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THE CHARGE FORM FACTOR AND THE NUCLEON MOMENTUM DISTRIBUTION OF $^4_2$He AND THEIR CENTRE-OF-MASS CORRECTION

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The problem of including approximate corrections, for the centre-of-mass motion in calculating the elastic form factor and the nucleon momentum distribution of the $^4_2$He nucleus, when nontranslationally invariant single particle wave functions are used, is addressed and results are discussed.

Рассматривается проблема и обсуждаются результаты приближенного учета движения центра масс в расчетах упругого формфактора и импульсного распределения нуклонов в ядре $^4_2$He для характерного случая использования одночастичных волновых функций, не удовлетворяющих условию трансляционной инвариантности.

As is well known, the measurements of the charge form factor $F_{\rm ch}$ of various nuclei in a range of momentum transfers has stimulated extensive theoretical work for the calculation of this quantity and also for the corresponding nucleon momentum distribution $\eta(p)$ [1–5]. The $^4_2$He nucleus has received particular attention, not only because of the relative simplicity of this nucleus and its importance in nuclear physics but also because its $F_{\rm ch}(q)$ has been measured in a wide region of momentum transfers q [6].

It was realized long ago that the simple harmonic oscillator (HO) model is not able to reproduce the diffraction minimum of the $F_{\rm ch}(q)$ which appears at $q^2 \cong 10$ fm^{-2}. If one wishes to reproduce that feature in the framework of a single particle potential model, thus avoiding more complicated calculations, a proper potential has to be used. It was found that potentials having a short range strong repulsion aiming at simulating in a way part of the effect of omitted characteristics, such as short-range nucleon correlations in the nuclear wave function, have that desirable feature. Thus, a square (or a Woods–Saxon) well with hard core or a Morse potential were used [7–9]. Alternatively, a «modified harmonic oscillator (MHO) potential» [10, 11], which has in addition considerable analytic advantages, appears to be quite suitable for that purpose. Using, however, a single particle potential, except the HO potential, one faces the serious problem of the treatment of the centre-of-mass (CM) motion [1–5, 12], in particular for a light nucleus such as $^4_2$He and also for high values of momentum transfer. It is the aim of this letter to address this problem and attempt an approximate treatment. Such an investigation seems to be rather appealing, in particular for the nucleon momentum distribution $\eta(p)$, since it appears that a sound CM correction for that quantity has been neglected so far in various treatments employing nontranslationally invariant wave functions, except those for the HO potential, in certain studies [13].

In the following, we discuss the single particle wave functions used in the present treatment, and subsequently, the expressions for the (point proton) elastic form factor in Born approximation and the nucleon momentum distribution $\eta(p)$, corrected for the CM motion. Finally, we give and discuss the basic numerical results obtained.

The single-particle (nontranslationally invariant) wave functions, used here, are the following:

Firstly, that for the ground state in the «modified harmonic oscillator» (MHO) potential

$$V(r) = -V_0 + (\hbar^2/2mb^4)r^2 + B/r^2 \quad , \qquad V_0 > 0, \quad B > 0. \tag{1}$$

For $B = 0$ this reduces to the usual HO potential.

The energy eigenvalues and eigenfunctions for this potential are given analytically. The (normalized) 1s-radial ($\phi(r) = rR(r)$) state needed for ${}^4_2$He is: $\phi_{00}^{\mathrm{MHO}}(r) = [2/b\Gamma(2\lambda_0 + 1/2)]^{1/2}(r/b)^{2\lambda_0}\mathrm{e}^{-r^2/2b^2}$, $\lambda_0 = (1/4)[1 + (1 + (8mB/\hbar^2))^{1/2}]$. Analytic expressions have also been given for the single (point) particle (the «body») density $\rho_s(r)$, the corresponding elastic form factor $F_s(q)$ and the nucleon momentum distribution $\eta_s(p)$ [10].

Secondly, the Radhakant, Khadkikar and Banerjee [14] (RKB) normalized radial wave function for the lowest single-particle state of ${}^4_2$He:

$$\phi^{\mathrm{RKB}}(r) = (1 + \beta^2)^{-1/2}(\phi_{00}(r) + \beta\phi_{10}(r)), \tag{2}$$

where ϕ_{00} and ϕ_{10} are the normalized HO radial orbitals with parameter b_{H} for the states with $n = 0$, $l = 0$ and $n = 1$, $l = 0$, respectively, and β is the mixing parameter. This wave function leads to simple analytic expressions for the quantities of interest, such as:

$$\eta_s(p) = \pi^{-3/2}b_{\mathrm{H}}^3(1 + \beta^2)^{-1}[1 + \beta(3/2)^{1/2}(-1 + (2/3)(b_{\mathrm{H}}p)^2)]^2\mathrm{e}^{-(b_{\mathrm{H}}p)^2}. \tag{3}$$

In the «fixed CM approximation», the Ernst, Shakin and Thaler (EST) prescription [14–16], the nuclear many-body wave function is written

$$\Psi = (2\pi)^{3/2}|\mathbf{P})\Phi_{\mathrm{intr}}^{\mathrm{EST}}. \tag{4}$$

A round bracket is used to represent a vector in the space of the CM coordinate only, so that, e. g., $|\mathbf{P})$ means the eigenstate of total momentum operator $\hat{\mathbf{P}}$. The EST intrinsic wave function

$$\Phi_{\mathrm{intr}}^{\mathrm{EST}} = (\mathbf{R} = 0|\Phi_s\rangle[\langle\Phi_s|\mathbf{R} = 0)(\mathbf{R} = 0|\Phi_s\rangle]^{-1/2} \tag{5}$$

is constructed from an arbitrary (in general, nontranslationally invariant) wave function Φ_s, by requiring that the CM coordinate $\mathbf{R}$ be equal to zero.

Use of a Slater determinant for Φ_s leads to the following expression of $F(q)$ — the elastic form factor for ${}^4_2$He corrected for the CM motion [14, 16]:

$$F(q) = \int F_s(|\mathbf{q} + \mathbf{u}|)F_s^3(u)d\mathbf{u} \,/ \int F_s^4(u)d\mathbf{u}. \tag{6}$$

This expression may be used to calculate $F(q)$ numerically. A convenient way to do this with the RKB wave function has been considered and such a calculation is reduced to one-dimensional integrals from 0 to 1 of polynomials and of other well-known functions of suitable arguments.

Pertaining to $\eta(\mathbf{p})$ with the «fixed CM approximation»:

$$\eta(\mathbf{p}) = \langle \Phi_s | (2\pi)^3 \delta(\mathbf{R}) \delta(\mathbf{p}_1 - \mathbf{P}/A - \mathbf{p}) | \Phi_s \rangle / \langle \Phi_s | (2\pi)^3 \delta(\mathbf{R}) | \Phi_s \rangle, \tag{7}$$

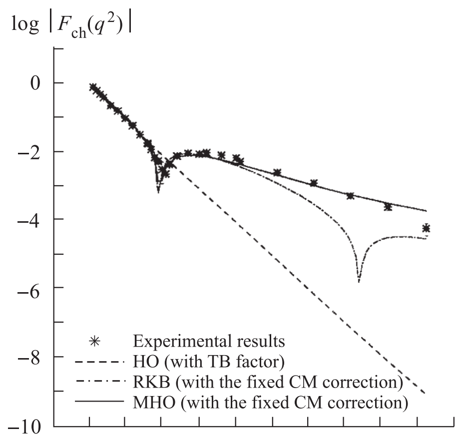
one can show that for ${}^4_2$He with the RKB wave function, $\eta(p)$ can be calculated again semianalytically (after a lengthy procedure) by reducing it to one-dimensional integrals of structure similar to those derived for $F(q)$.

In view of the valuable advantages of $\Phi^{\rm RKB}(r)$, the approximation of $\Phi^{\rm MHO}$ by $\Phi^{\rm RKB}$ has been investigated. This was achieved through a best approximation in the mean, that is by requiring $\epsilon_2 = \int |\phi^{\rm MHO}(r) - \phi^{\rm RKB}(r)|^2 dr$ to be minimum, which leads to expressions of β and $b_{\rm H}$ in terms of b and λ_0.

We give first the results of the charge form factor of ${}^4_2$He with the MHO model and the RKB wave function by fitting to the known experimental values [6] using the «fixed CM approximation» and considering for the finite proton size $f_p(q)$ the Chandra and Sauer prescription [17]. The two parameters in each case are determined by least-squares fit. The results for $\log|F_{\rm ch}(q^2)|$ are shown in Fig. 1. The most satisfactory fit is with the MHO model. Also the fit with the RKB wave function is very good apart from the higher q values, where a second diffraction minimum is predicted at $q^2 \simeq 37$ fm^{-2}, which does not seem to be indicated there by the data. The results with the HO and Tassie and Barker (TB) factor are shown as well. The quality of the fit is very poor in this case.

Having determined the parameters in the described way, the values of $\eta(p)$ were calculated with the $\phi^{\rm RKB}(r)$ wave function without and with CM correction (see Fig. 2 for the plots of $\log \eta(p)$). The parameters in the $\Phi^{\rm RKB}$ used were determined directly from the fit of $F_{\rm ch}(q)$. In the same figure the corresponding quantities for the HO potential, namely:

$$\log \eta^{\rm HO}(p) = \log \frac{b_{\rm HO}^3}{\pi\sqrt{\pi}} \exp(-b_{\rm HO}^2 p^2) \tag{8}$$



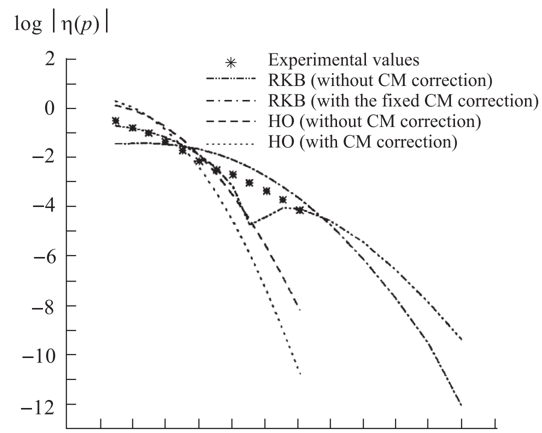


Fig. 1. The $\log|F_{\rm ch}(q^2)|$ versus q^2 for various cases. For the abbreviations see the text

Fig. 2. The $\log \eta(p)$ versus p for various cases. For the abbreviations see the text

and

$$\log \eta_{\rm cm}^{\rm HO}(p) = \log \left(\frac{4}{3}\right)^{3/2} \frac{b_{\rm HO}^3}{\pi\sqrt{\pi}} \exp \left[-\frac{4}{3} b_{\rm HO}^2 p^2\right], \tag{9}$$

are also shown for comparison. It is seen that a considerable improvement is mostly observed in comparison with the HO case if we consider the «experimental points» (which are model dependent). It should be also noted that if $\phi^{\rm RKB}$ is determined by the minimization of ϵ_2, it approximates very well the $\phi^{\rm MHO}$ since the minimum value of ϵ_2 is very small ($\epsilon_2 \simeq 0{,}00163$).

In conclusion, the present analysis shows that the approximate treatment of the CM motion with the fixed CM method for the $\eta(p)$ of ^{4}He is feasible, through the RKB wave function, although quite cumbersome.

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