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SOLAR NEUTRINO PROBLEM ACCOUNTING FOR SELF-CONSISTENT MAGNETOHYDRODYNAMICS SOLUTION FOR SOLAR MAGNETIC FIELDS

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The analysis of the resonant spin-flavour (RSF) solutions to the solar neutrino problem in the framework of simplest analytic solutions to the solar magnetohydrodynamics (MHD) equations is presented. We performed the global fit of the recent solar neutrino data, including event rates as well as zenith angle distributions and recoil electron spectra induced by solar neutrino interactions in Superkamiokande. We compare quantitatively our simplest MHD-RSF fit with vacuum oscillation (VAC) and MSW-type (SMA, LMA and LOW) solutions to the solar neutrino problem using a common well-calibrated theoretical calculation and fit procedure and find MHD-RSF fit to be somewhat better than those obtained for the favored neutrino oscillation solutions. We made the predictions for future experiments (e.g., SNO) to disentangle the MHD-RSF scenario from other scenarios.

Механизм резонансной спин-ароматной конверсии нейтрино для решения проблемы солнечных нейтрино пересмотрен в рамках аналитического решения уравнений солнечной магнитной гидродинамики (МГД). Учитывая математическую самосогласованность уравнений и существующие физические ограничения, мы предлагаем простейшую схему конверсии нейтрино в рамках реалистичных статических решений МГД-уравнений. Мы провели количественное сравнение полученных результатов с другими механизмами, такими как вакуумные осцилляции и МСВ-эффект, используя единую схему вычислений, и показали, что наш сценарий дает лучшее согласие с экспериментальными данными. В работе сделаны предсказания для нейтринного эксперимента в Садбэри.

INTRODUCTION

The persistent disagreement between solar neutrino data and theoretical expectations has been a long-standing problem in physics. The most popular solutions of the solar neutrino anomalies are based on the idea of neutrino oscillations, either in vacuum or in the Sun due to the enhancement arising from matter effects [1]. In addition there is considerable interest in

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alternative interpretations. Here we will re-analyse the status of resonant spin-flavour (RSF) solutions [2,3] to the solar neutrino problem in the light of the most recent global set of solar neutrino data. In contrast to previous attempts [2–4] we will adopt the general framework of self-consistent magnetohydrodynamic (MHD) models of the Sun [5]. For definiteness we will concentrate on the recent proposal of Ref. [6]. We perform global fits of solar neutrino data for realistic solutions to the magnetohydrodynamics equations inside the Sun. This way and by neglecting neutrino mixing we obtain the simplest MHD-RSF solution to the solar neutrino problem, characterized by two effective parameters, Δm^2 and $\mu_\nu B_{\perp\max}$, $B_{\perp\max}$ being the maximum magnitude of the magnetic field inside the convective region. We find that our simplest two-parameter MHD-RSF fits to the solar neutrino data are slightly better than those for the oscillation solutions. The required best fit points correspond to maximum magnetic field magnitudes in the convective zone smaller than 100 kG. We briefly discuss the prospects to distinguish our simplest MHD-RSF scenario from the neutrino oscillation solutions to the solar neutrino problem.

1. STATIC MAGNETIC FIELD PROFILES IN THE SUN

In solar magnetohydrodynamics [7] (MHD) the corresponding magnetic field profiles are rather complicated and difficult to extract. However, there are stationary solutions which are known analytically in terms of relatively simple functions [6].

We consider the magnetic field profiles which are only solutions to the equation for a static MHD plasma configuration in a gravitational field, given by

$$\nabla p - \frac{1}{c} \mathbf{j} \times \mathbf{B} + \rho \nabla \Phi = 0. \quad (1)$$

This static MHD equations correspond to a quiet Sun and they admit axially symmetric solutions in the spherically symmetric gravitational field which can be simply expressed in terms of spherical Bessel functions and were first discussed in Ref. [6]. The model magnetic field depends on z_k , the roots of the spherical Bessel function $f_{5/2} = \sqrt{z} J_{5/2}(z)$, to ensure the boundary condition that \mathbf{B} vanishes on the solar surface

$$\begin{aligned} B_r^k(r, \theta) &= 2\hat{B}^k \cos \theta \left[1 - \frac{3}{r^2 z_k \sin z_k} \left(\frac{\sin(z_k r)}{z_k r} - \cos(z_k r) \right) \right], \\ B_\theta^k(r, \theta) &= -\hat{B}^k \sin \theta \left[2 + \frac{3}{r^2 z_k \sin z_k} \left(\frac{\sin(z_k r)}{z_k r} - \cos(z_k r) - z_k r \sin(z_k r) \right) \right], \\ B_\phi^k(r, \theta) &= \hat{B}^k z_k \sin \theta \left[r - \frac{3}{r z_k \sin z_k} \left(\frac{\sin(z_k r)}{z_k r} - \cos(z_k r) \right) \right], \end{aligned} \quad (2)$$

where the coefficient $\hat{B}^k(B_{\text{core}})$ is given by

$$\hat{B}^k = \frac{B_{\text{core}}}{2(1 - z_k / \sin z_k)}. \quad (3)$$

The distance r has been normalized to $R_\odot = 1$. In our calculations we have averaged over polar angle θ .

The parameter B_{core} is a central magnetic field. In Fig. 1 we display the perpendicular component of \mathbf{B} for various k values, 1, 3 and 10, which correspond to the roots $z_1 = 5.7$, $z_3 = 12.3$ and $z_{10} = 34.5$, respectively.

We now discuss the astrophysical restrictions on the free parameters B_{core} and k characterizing the model. The magnitude of a magnetic field at the center of the Sun is constrained by the Fermi–Chandrasekhar limit [8] which requires an upper bound on $B_{\text{core}} \lesssim 2$ MG.

The possible values of k can be constrained by taking into account that, in order to justify the use of a stationary solution, it is necessary that the diffusion time due to ohmic dissipation must be less than solar lifetime. The simple estimations give us that reasonable values of k are less than 10.

It is commonly accepted that magnetic fields measured at the surface of the Sun are weaker than within the convective zone interior where this field is supposed to be generated. On the other hand, the general knowledge of the solar magnetic field models is that the magnetic field increases at the overshoot layer, while being small at the solar interior, a picture rather opposite to the one we have seen in Fig. 1. The correct way is to use the linear nature of the basic equilibrium MHD equation in Eq. (1). This implies that any linear combination of solutions \mathbf{B}^k ($k = 1, 2, \dots, k_M$, for some fixed number $k_M < 10$) $\mathbf{B} = c_1\mathbf{B}_1 + c_2\mathbf{B}_2 + \dots + c_M\mathbf{B}_M$ is also a solution. We will require that combined magnetic field is equal to zero in the center of the Sun and its total energy must be minimal in the region below the bottom of the convective zone, characterized by a certain value of r_0 .

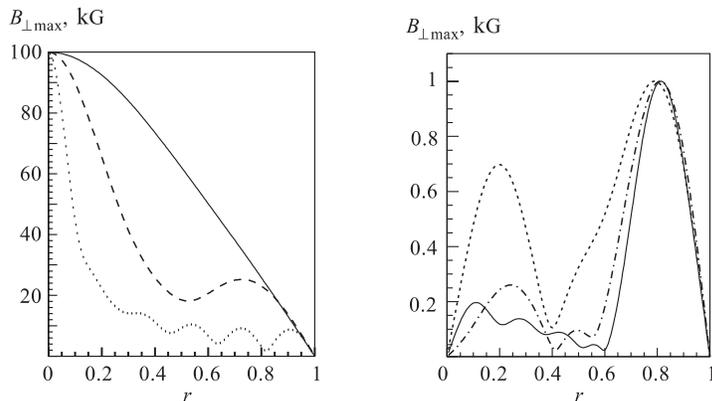


Fig. 1. The perpendicular component of \mathbf{B} for various k values, 1 (solid), 3 (dashed) and 10 (dotted)

Fig. 2. Magnetic field configurations obtained by combining individual modes for different k_M values, 5 (dotted), 6 (dash-dotted) and 10 (solid). Summing up to higher modes achieves better localization of the field in the convective region

The procedure sketched above provides a consistent method for combining individual mode solutions \mathbf{B}_k of the static MHD equation (Fig. 2).

2. FITTING THE SOLAR NEUTRINO DATA

We will neglect neutrino mixing in what follows and consider the case of active-active neutrino conversions. In this case the $\nu_e \rightarrow \bar{\nu}_\ell$ conversions are described by the master

Schrödinger evolution equation

$$i \begin{pmatrix} \dot{\nu}_e \\ \dot{\bar{\nu}}_\ell \end{pmatrix} = \begin{pmatrix} V_e - \delta & \mu_\nu B_+ \\ \mu_\nu B_- & -V_\ell + \delta \end{pmatrix} \begin{pmatrix} \nu_e \\ \bar{\nu}_\ell \end{pmatrix}, \quad (4)$$

where μ_ν denotes the neutrino transition magnetic moment [9] in units of $10^{-11} \mu_B$, ℓ denoting either μ or τ . Here $B_\pm = B_x \pm iB_y$ and $\delta = \Delta m^2/4E$ is the neutrino mass parameter; $V_e(t) = G_F \sqrt{2}(\rho(t)/m_p)(Y_e - Y_n/2)$ and $V_\ell(t) = G_F \sqrt{2}(\rho(t)/m_p)(-Y_n/2)$ are the neutrino vector potentials for ν_e and ν_ℓ in the Sun given by the abundances of the electron ($Y_e = m_p N_e(t)/\rho(t)$) and neutron ($Y_n = m_p N_n(t)/\rho(t)$) components. In our numerical study of solar neutrino data we adopt the Standard Solar Model density profile of Ref. [10].

We solve Eq. (4) numerically by finding a solution of the Cauchy problem in the form of a set of wave functions $\nu_a(t) = |\nu_a(t)\rangle e^{i\Phi_a(t)}$ from which the neutrino survival probabilities $P_{aa}(t) = \nu_a^* \nu_a$ are calculated. They obey the unitarity condition $\sum_a P_{aa} = 1$, where the subscript a denotes $a = e$ for ν_e and $a = \ell$ for $\bar{\nu}_\ell$, respectively.

The typical neutrino survival probability P_{ee} calculated in the MHD-RSF scheme from Eq. (4) plotted versus $E/\Delta m^2$ in Fig. 3.

To determine the possible values of the parameters we have first used the data on the total event rates measured at the Chlorine experiment in Homestake [11], at the two Gallium experiments GALLEX and SAGE [12, 13] and the 1117-day Superkamiokande data [14], as given in Table 1.

Table 1. Solar neutrino rates measured in the Chlorine, Gallium and Superkamiokande experiments

Experiment	Rate	Ref.	Units	R_i^{BP08}
Homestake	2.56 ± 0.23	[11]	SNU	7.8 ± 1.1
GALLEX + SAGE	74.66 ± 5.2	[12, 13]	SNU	130 ± 7
Superkamiokande	2.40 ± 0.08	[14]	$10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}$	5.2 ± 0.9

We have found that allowed regions of neutrino parameters are pretty stable and do not depend significantly on the choice of k_M and r_0 allowed by astrophysics. In Fig. 4 we display the region of MHD-RSF parameters allowed by the solar neutrino rates for the case $M = 6$ and $r_0 = 0.6R_\odot$. We can see that there are several allowed regions for different values of the magnetic field.

Apart from total event rates the water Cerenkov experiment also measures the zenith angle distribution of solar neutrino events as well as their electron recoil energy spectrum with their recent 1117-day data sample [14]. The predicted spectrum is essentially flat except for the upper part of the Δm^2 region. As an example, we show in Fig. 5 the excluded region at 99% C.L. for the case $k_M = 6$ and $r_0 = 0.6$. For this reason, the allowed regions are slightly modified by the inclusion of the zenith angular dependence and the energy spectrum data. In Fig. 6 we have presented the results of global fit analysis.

We now move to the case of active-sterile MHD-RSF conversions. In the rate only fit, the χ_{rates}^2 is worse than for the active-active case, essentially due to the neutral current contribution in the Superkamiokande experiment.

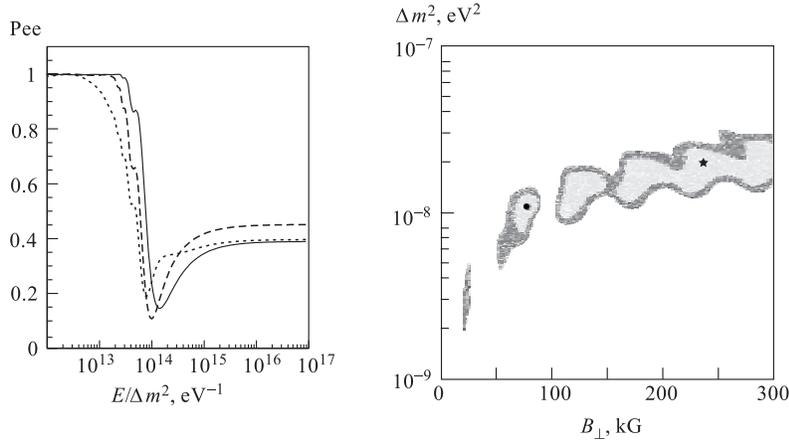


Fig. 3. Typical MHD-RSF neutrino survival probability P_{ee} versus $E/\Delta m^2$

Fig. 4. MHD-RSF 90 % C.L. (light) and 99 % C.L. (dark) regions of Δm^2 versus $B_{\perp\max}$ (kG) allowed by the rates given in Table 1, for $r_0 = 0.6$ and $k_M = 6$

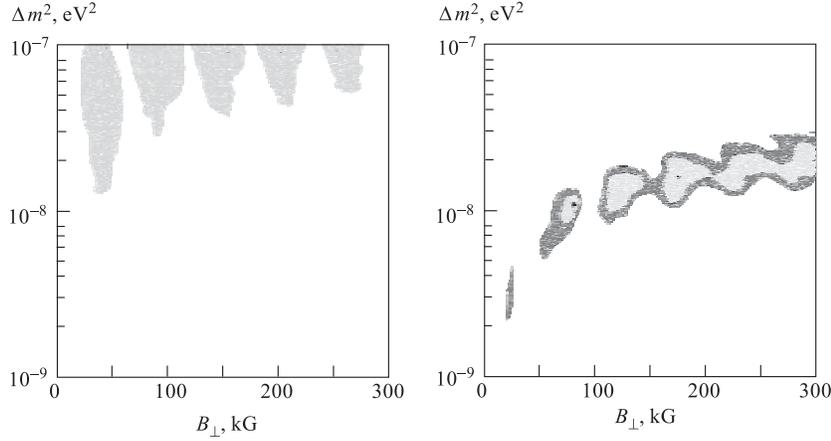


Fig. 5. MHD-RSF 99 % C.L. regions of Δm^2 versus $B_{\perp\max}$ forbidden by the recoil electron spectrum data given in Table 2 for $r_0 = 0.6$ and $k_M = 6$

Fig. 6. 90 % C.L. (light) and 99 % C.L. (dark) allowed MHD-RSF regions in Δm^2 and $B_{\perp\max}$ from the measurements of rates combined with the zenith angle distribution and the recoil energy spectrum in Superkamiokande, for $r_0 = 0.6$ and $k_M = 6$

3. DISCUSSION AND CONCLUSIONS

From the results of the previous section it follows that our MHD-RSF solution to the solar neutrino problem provides a good description of the most recent solar neutrino data, including event rates as well as zenith angle distributions and recoil electron spectra induced by solar neutrino interactions in Superkamiokande. We have shown that our procedure is quite robust in the sense that the magnetic field profile has been determined in an essentially unique way.

Table 2. Best fit points and the corresponding probabilities for different solutions to the solar neutrino problem

Solution	Δm^2	$B_{\perp\max}$	χ^2_{\min} (Prob. %)	Ref.
MHD-RSF _A	$1.1 \cdot 10^{-8}$ Δm^2	80 $\sin^2(2\vartheta)$	25.7 (32) χ^2_{\min} (Prob. %)	the present work
SMA _A	$5.2 \cdot 10^{-6}$	$4.7 \cdot 10^{-3}$	29.7 (16)	[15, 16]
LMA	$2.4 \cdot 10^{-5}$	0.78	27.0 (26)	[15, 16]
LOW	$1.0 \cdot 10^{-7}$	0.93	32.0 (10)	[15, 16]
SMA _S	$5.2 \cdot 10^{-6}$	$4.7 \cdot 10^{-3}$	32.0 (10)	[15, 17]
VAC	$4.4 \cdot 10^{-10}$	0.9	34.3 (6)	[17]
no-osc.			87.9 ($6 \cdot 10^{-7}$)	[15]

This effectively substitutes the neutrino mixing which characterizes the oscillation solutions by a single parameter $B_{\perp\max}$, characterizing the maximum magnitude of the magnetic field inside the convective region. The value of k_M , characterizing the maximum number of individual modes superimposed in order to obtain a realistic profile, and the parameter r_0 , characterizing the location of the convective region, are severely restricted. The allowed k_M values are

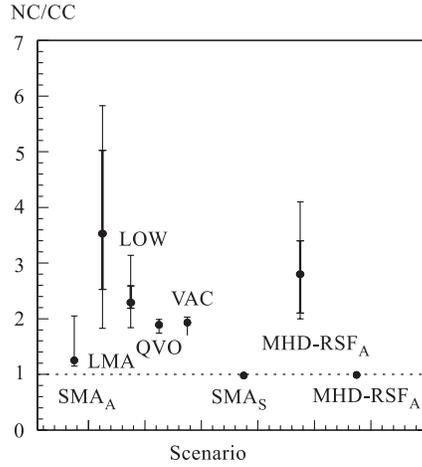


Fig. 7. Neutral-to-charged-current event ratio expected at SNO for different solutions to the solar neutrino problem at 90% C.L. and 99% C.L. The no-oscillation or SM case is denoted by the horizontal line at $NC/CC = 1$

minima for $B_{\perp\max} < 100$ kG for the case

restricted by ohmic dissipation arguments to be lower than 10 or so, while r_0 is close to $0.6R_{\odot}$. We have found that our solar neutrino fits are pretty stable as long as k_M exceeds 5 and r_0 lies in the relevant narrow range. Therefore our fits are effectively two-parameter fits (Δm^2 and $B_{\perp\max}$) whose quality can be meaningfully compared with that of the fits obtained for the favored neutrino oscillation solutions to the solar neutrino problem. In Table 2 we compare the various solutions of the solar neutrino problem with the MHD-RSF solutions for the lower magnetic field presented here. Clearly the MHD-RSF fits seem somewhat better (though not in a statistically significant way) than those obtained for the MSW effect [15] as well as just-so solutions [17].

We determined the expected solar neutrino rates at SNO within the framework of our MHD-RSF solution to the solar neutrino problem. We used the cross sections of the CC and NC νd reactions given by Ref. [18] and the best fit points we have determined in the present paper. For definiteness we have considered the global best fit points and local minima for $k_M = 6$ and $r_0 = 0.6$ and active-active MHD-RSF

We have calculated the neutral-to-charged-current event ratio (NC/CC) and our results are presented in Fig. 7. It is clear from Fig. 7 that there is a substantial overlap between our MHD-RSF predictions and those found for each of the oscillation solutions (SMA, LMA, LOW, VAC). The overlap is especially large between the LMA and the MHD-RSF solutions. Taking into account the present theoretical uncertainties and a reasonable estimate of the experimental errors attainable, it follows that an unambiguous discrimination between our MHD-RSF solution and the neutrino oscillation-type solutions to the solar neutrino problem on the basis of the averaged event rates seems rather difficult.

Note added: As we finished our paper there appeared the paper by E.K.Akhmedov and J.Pulido, hep-ph/0005173, which also considers predictions for some SNO observables in the conventional RSF scheme employing ad hoc magnetic field profiles used in Ref. [4].

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