УДК 539.142.3+539.172.2

ROLE OF THE COULOMB DISTORTION IN FORM-FACTOR CALCULATIONS FOR ^{12}C WITH ALPHA-CLUSTERIZATION AND NUCLEON-NUCLEON CORRELATIONS

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The elastic form factor of ^{12}C is calculated in the plane-wave Born approximation (BA) and also by accounting for distortions of electron waves in the nuclear Coulomb field both within the high-energy approximation (HEA) and by numerical solving of the Dirac equation (SDE). The nuclear wave function includes peculiarities associated with the alpha-clusterization and short-range correlations of nucleons. It is shown that these correlations affect the form factors at comparably large transfer momenta, where a considerable difference takes place between different schemes of calculations, namely, BA, HEA and SDE methods. It is concluded that the SDE method is preferable when studying the influence on the form factors of the short- and middle-range nucleon correlations in nuclei.

Формфакторы ядра ^{12}C рассчитаны в плосковолновом борновском приближении (БП), а также при учете искажения электронных волн в кулоновском поле ядра как в рамках высокоэнергетического приближения (ВЭП), так и путем численного решения уравнения Дирака (РУД). Ядерные волновые функции включают в себя особенности, связанные с альфа-кластеризацией и короткодействующими корреляциями нуклонов. Показано, что учет таких корреляций оказывает влияние на формфакторы в области сравнительно больших переданных импульсов, где значительные различия имеют место также при использовании БП-, ВЭП-, РУД-методов расчета электронных волн. Сделан вывод о том, что метод РУД является предпочтительным при изучении влияния на формфакторы корреляций нуклонов на малых и средних расстояниях.

1. The residual NN correlations in nuclei are of significant interest in investigations of nuclear physics, especially of processes at large momentum transfers. In this connection, the clusterization problems related to the middle-range correlations and the repulsion correlations of nucleons in nuclei at short distances are widely discussed in the last decade (see, e. g., [1,2]). In our recent papers [3], we studied these problems when calculating form factors of ^{12}C. For this aim, the well-grounded Brink alpha-cluster model [4] has been employed for calculating form factors basing on [5]. At the same time, we tested, for alpha clusters themselves, the model where the method of one-body density matrix [6–9] has been applied to describe experimental data on the momentum and spatial distributions of nucleons in ^{4}He. It was found that the successful explanation of the data can be achieved by suggesting various constructions of the respective density matrix, and, in particular, the d-shell admixture in the ^{4}He wave

function can be introduced to explain the data. Thus, the problem of ambiguity of the ^{4}He wave functions corresponding to different models including short-range correlations has to be resolved by invoking the information from other data. In this connection, investigations of form factors of the alpha-cluster nucleus ^{12}C can be useful to distinguish between different models of the ^{4}He wave functions. Indeed, results of calculations in [3] show that one can distinguish their behavior studying the ^{12}C form factors at comparably large momentum transfers $q \geq 3$ fm^{-1}. In this region, one can see visible differences between form factors of an order of magnitude like that of $|F(q)|^2$.

However, at large transfer momenta, there appears the problem of a proper taking account of the Coulomb distortion of electron waves in a process of scattering from nuclei. To this end, we have used the high-energy approximation (HEA) method [10] which «fills» the minima of form factors in the Born approximations (BA). Nevertheless, to be sure in further conclusions on the nuclear structure effects, one should make an additional test by applying the exact method of numerical solving of the Dirac equation (SDE) for an electron in the electrostatic potential of a spherically symmetrical charge distribution.

In this paper, calculations of the elastic form factor of ^{12}C are performed. The two typical kinds of the nucleon wave functions in the ^{4}He cluster are tested, the results of calculations within BA, HEA and SDE methods are compared and discussed, and the respective conclusions on the comparative role of the nuclear structure and the distortion of electron-waves effects are drawn.

2. The form factors in the HEA can be represented in the form [11]:

$$|F(q)|^2 = G(q) \left| \int d\mathbf{r} \frac{q^2}{\tilde{q}^2} g(\mathbf{r}) \exp\, (i\mathbf{qr} + i\phi(\mathbf{r}))\rho(r) \right|^2, \tag{1}$$

where functions $\tilde{q}$, $g(\mathbf{r})$, and $\phi(\mathbf{r})$ incorporate the distortion of the plane waves. In the plane-wave Born approximation, $\tilde{q} = q$, $\phi = 0$ and $g = 1$; $G(q)$ accounts the proton size and the centre-of-mass motion:

$$G(q) = \exp\, \left[-\frac{q^2}{3} \left(\bar{R}^2(p) - \frac{\bar{R}^2(\alpha)}{A} \right) \right], \tag{2}$$

where $\bar{R}(p)$ and $\bar{R}(\alpha)$ are the r. m. s charge radii of the proton and the alpha particle in the target nucleus A.

As to the Dirac equation, the latter contains the central potential arising from the ground-state nuclear-charge distribution,

$$V(r) = -\alpha \int \frac{1}{r_>} \rho(r')\; r'^2\; dr', \tag{3}$$

with $\alpha = e^2/\hbar c$ and $\rho(r)$ normalized to Z. The code was performed following [12], where a partial wave decomposition of the Dirac equation neglecting the electron mass yields

$$\frac{dG_j}{dx} - \frac{j + \frac{1}{2}}{x}\, G_j + \left[1 - \frac{V(x/E)}{E} \right]\, F_j = 0, \tag{4}$$

$$\frac{dF_j}{dx} - \frac{j+\frac{1}{2}}{x}\,F_j - \left[1 - \frac{V(x/E)}{E}\right]\,G_j = 0. \tag{5}$$

Here $x = rE$; F_j and G_j are appropriate jth radial partial-wave functions with the asymptotic behavior at $r \gg R$

$$G_j \sim \sin\,\left(x - \frac{\pi}{2}(j-1/2) + \eta_j + Z\alpha \ln\,2x\right). \tag{6}$$

For numerical integration of the radial Dirac equations (4), (5), a step-by-step procedure is used based on the 4th accuracy order Runge–Kutt algorithm. Solutions at the initial point are taken in the form of a power expansion around the origin [12]. Alternatively, one can use closed expressions for relativistic Coulomb functions as they are taken, e. g., in [13]. Integration runs from the initial point to a point outside the twice nuclear radius where the charge density falls down noticeably. Matching the numerical solutions of (4), (5) in asymptotics with the analytic ones (6), one finds the phase shifts η_j and the scattering amplitude

$$f(\vartheta) = -\frac{i}{E}\sum_j (2j+1)\exp\,(2i\eta_j)\left[P_{j+\frac{1}{2}}(\cos\,\vartheta) + P_{j-\frac{1}{2}}(\cos\,\vartheta)\right], \tag{7}$$

and then the form factor $|F(q)|^2 = G(q)d\sigma/d\sigma_M$, where the differential cross section is $d\sigma/d\Omega = \cos^{-2}(\vartheta/2)\,|f(\vartheta)|^2$, and the $d\sigma_M/d\Omega$ is the Mott cross section. The above procedure was programmed in the FORTRAN code, and was tested by comparisons with results of calculations of form factors presented in Ref. 12. Calculations were performed at the multiprocessor computer SPP-2000 using the Message Passing Interface (MPI) system to organize the parallel regime of computing. As a result, one gets «filling» zeroes of the Born-approximation form factors squared, and also slight «shifting» patterns of angular distributions at large q predicted by BA and HEA calculations.

3. For calculating the ^{12}C charge density distribution $\rho(r)$, in the model [4] the projection procedure is used to generate rotational states $|JMK\pi\rangle$ of the total nuclear wave function $|JM\pi\rangle = \sum_K C_K^J\,|JMK\pi\rangle$ from the many-body wave function $U_\pi(\mathbf{R}) = U(\mathbf{R}) + \pi U_\pi(-\mathbf{R})$ where the parity is $\pi = \pm 1$ and vectors $\mathbf{R} \equiv \{\mathbf{R_i}\}$ stand for the positions of alpha clusters at the vertices of the equlibrium triangle configuration prescribed for the ^{12}C nucleus. In [5], the equation for the density distribution function is written as follows

$$\rho(r) = \frac{C_0^0}{4\pi^2\pi N_{00\pi_0}}\int d\Theta\big[\langle U(\mathbf{R})|\mathcal{P}|U(\mathbf{S})\rangle + P_{\pi_0}\langle U(\mathbf{R})|\mathcal{P}|U(-\mathbf{S})\rangle\big], \tag{8}$$

where the vector $\mathbf{S}$ is the position of $\mathbf{R}$ after the Wigner rotation by the angle Θ. The antisymmetrized many-body wave function of the nucleus is as follows:

$$U(\mathbf{R}) = \sum_p \epsilon_p \prod_{a=1}^{A}\,u_a(pa), \qquad u_a(pa) = u_a(\mathbf{r} - \mathbf{R_i}) = \chi_\sigma\chi_\tau u(|\mathbf{r} - \mathbf{R_i}|), \tag{9}$$

with $\epsilon_p = +1$ or -1 corresponding to even or odd permutations of nucleons, respectively, and the wave functions of nucleons u_p are related to the centres i of alpha clusters in the

nucleus. The density operator is in the usual form

$$\mathcal{P} = \sum_{k=1}^{Z} \frac{e}{Z} \delta(r - r_k) \frac{1}{r^2}, \tag{10}$$

C_0^0 is obtained from the symmetry properties of the triangular configuration, and $N_{00\pi_0}$ is the normalization constant of the total wave function. Thus, the problem is reduced to calculations of the multiparticle matrix elements with the wave functions of nucleons $u(r)$ belonging to alpha clusters. They can be performed in the impulse representation of functions in integrands of the respective multiparticle matrix elements (for details see Ref. 5).

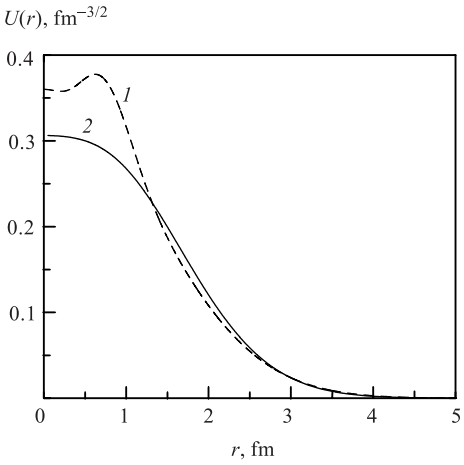

Fig. 1. The nucleon s. p. wave functions: the function $R(r)$ (MHO, curve *1*) from (13) with $\lambda_{1s} = 0.86$, $\lambda_{1d} = 0.028$, $\hbar\omega_{1s} = 25$ MeV and $\hbar\omega_{1d} = 150$ MeV and the overlap function (OV, curve *2*) obtained following [7]

4. As was mentioned before, we aim at testing in ^{12}C the realistic one-particle wave functions $u(r)$ fitted to the data on form factors and the momentum distribution of ^{4}He. In [3], we have used several forms of $u(r)$ which correspond to different approaches in constructing the respective one-body density matrix (OBDM).

The first approach uses the representation of the A-nucleus ground-state wave function $\Psi^{(A)}$ with the help of the natural orbitals $\psi_\alpha(\mathbf{r})$ [14] defined as a complete orthogonal set of functions which diagonalize the OBDM:

$$\rho(\mathbf{r}, \mathbf{r}') = \langle \Psi^{(A)} | a^\dagger(\mathbf{r}) a(\mathbf{r}') | \Psi^{(A)} \rangle = \\ = \sum_\alpha N_\alpha \psi_\alpha^*(\mathbf{r}) \psi_\alpha(\mathbf{r}'). \tag{11}$$

In [9], the proper natural orbitals and occupation numbers N_α have been determined by the requirement for them to give a realistic description of the density distribution

$$\rho(r) = \frac{1}{4\pi}(2\lambda_{1s}|R_{1s}(r)|^2 + 10\lambda_{1d}|R_{1d}(r)|^2), \tag{12}$$

and the corresponding momentum distributions $n(k)$ in ^{4}He. In (12), one has $2\lambda_{1s} + 10\lambda_{1d} = 2$, where λ_{1s} and λ_{1d} are the occupation numbers, and $R_{1s}(r)$ and $R_{1d}(r)$ are the natural orbitals taking account of the short-range NN correlations. These orbitals have been chosen to be single particle wave functions corresponding to the multiharmonic Hamiltonian of the isomorphic shell model [15]. The radial part of the nucleon wave function in ^{4}He related to the density distribution (12) and normalized to unity is chosen in our work to be

$$R(r) = \sqrt{2\pi\rho(r)}. \tag{13}$$

The function $R(r)$ uses a set of $1s$- and $1d$-oscillator parameters and occupation numbers obtained by fitting both density and momentum distributions, as well as by fitting the charge

r. m. s radius and the mean kinetic energy in ^{4}He [9]. This function is presented in Fig. 1 by MHO (coming from «the multiharmonic oscillator approach»).

The second type of single-particle wave functions has been chosen to be the overlap (OV) functions $\phi_\alpha(\mathbf{r})$ which realize the representation of OBDM as follows

$$\rho(\mathbf{r},\mathbf{r}') = \sum_\alpha \phi^*_\alpha(\mathbf{r})\phi_\alpha(\mathbf{r}'), \qquad \phi_\alpha(\mathbf{r}) = \langle \Psi^{(A-1)}_\alpha | a(\mathbf{r}) | \Psi^{(A)} \rangle, \tag{14}$$

where α stands for the corresponding state of the residual nucleus. Following [16], the OV functions have been calculated in [7] using the OBDM obtained with including the short-range Jastrov correlation factor $1 - \exp(-\beta^2 r^2)$. The radial part of the lowest $\phi_{n_0lj}(r)$ neuteron bound-state OV can be obtained from a definition of the radial part of OBDM for $r \equiv a \to \infty$, where $\phi_{n_0lj}(a) = B_{n_0lj} \exp(-k_{n_0lj}a)/a$. Thus,

$$\phi_{n_0lj}(r) = \frac{\rho_{lj}(r,a)}{B_{n_0lj}\ \exp(-k_{n_0lj}a)/a}. \tag{15}$$

Here $k_{nlj} = \hbar^{-1}\sqrt{2m(E^{(A-1)}_{nlj} - E^{(A)}_0)}$, and the coefficient B_{n_0lj} can be obtained from the asymptotic form of the diagonal part of the radial OBDM $\rho_{lj}(a,a)$.

The overlap function $\phi_{1s_{1/2}}$ (15) for ^{4}He, taken from [3], is displayed (as an OV curve) in Fig. 1. Both functions (13) and (15) are used in the present work to calculate the charge density function and the elastic form factor in the ^{12}C nucleus.

5. The elastic form factor of the ^{12}C nucleus has been calculated by using the theoretical scheme, briefly outlined in Secs. 2, 3 and nucleon wave functions given in Sec. 4. The results of the predictions using the BA, HEA and SDE methods are shown in Fig. 2, *a*, *b*. The

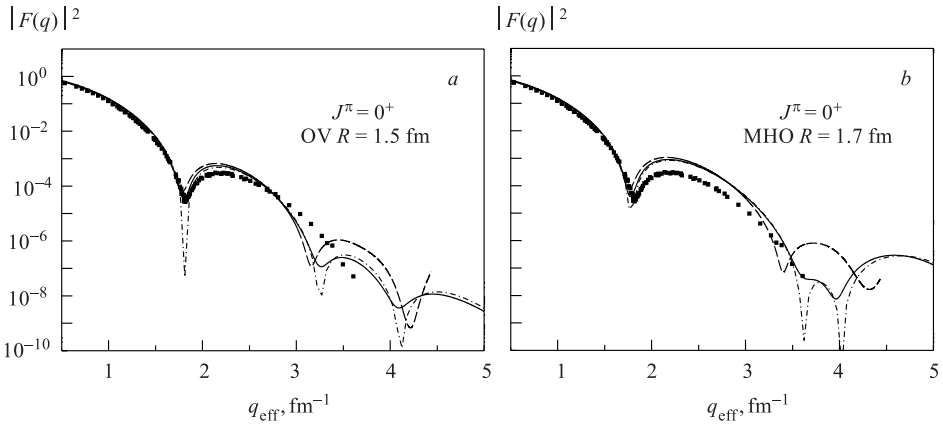

Fig. 2. The elastic form factors of ^{12}C calculated in the Born approximation (dot-dashed line), in HEA (solid line), and in the Dirac-equation method (dashed line): *a*) by using the overlap function (OV) obtained following [7], experimental data (■) are taken from [17]; *b*) by using the nucleon wave function $R(r)$ from (13) with the same set of parameters as in Fig. 1

experimental data are taken from [17]. As can be seen, there are no substantial differences at small and intermediate values of q between the results obtained with the use of HEA and SDE methods. Both the methods lead to «filling in» the minima in the Born-approximation form factor. However, at comparatively large momentum transfers, one can see disagreements between their predictions. The best value of the parameter R giving the distance between the centre of the equilateral triangle and the alpha cluster for each case is presented, too. The form factor is plotted versus the effective momentum transfer

$$q_{\text{eff}} = q\left(1 + \frac{4}{3}\frac{Z}{137}\frac{1}{\bar{R}(A)E}\right), \tag{16}$$

which makes it possible to take into account its dependence on the energy E of the incident electrons, $\bar{R}(A)$ being the nucleus r. m. s radius. The radial densities calculated within the use of the nucleon wave functions, leading to the form factors given in Fig. 2, *a*, *b*, are presented in Fig. 3.

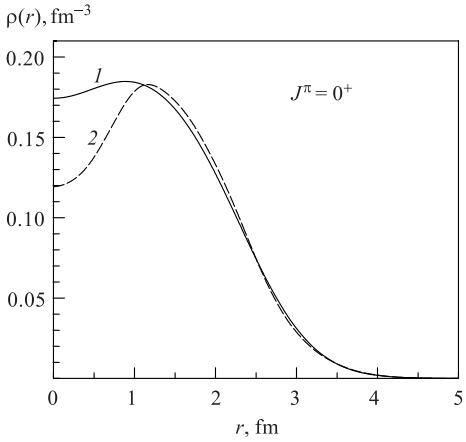

Fig. 3. The radial density distribution of the ground state in ^{12}C calculated with the nucleon wave functions OV (curve *1*, $R = 1.5$) and MHO (curve *2*, $R = 1.7$) given in Fig. 1

It is seen from Fig. 2, *a*, *b*, that in the region of transfer momenta $q_{\text{eff}} \leq 2$ fm^{-1} (lower than the first minimum), one can get a satisfactory overall agreement between form factors, calculated within the BA, HEA and SDE methods and by using the same OV function and MHO-natural orbital as well as with the experimental data in ^{12}C. For larger q_{eff} from 2 to 3 fm^{-1}, we have approximately the same behavior of the HEA and SDE theoretical curves for the OV functions, and, separately, for MHO-natural orbitals, but both bunches of theoretical curves deviate from experimental data. When comparing, in general, calculations with OV and MHO functions, a noticeably better agreement with the data is found for the multiharmonic oscillator nucleon wave function $R(r)$ (see Eq. (13)) shown in Fig. 2, *b*. Concerning the region of $q_{\text{eff}} > 3$ fm^{-1}, one can conclude that here all the three calculation methods give very different form factors. Moreover, their deviations from one another are of the same order of magnituide as the difference associated with peculiarities in behavior of the nucleon wave functions themselves. Thus, if one wants to study the subtle structure effects on form factors caused by, e. g., the short-range NN correlations in nuclei, then the precise SDE method is preferable to be used for calculations at large transfer momenta.

We can note that in the structure calculations, we have used, as the single-particle wave functions, the natural orbitals and overlap functions related to the realistic density matrices for the ground state of the ^{4}He nucleus, and thus effects of the short-range NN correlations are incorporated in predictions of the ^{12}C form factors. The only free parameter of our theoretical scheme is the distance R between the centre and vertices of the triangular frame for clusters in ^{12}C. Thus, to exclude some disagreements of calculations with the experimental

data, there still remains the possibility to add an admixture of the linear cluster configuration to the total wave function of ^{12}C and to take account of a possible change of the structure of alpha-clusters themselves as compared to the structure of a free ^{4}He.

Acknowledgments. Four of the authors (D. N. K., A. N. A., K. S., and G. S. A.) would like to thank the Laboratory of Theoretical Physics of JINR for support and warm hospitality during their visits to Dubna. Three of the authors (D. N. K., A. N. A., and K. S.) thank the Bulgarian National Fund for Scientific Research for partial financial support of this work under Contracts No. Phi-809, No. Phi-905 and No. MU-F-02/98 (for D. N. K.). Also V. K. L. is grateful to the RFBR Grants No. 00-15-96737, and E. V. Z. to the RFBR Grant No. 0001-00617 for partial support.

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Received on February 26, 2002.