\Delta_C^D S_D^E(y-z)\,\Delta_E^F S_F^A(z-x) - \\
&-\left(\frac{1}{4}-\frac{1}{2}\right)\int dy\int dz\int dw\,\Delta_A^B S_B^C(x-y)\,\Delta_C^D S_D^E(y-z)\times \\
&\times\Delta_E^F S_F^G(z-w)\,\Delta_G^H S_H^A(w-x) + ... = \mathrm{Tr}\Big\{\left(1-\frac{1}{2}\right)\Delta\, S(0) + \\
&+\left(\frac{1}{3}-\frac{1}{2}\right)\Delta\int dy\int dz S(x-y)\,\Delta\, S(y-z)\,\Delta\, S(z-x) - \\
&-\left(\frac{1}{4}-\frac{1}{2}\right)\Delta\int dy\int dz\int dw\, S(x-y)\,\Delta S(y-z)\times \\
&\times\Delta\ S(z-w)\,\Delta\, S(w-x) + ...\Big\} = \\
&= \frac{1}{\beta}\sum_m \frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{Tr}\left\{\left(1-\frac{1}{2}\right)\Delta\,\widetilde{S}(\mathbf{p},\varepsilon_m) + \right.\\
&\left. +\left(\frac{1}{3}-\frac{1}{2}\right)\left[\Delta\,\widetilde{S}(\mathbf{p},\varepsilon_m)\right]^3 - \left(\frac{1}{4}-\frac{1}{2}\right)\left[\Delta\,\widetilde{S}(\mathbf{p},\varepsilon_m)\right]^4 + ...\right\}. \qquad (95)
\end{aligned}
$$

Summing up the infinite series, we obtain

$$
F[\Delta] = \frac{1}{\beta}\sum_m \frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{Tr}\left\{\widetilde{\Delta}(\mathbf{p},\varepsilon_m)\left[\int_0^1 \widetilde{\mathbf{S}}^\xi(\mathbf{p},\varepsilon_m)\,d\xi - \frac{1}{2}\widetilde{\mathbf{S}}(\mathbf{p},\varepsilon_m)\right]\right\}, \qquad (96)
$$

where $\widetilde{\mathbf{S}}(\mathbf{p},\varepsilon_m)$ satisfies the equation (93) and $\widetilde{\mathbf{S}}^\xi(\mathbf{p},\varepsilon_m)$ is determined by a similar one with the replacement of Δ by $\xi\Delta$:

$$
\frac{1}{\widetilde{\mathbf{S}}^\xi(\mathbf{p},\varepsilon_m)} = \frac{1}{\widetilde{S}(\mathbf{p},\varepsilon_m)} + \xi\Delta. \qquad (97)
$$

The order parameters Δ_A^B have the form

$$
\Delta_A^B = \Delta_{(ai\alpha)}^{(bj\beta)} = \delta_\alpha^\beta \Delta_{(ai)}^{S(bj)} + (\gamma_5)_\alpha^\beta\,\Delta_{(ai)}^{P(bj)}. \qquad (98)
$$

The nonvanishing order parameters $\Delta_{(ai)}^{P(bj)}$ lowering the free energy would mean the spontaneous parity conservation violation. If $M = 0$ and the interaction Lagrangian (104) is invariant under the chiral transformations, then the existence of nonvanishing order parameters $\Delta_{(ai)}^{S(bj)}$ and/or $\Delta_{(ai)}^{P(bj)}$ lowering the free energy would signify the spontaneous breaking of the chiral invariance. If $\Delta_{(ai)}^{S(bj)}$ and/or $\Delta_{(ai)}^{P(bj)}$ are not the singlets of the color and/or flavor group, then the color and/or flavor symmetries are spontaneously broken. For the system with the isomorphic color and flavor groups $SU(N)_c$ and $SU(N)_f$ there may exist the superconducting order parameters which are the irreducible spinor representations of the groups $SU(N)_c$ and $SU(N)_f$ but the singlet of the «diagonal» $SU(N)$ subgroup of the direct product $SU(N)_c \otimes SU(N)_f$. In this case we have the «color-flavor locking».

Now we consider the quark-antiquark pairing generated by the effective nonlocal interaction of the quark fields due to the gluon exchange. For this purpose we rewrite the partition function (42) in the appropriate form

$$Z = \int [D\psi] \left[D\overline{\psi}\right] \exp\left\{-\int dx \overline{\psi}^A(x) L_A^B \psi_B(x)\right\} \times \\ \times \exp\left\{\frac{1}{2}\int dx \int dy \overline{\psi}^A(x)\psi_B(y) U_{AC}^{BD}(x-y)\overline{\psi}^C(y)\psi_D(x)\right\}. \quad (99)$$

with the new notations

$$U_{AC}^{BD}(x-y) = -V_{AC}^{DB}(x-y), \quad (100)$$

$V_{AC}^{DB}(x-y)$ being given in formula (43). Introduce the hermitian bilocal bosonic fields $\Phi_B^A(x,y)$ and the functional integral

$$Z_0^\Phi = \int [D\Phi] \exp\left\{-\frac{1}{2}\int dx \int dy \Phi_B^A(x,y) U_{AC}^{BD}(x-y)\Phi_D^C(y,x)\right\}. \quad (101)$$

By shifting the functional integration variables, we obtain

$$\exp\left\{\frac{1}{2}\int dx \int dy \overline{\psi}^A(x)\psi_B(y) U_{AC}^{BD}(x-y)\overline{\psi}^C(y)\psi_D(x)\right\} = \\ = \frac{1}{Z_0^\Phi}\int [D\Phi] \exp\left\{-\frac{1}{2}\int dx \int dy \Phi_B^A(x,y) U_{AC}^{BD}(x-y)\Phi_D^C(y,x)\right\} \times \\ \times \exp\left\{-\int dx \int dy \overline{\psi}^A(x)\psi_B(y)\Delta_A^B(x,y)\right\}, \quad (102)$$

where

$$\Delta_A^B(x,y) = U_{AC}^{BD}(x-y)\Phi_D^C(y,x), \quad (103)$$

and rewrite the partition function in the form (78) with the effective action

$$S_{\text{eff}}[\Phi] = -\frac{1}{2}\int dx \int dy \Phi_B^A(x,y) U_{AC}^{BD}(x-y) \Phi_D^C(y,x) + W[\Delta], \qquad (104)$$

$$\exp\{W[\Delta]\} = 1 + \sum_{n=1}^{\infty} \Gamma^{(n)}[\Delta] =$$
$$= \left\langle \mathrm{T}\left[\exp\left\{-\int dx \int dy \overline{\psi}^A(x) \psi_B(y) \Delta_A^B(x,y)\right\}\right]\right\rangle. \qquad (105)$$

Calculations give

$$\Gamma^{(1)}[\Delta] = W^{(1)}[\Delta] = \int dx \int dy\, \Delta_A^B(x,y) S_B^A(y-x), \qquad (106)$$

$$\Gamma^{(2)}[\Delta] = \frac{1}{2}\left(W^{(1)}[\Delta]\right)^2 + W^{(2)}[\Delta], \qquad (107)$$

$$W^{(2)}[\Delta] = -\frac{1}{2}\int dx_1 \int dy_1 \int dx_2 \int dy_2 \Delta_{A_1}^{B_1}(x_1,y_1) S_{B_1}^{A_2}(y_1-x_2) \times$$
$$\times \Delta_{A_2}^{B_2}(x_2,y_2) S_{B_2}^{A_1}(y_2-x_1), \qquad (108)$$

$$\Gamma^{(3)}[\Delta] = \frac{1}{3!}\left(W^{(1)}[\Delta]\right)^3 + W^{(1)}[\Delta] W^{(2)}[\Delta] + W^{(3)}[\Delta], \qquad (109)$$

$$W^{(3)}[\Delta] = \frac{1}{3}\int dx_1 \int dy_1 ... \int dx_3 \int dy_3 \Delta_{A_1}^{B_1}(x_1,y_1) S_{B_1}^{A_2}(y_1-x_2) \times$$
$$\times \Delta_{A_2}^{B_2}(x_2,y_2) S_{B_2}^{A_3}(y_2-x_3) \Delta_{A_3}^{B_3}(x_3,y_3) S_{B_3}^{A_1}(y_3-x_1),$$
$$.............. \qquad (110)$$

The field equation is derived from the variational principle

$$\frac{\delta S_{\text{eff}}[\Phi]}{\delta \Phi_D^C(y,x)} = 0 \qquad (111)$$

and has the form

$$\Delta_C^D(y,x) = U_{CA}^{DB}(x-y) \frac{\delta W[\Delta]}{\delta \Delta_A^B(x,y)}. \qquad (112)$$

Using the expressions for $W^{(n)}[\Delta]$, we obtain the explicit equation

$$\Delta_C^D(y,x) = U_{CA}^{DB}(x-y)\Big\{ S_B^A(y-x) - \\ - \int dx_2 \int dy_2 S_B^{A_2}(y-x_2)\,\Delta_{A_2}^{B_2}(x_2,y_2)\,S_{B_2}^A(y_2-x) + \\ + \int dx_2 \int dy_2 \int dx_3 \int dy_3 S_B^{A_2}(y-x_2)\,\Delta_{A_2}^{B_2}(x_2,y_2) \times \\ \times S_{B_2}^{A_3}(y_2-x_3)\,\Delta_{A_3}^{B_3}(x_3,y_3) S_{B_3}^A(y_3-x) + \ldots \Big\}. \quad (113)$$

Introduce the two-point Green function of quark field in the presence of the quark-antiquark pairing

$$\mathbf{S}_B^A(y,x) = S_B^A(y-x) - \\ - \int dx_2 \int dy_2\, S_B^{A_2}(y-x_2)\,\Delta_{A_2}^{B_2}(x_2,y_2)\,S_{B_2}^A(y_2-x) + \\ + \int dx_2 \int dy_2 \int dx_3 \int dy_3\, S_B^{A_2}(y-x_2)\,\Delta_{A_2}^{B_2}(x_2,y_2)\,S_{B_2}^{A_3}(y_2-x_3) \times \\ \times \Delta_{A_3}^{B_3}(x_3,y_3)\,S_{B_3}^A(y_3-x) + \ldots \quad (114)$$

It satisfies the Schwinger–Dyson equation

$$\mathbf{S}_B^A(y,x) = S_B^A(y-x) - \int dz \int dw\, S_B^C(y-z)\,\Delta_C^D(z,w)\,\mathbf{S}_D^A(w,x)\,. \quad (115)$$

Then the field equation (113) becomes

$$\Delta_C^D(y,x) = U_{CA}^{DB}(x-y)\,\mathbf{S}_B^A(y,x)\,. \quad (116)$$

Consider the solution of this field equation in the special class of functions depending only on the difference of the coordinates

$$\Delta_A^B(x,y) = \Delta_A^B(x-y)\,. \quad (117)$$

In this case the function $\mathbf{S}_B^A(x,y)$ depends also only on the difference of the coordinates

$$\mathbf{S}_B^A(x,y) = \mathbf{S}_B^A(x-y)\,. \quad (118)$$

Performing the Fourier transformation

$$\Delta_A^B(\mathbf{x}-\mathbf{y},\tau-\sigma) = \frac{1}{\beta}\sum_m \mathrm{e}^{i\varepsilon_m(\tau-\sigma)}\frac{1}{(2\pi)^3}\int d\mathbf{p}\,\mathrm{e}^{i\mathbf{p}(\mathbf{x}-\mathbf{y})}\widetilde{\Delta}_A^B(\mathbf{p},\varepsilon_m), \\ \varepsilon_m = (2m+1)\,\frac{\pi}{\beta}, \quad (119)$$

and introducing the matrices $\widetilde{\Delta}(\mathbf{p},\varepsilon_m)$ with the elements $\widetilde{\Delta}_A^B(\mathbf{p},\varepsilon_m)$, we rewrite the integral relation (114) and the integral equation (115) in the form of the algebraic ones:

$$\widetilde{\mathbf{S}}\left(\mathbf{p},\varepsilon_m\right)=\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)-\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)+ \\ +\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)+... \quad (120)$$

and

$$\widetilde{\mathbf{S}}\left(\mathbf{p},\varepsilon_m\right)=\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)-\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right)\widetilde{\mathbf{S}}\left(\mathbf{p},\varepsilon_m\right) \quad (121)$$

or

$$\frac{1}{\widetilde{\mathbf{S}}\left(\mathbf{p},\varepsilon_m\right)}=\frac{1}{\widetilde{S}\left(\mathbf{p},\varepsilon_m\right)}+\widetilde{\Delta}\left(\mathbf{p},\varepsilon_m\right). \quad (122)$$

The field equation (116) becomes

$$\widetilde{\Delta}_C^D\left(\mathbf{p},\varepsilon_m\right)=\frac{1}{\beta}\sum_n\frac{1}{(2\pi)^3}\int d\mathbf{q}\widetilde{U}_{CA}^{DB}\left(\mathbf{p}-\mathbf{q},\varepsilon_m-\varepsilon_n\right)\widetilde{\mathbf{S}}_B^A\left(\mathbf{q},\varepsilon_n\right), \quad (123)$$

where $\widetilde{U}_{CA}^{DB}\left(\mathbf{p}-\mathbf{q},\varepsilon_m-\varepsilon_n\right)$ is the Fourier transform of $U_{CA}^{DB}\left(x-y\right)$,

$$U_{CA}^{DB}(\mathbf{x}-\mathbf{y},\tau-\sigma)=\frac{1}{\beta}\sum_m \mathrm{e}^{i\omega_m(\sigma-\tau)}\frac{1}{(2\pi)^3}\times \\ \times\int d\mathbf{p}\,\mathrm{e}^{i\mathbf{p}(\mathbf{y}-\mathbf{x})}\widetilde{U}_{CA}^{DB}(\mathbf{p},\omega_m),\quad \omega_m=2m\frac{\pi}{\beta}. \quad (124)$$

Using the field equation (112), we obtain the value of the effective action (104)

$$S_{\text{eff}}\left[\Phi\right]=W[\Delta]-\frac{1}{2}\int dx\int dy\Delta_A^B\left(x,y\right)\frac{\delta\,W\left[\Delta\right]}{\delta\,\Delta_A^B\left(x,y\right)}= \\ =\left(1-\frac{1}{2}\right)\int dx\int dy\Delta_A^B\left(x,y\right)S_B^A\left(y-x\right)+ \\ +\left(\frac{1}{3}-\frac{1}{2}\right)\int dx_1\int dy_1...\int dx_3\int dy_3\Delta_{A_1}^{B_1}\left(x_1,y_1\right)S_{B_1}^{A_2}\left(y_1-x_2\right)\times \\ \times\Delta_{A_2}^{B_2}\left(x_2,y_2\right)S_{B_2}^{A_3}\left(y_2-x_3\right)\Delta_{A_3}^{B_3}\left(x_3,y_3\right)S_{B_3}^{A_1}\left(y_3-x_1\right)- \\ -\left(\frac{1}{4}-\frac{1}{2}\right)\int dx_1\int dy_1...\int dx_4\int dy_4\Delta_{A_1}^{B_1}\left(x_1,y_1\right)S_{B_1}^{A_2}\left(y_1-x_2\right)\times \\ \times\Delta_{A_2}^{B_2}\left(x_2,y_2\right)...\Delta_{A_4}^{B_4}\left(x_4,y_4\right)S_{B_4}^{A_1}\left(y_4-x_1\right)+...= \\ =\mathrm{Tr}\left\{\left(1-\frac{1}{2}\right)\int dx\int dy\Delta\left(x,y\right)S\left(y-x\right)+\right.$$

$$
+\left(\frac{1}{3}-\frac{1}{2}\right)\int dx_1\int dy_1 ... \int dx_3\int dy_3\,\Delta\left(x_1,y_1\right)S\left(y_1-x_2\right)\times
$$
$$
\times\,\Delta\left(x_2,y_2\right)S\left(y_2-x_3\right)\Delta\left(x_3,y_3\right)S\left(y_3-x_1\right)-
$$
$$
-\left(\frac{1}{4}-\frac{1}{2}\right)\int dx_1\int dy_1 ... \int dx_4\int dy_4\Delta\left(x_1,y_1\right)S\left(y_1-x_2\right)\times
$$
$$
\left.\times\,\Delta\left(x_2,y_2\right)...\,\Delta\left(x_4,y_4\right)S\left(y_4-x_1\right)+...\right\}, \quad (125)
$$

where $\Delta\left(x,y\right)$ is the matrix with the elements $\Delta_A^B\left(x,y\right)$. For the fields in the special class (117) the effective action is expressed in terms of the free energy density $F[\mathbf{x};\Delta]$:

$$
S_{\text{eff}}[\Phi]=-\beta\int d\mathbf{x}F[\mathbf{x};\Delta]. \quad (126)
$$

It follows that

$$
F\left[\mathbf{x};\Delta\right]=F\left[\Delta\right]=\left(1-\frac{1}{2}\right)\int dy\Delta_A^B\left(x-y\right)S_B^A\left(y-x\right)+
$$
$$
+\left(\frac{1}{3}-\frac{1}{2}\right)\int dy\int dx_2\int dy_2\,\int dx_3\int dy_3\Delta_{A_1}^{B_1}\left(x-y_1\right)S_{B_1}^{A_2}\left(y_1-x_2\right)\times
$$
$$
\times\,\Delta_{A_2}^{B_2}\left(x_2-y_2\right)S_{B_2}^{A_3}\left(y_2-x_3\right)\Delta_{A_3}^{B_3}\left(x_3-y_3\right)S_{B_3}^{A_1}\left(y_3-x\right)-
$$
$$
-\left(\frac{1}{4}-\frac{1}{2}\right)\,\int dy\int dx_2\int dy_2 ... \int dx_4\int dy_4\Delta_{A_1}^{B_1}\left(x-y_1\right)S_{B_1}^{A_2}\left(y_1-x_2\right)\times
$$
$$
\times\,\Delta_{A_2}^{B_2}\left(x_2-y_2\right)...\,S_{B_3}^{A_4}\left(y_3-x_4\right)\Delta_{A_4}^{B_4}\left(x_4-y_4\right)S_{B_4}^{A_1}\left(y_4-x\right)+...\;=
$$
$$
=\text{Tr}\left\{\left(1-\frac{1}{2}\right)\int dy\Delta\left(x-y\right)S\left(y-x\right)+\right.
$$
$$
+\left(\frac{1}{3}-\frac{1}{2}\right)\int dy\int dx_2\int dy_2\,\int dx_3\int dy_3\Delta(x-y_1)S\left(y_1-x_2\right)\times
$$
$$
\times\,\Delta\left(x_2-y_2\right)S\left(y_2-x_3\right)\Delta\left(x_3-y_3\right)S\left(y_3-x\right)-
$$
$$
-\left(\frac{1}{4}-\frac{1}{2}\right)\,\int dy\int dx_2\int dy_2 ... \int dx_4\int dy_4\Delta(x-y_1)S(y_1-x_2)\times
$$
$$
\left.\times\,\Delta(x_2-y_2) ...\, S(y_3-x_4)\Delta\left(x_4-y_4\right)S(y_4-x)+...\right\}=
$$
$$
=\frac{1}{\beta}\sum_m\frac{1}{(2\pi)^3}\int d\mathbf{p}\,\text{Tr}\left\{\left(1-\frac{1}{2}\right)\widetilde{\Delta}(\mathbf{p},\varepsilon_m)\widetilde{S}(\mathbf{p},\varepsilon_m)\,+\right.
$$
$$
+\left(\frac{1}{3}-\frac{1}{2}\right)\left[\widetilde{\Delta}(\mathbf{p},\varepsilon_m)\widetilde{S}(\mathbf{p},\varepsilon_m)\right]^3-
$$

$$- \left(\frac{1}{4} - \frac{1}{2} \right) \left[\widetilde{\Delta} \left(\mathbf{p}, \varepsilon_m \right) \widetilde{S} \left(\mathbf{p}, \varepsilon_m \right) \right]^4 + ... \Bigg\} . \tag{127}$$

Summing up the infinite series, we obtain

$$F \left[\Delta \right] = \frac{1}{\beta} \sum_m \frac{1}{(2\pi)^3} \int d\mathbf{p} \, \mathrm{Tr} \left\{ \widetilde{\Delta} \left(\mathbf{p}, \varepsilon_m \right) \left[\int_0^1 \widetilde{\mathbf{S}}^\xi \left(\mathbf{p}, \varepsilon_m \right) d\xi - \frac{1}{2} \widetilde{\mathbf{S}} \left(\mathbf{p}, \varepsilon_m \right) \right] \right\} , \tag{128}$$

where $\widetilde{\mathbf{S}}^\xi \left(\mathbf{p}, \varepsilon_m \right)$ is determined by the equation of the form (122) with the replacement of $\widetilde{\Delta} \left(\mathbf{p}, \varepsilon_m \right)$ by $\xi \widetilde{\Delta} \left(\mathbf{p}, \varepsilon_m \right)$

$$\frac{1}{\widetilde{\mathbf{S}}^\xi \left(\mathbf{p}, \varepsilon_m \right)} = \frac{1}{\widetilde{\mathbf{S}} \left(\mathbf{p}, \varepsilon_m \right)} + \xi \widetilde{\Delta} \left(\mathbf{p}, \varepsilon_m \right) . \tag{129}$$

Introduce 4-vector p_μ with $p_4 = \varepsilon_m$ and denote $\widetilde{\Delta}_A^B \left(\mathbf{p}, \varepsilon_m \right)$ by $\widetilde{\Delta}_A^B (p)$. These order parameters have following most general form

$$\widetilde{\Delta}_A^B \left(p \right) = \delta_\alpha^\beta \widetilde{\Delta}_{(ai)}^{S(bj)} \left(p \right) + \left(\gamma_5 \right)_\alpha^\beta \widetilde{\Delta}_{(ai)}^{P(bj)} \left(p \right) + \left(\gamma_\mu \right)_\alpha^\beta \widetilde{\Delta}_{\mu(ai)}^{V(bj)} \left(p \right) + \\ + \left(\gamma_\mu \gamma_5 \right)_\alpha^\beta \widetilde{\Delta}_{\mu(ai)}^{A(bj)} \left(p \right) + \left(\sigma_{\mu\nu} \right)_\alpha^\beta \widetilde{\Delta}_{\mu\nu(ai)}^{t(bj)} \left(p \right) . \tag{130}$$

The existence of the nonvanishing order parameters with definite transformation properties would mean the spontaneous breaking of the corresponding symmetries.

3. FORMATION OF TRIQUARKS

It is straightforward to generalize the method presented in preceding Sections for applying to the problem of the formation of triquarks — the bound states of three quarks. We note that the fundamental interaction mechanisms in QCD (instanton induced, gluon exchange, etc.) always lead to some effective (nonlocal, in general) six-fermion interaction between quark fields with the effective interaction action of the general form

$$S_{\text{int}} = \frac{1}{6} \int dx \int dy \int dz \int du \int dv \int dw \overline{\psi}^F \left(w \right) \overline{\psi}^E \left(v \right) \overline{\psi}^D \left(u \right) \times \\ \times V_{DEF}^{CBA} \left(u, v, w; z, y, x \right) \psi_A \left(x \right) \psi_B \left(y \right) \psi_C \left(z \right) ,$$

$$V_{DEF}^{CBA} \left(u, v, w; z, y, x \right) = -V_{DEF}^{CAB} \left(u, v, w; z, y, x \right) = \\ = -V_{EDF}^{CBA} \left(u, v, w; z, y, x \right) = ... \tag{131}$$

The formfactors $V_{DEF}^{CBA}(u,v,w;z,y,x)$ depend only on the differences of the space-time coordinates. The partition function of the many-quark system with this effective six-quark interaction equals

$$Z = \int [D\psi]\,[D\overline{\psi}]\exp\left\{-\int dx\overline{\psi}^A(x)\,L_A^B\psi_B(x)\right\}\times$$
$$\times\exp\left\{\frac{1}{6}\int dx\int dy\int dz\int du\int dv\int dw\overline{\psi}^F(w)\,\overline{\psi}^E(v)\,\overline{\psi}^D(u)\times\right.$$
$$\left.\times\; V_{DEF}^{CBA}(u,v,w;z,y,x)\,\psi_A(x)\,\psi_B(y)\,\psi_C(z)\right\}. \quad (132)$$

In order to describe the triquarks we introduce some trispinor trilocal field $\Phi_{ABC}(x,y,z)$ as well as its conjugate $\overline{\Phi}^{CBA}(z,y,x)$ and set

$$Z_0^\Phi = \int [D\Phi]\,[D\overline{\Phi}]\exp\left\{\frac{1}{6}\int dx\int dy\int dz\int du\int dv\int dw\times\right.$$
$$\left.\times\;\overline{\Phi}^{FED}(w,v,u)\,V_{DEF}^{CBA}(u,v,w;z,y,x)\,\Phi_{ABC}(x,y,z)\right\}. \quad (133)$$

By shifting the functional integration variables

$$\Phi_{ABC}(x,y,z) \rightarrow \Phi_{ABC}(x,y,z)+\frac{1}{\sqrt{2}}\psi_A(x)\,\psi_B(y)\,\psi_C(z),$$
$$\overline{\Phi}^{FED}(w,v,u) \rightarrow \overline{\Phi}^{FED}(w,v,u)+\frac{1}{\sqrt{2}}\overline{\psi}^F(w)\,\overline{\psi}^E(v)\,\overline{\psi}^D(u),$$

we establish the Hubbard–Stratonovich transformation

$$\exp\left\{\frac{1}{6}\int dx\int dy\int dz\int du\int dv\int dw\overline{\psi}^F(w)\,\overline{\psi}^E(v)\,\overline{\psi}^D(u)\times\right.$$
$$\left.\times\; V_{DEF}^{CBA}(u,v,w;z,y,x)\,\psi_A(x)\,\psi_B(y)\,\psi_C(z)\right\}=$$
$$= \frac{1}{Z_0^\Phi}\int [D\Phi]\,[D\overline{\Phi}]\exp\left\{-\frac{1}{3}\int dx\int dy\int dz\int du\int dv\int dw\times\right.$$
$$\left.\times\;\overline{\Phi}^{FED}(w,v,u)\,V_{DEF}^{CBA}(u,v,w;z,y,x)\,\Phi_{ABC}(x,y,z)\right\}\times$$
$$\times\exp\left\{-\frac{1}{\sqrt{18}}\int dx\int dy\int dz\left[\overline{\Delta}^{CBA}(z,y,x)\,\psi_A(x)\,\psi_B(y)\,\psi_C(x)+\right.\right.$$
$$\left.\left.+\;\overline{\psi}^C(z)\,\overline{\psi}^B(y)\,\overline{\psi}^A(x)\,\Delta_{ABC}(x,y,z)\right]\right\}, \quad (134)$$

where

$$\overline{\Delta}^{CBA}(z,y,x) = \int du \int dv \int dw \overline{\psi}^F(w)\overline{\psi}^E(v)\overline{\psi}^D(u) V_{DEF}^{CBA}(u,v,w;z,y,x),$$
$$\Delta_{ABC}(x,y,z) = \int du \int dv \int dw V_{DEF}^{CBA}(u,v,w;z,y,x)\psi_D(u)\psi_E(v)\psi_F(w), \tag{135}$$

and transform, after lengthy calculations, the partition function (132) into the form (23) of a functional integral over the trilocal fields $\Phi_{ABC}(x,y,z)$ and $\overline{\Phi}^{CBA}(z,y,x)$ with the effective action

$$S_{\text{eff}}\left[\Phi,\overline{\Phi}\right] = -\frac{1}{3}\int dx \int dy \int dz \overline{\Phi}^{FDE}(w,v,u)\, V_{DEF}^{CBA}(u,v,w;z,y,x) \times$$
$$\times \Phi_{ABC}(x,y,z) + W\left[\Delta,\overline{\Delta}\right] \tag{136}$$

and the functional $W\left[\Delta,\overline{\Delta}\right]$ of the form

$$W\left[\Delta,\overline{\Delta}\right] = \sum_{n=1}^{\infty} W^{(2n)}\left[\Delta,\overline{\Delta}\right], \tag{137}$$

where $W^{(2n)}\left[\Delta,\overline{\Delta}\right]$ is a functional of the n-th order with respect to each type of trilocal fields $\Delta_{ABC}(x,y,z)$and $\overline{\Delta}^{CBA}(z,y,x)$, for example,

$$W^{(2)}\left[\Delta,\overline{\Delta}\right] = \frac{1}{3}\int dx \int dy \int dz \int du \int dv \int dw \times$$
$$\times \overline{\Delta}^{FED}(w,v,u)\, S_F^C(w-z) S_E^B(v-y) S_D^A(u-x)\Delta_{ABC}(x,y,z), \tag{138}$$

$$W^{(4)}\left[\Delta,\overline{\Delta}\right] = -\frac{1}{2}\int dx_1 \int dy_1 \int dz_1 ... \int du_2 \int dv_2 \int dw_2 \times$$
$$\times \overline{\Delta}^{F_1E_1D_1}(w_1,v_1,u_1)\,\overline{\Delta}^{F_2E_2D_2}(w_2,v_2,u_2)\, S_{F_2}^{C_1}(w_2-z_1)\, S_{E_1}^{B_1}(v_1-y_1) \times$$
$$\times S_{D_2}^{A_1}(u_1-x_1) S_{F_1}^{C_2}(w_1-z_2) S_{E_2}^{B_2}(v_2-y_2)\, S_{D_2}^{A_2}(u_2-x_2) \times$$
$$\times \Delta_{A_2B_2C_2}(x_2,y_2,z_2)\,\Delta_{A_1B_1C_1}(x_1,y_1,z_1),$$
$$\ldots\ldots\ldots\ldots \tag{139}$$

From the variational principle we derive the field equation

$$\frac{1}{3}\Delta_{ABC}(x,y,z) = \int du \int dv \int dw V_{ABC}^{FED}(u,v,w;z,y,x) \times$$
$$\times \sum_{n=1}^{\infty} \frac{\delta W^{(n)}\left[\Delta,\overline{\Delta}\right]}{\delta\overline{\Delta}^{FED}(w,v,u)}. \tag{140}$$

Using explicit expression of $W^{(2n)}\left[\Delta,\overline{\Delta}\right]$, we have shown that up to the 6th order of the pertubation theory there exists the following system of integral equations

$$\Delta_{ABC}(x,y,z)=\int du\int dv\int dw\int dx'\int dy'\int dz' V_{ABC}^{FED}(u,v,w;z,y,x)\times \\ \times G_F^{C'}(w,z')\,G_E^{B'}(v,y')\,G_D^{A'}(u,x')\,\Delta_{A'B'C'}(x',y',z'), \quad (141)$$

$$G_A^D(x,u)=S_A^D(x-u)+\int dy\int dz S_A^B(x-y)\,\Sigma_B^C(y,z)\,G_C^D(z,u), \quad (142)$$

$$\Sigma_C^F(z,w)=\int dx\int dy\int du\int dv\overline{\Delta}^{FED}(w,v,u)\times \\ \times G_D^A(u,x)\,G_E^B(v,y)\,\Delta_{ABC}(x,y,z). \quad (143)$$

It is easy to verify that $G_A^D(x,u)$ is the two-point Green function of the quark field in the presence of its interaction with the «external» trilocal fields $\Delta_{ABC}(x,y,z)$ and $\overline{\Delta}^{FED}(w,v,u)$. It is determined by the Schwinger–Dyson equation represented by the Feynman diagram in Fig. 1 with the self-energy part (143) reprsented by the Feynman diagram in Fig. 2.

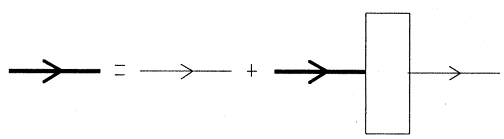

Fig. 1.

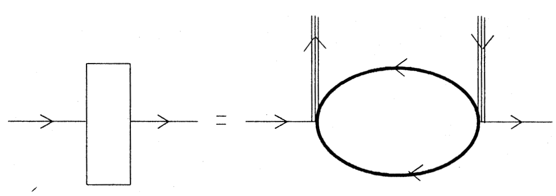

Fig. 2.

The solutions of the system of integral equations (141)–(143) in the class of the fields depending only on the differences of the coordinates can be considered as the anticommuting order parameters of the ground state of the many-quark system with the binding of the quarks into the triquarks. This means that in the QCD dense quark matter there might exist a phase transition with the anticommuting order parameters. Note that there is no condensation of the triquarks, because these composite particles are fermions.

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