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LEPTON NUMBER VIOLATING PROCESSES AND MAJORANA NEUTRINOS

C.Dib, V.Gribanov[1]*, S.Kovalenko*[1]*, I.Schmidt*

Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

We discuss some generic properties of lepton number violating ($\not{L}$) processes and their relation to different entries of the Majorana neutrino mass matrix. Present and near future experiments searching for these processes, except the neutrinoless double beta decay, are unable to probe light (eV mass region) and heavy (hundred GeV mass region) neutrinos. On the other hand, due to the effect of a resonant enhancement, some of $\not{L}$ decays can be very sensitive to the intermediate-mass neutrinos with typical masses in the hundred MeV region. These neutrinos may appear as admixtures of the three active and an arbitrary number of sterile neutrino species. We analyze the experimental constraints on these massive neutrino states and discuss their possible cosmological and astrophysical implications.

Обсуждаются некоторые общие свойства процессов с нарушением закона сохранения лептонного числа и их связь с различными параметрами майорановской матрицы смешивания нейтрино. Современные эксперименты и эксперименты ближайшего будущего, нацеленные на поиск таких процессов, за исключением безнейтринного двойного бета-распада, не способны исследовать область масс легких (порядка нескольких эВ) и тяжелых (порядка сотен ГэВ) нейтрино. С другой стороны, благодаря резонансному усилению некоторые из нарушающих лептонное число распадов могут оказаться очень чувствительными к промежуточным значениям масс нейтрино (порядка нескольких сотен МэВ). Эти нейтрино могут представлять собой смесь трех активных типов нейтрино и произвольного числа стерильных нейтрино. Проанализированы экспериментальные ограничения на такие массивные состояния нейтрино, и обсуждаются их возможные космологические и астрофизические следствия.

INTRODUCTION

Recent evidence for neutrino oscillations leaves nearly no room for doubts that neutrinos are massive particles. After all, this point of view is becoming conventional. Solar neutrino deficit, atmospheric neutrino anomaly and results of the LSND neutrino oscillation experiment all can be explained in terms of neutrino oscillations implying non-zero neutrino masses and mixings [1]. On the theoretical side [2] there are also many indications in favour of non-zero neutrino masses following from almost all phenomenologically viable models of the physics beyond the Standard Model (SM). These models typically predict Majorana-type neutrino masses, suggesting that neutrinos are truly neutral particles.

The neutrino oscillation searches fix both the neutrino mass square difference $\delta m_{ij}^2 = m_i^2 - m_j^2$ and the neutrino mixing angles, leaving the overall mass scale and the CP-phases arbitrary. Since the latter has no effect on neutrino oscillations, the important question of whether neutrinos are Majorana or Dirac particles cannot be answered by these searches.

[1]On leave from the Joint Institute for Nuclear Research, Dubna.

Majorana masses violate total lepton number conservation by two units $\Delta L = 2$. Thus lepton number violating ($\not{L}$) processes represent a most appropriate tool to address the question of the Majorana nature of neutrinos. A celebrated example of $\not{L}$ process, most advanced experimentally and theoretically, is the neutrinoless nuclear double beta ($0\nu\beta\beta$) decay (for a review see [3, 4]). The $0\nu\beta\beta$ experiments achieved unprecedented sensitivity to the so-called effective Majorana neutrino mass $\langle m_\nu \rangle_{ee}$ [5], which in the presence of only light neutrinos coincides with the entry of the Majorana neutrino mass matrix $\langle m_\nu \rangle_{ee} = M^{(\nu)}_{ee}$. One may hope to infer information on the other entries from the other $\not{L}$ processes. Many of them have been studied in the literature in this respect from both theoretical and experimental sides. Among them there are the decay $K^+ \to \mu^+\mu^+\pi^-$ [6–11], the nuclear muon-to-positron [12] or to-antimuon [13] conversion, tri-muonium production in neutrino muon scattering [14], and the process $e^+p \to \bar{\nu} l_1^+ l_2^+ X$ relevant for HERA [15], as well as direct production of heavy Majorana neutrinos at various colliders [16]. Unfortunately sensitivities of the current experiments searching for these processes are much less than in the case of $0\nu\beta\beta$ decay. The analysis made in the literature [8, 17] leads to the conclusion that if these processes are mediated by Majorana neutrino exchange then, except $0\nu\beta\beta$ decay, they can hardly be observed experimentally. This analysis relies on the current neutrino oscillation data, and on certain assumptions related to the neutrino mass matrix. We will show that, despite the above conclusion being true for contributions of the neutrino states much lighter or much heavier than the typical energy of a certain $\not{L}$ process, there are still special windows in the neutrino sector which can be efficiently probed by searching for some of these processes. For the $K^+ \to \mu^+\mu^+\pi^-$ decay this window lies in the neutrino mass range $245 \leq m_{\nu_j} \leq 389$ MeV, where the s-channel neutrino contribution to the $K^+ \to \mu^+\mu^+\pi^-$ decay is resonantly enhanced, therefore making this decay very sensitive to neutrinos in this mass domain. If neutrinos with these masses exist, then from the present experimental data we can extract stringent limits on their mixing with ν_μ.

Recently some phenomenological, cosmological and astrophysical issues of the intermediate-mass neutrinos in the MeV mass region have been addressed [18]. This was stimulated by the attempts of explanation of the KARMEN anomaly in terms of these massive neutrino states [19]. Although recent data of the KARMEN collaboration [20] have not confirmed this anomaly, the possible existence of these massive neutrinos remains open, motivated by the idea of sterile neutrinos ν_s required for the explanation of all the neutrino oscillation data, including the LSND results. The sterile species ν_s may mix with the active ones $\nu_{e,\mu,\tau}$ to form massive states with *a priori* arbitrary masses. Their existence is the subject of experimental searches as well as cosmological and astrophysical constraints.

The paper is organized as follows. In section 1 we discuss a model with sterile neutrinos and possible spectrum of massive neutrino states. Section 2 is devoted to some general features of constraints on the neutrino mass matrix, derivable from the $\not{L}$ processes. In section 3 we give the theoretical framework for the $K^+ \to \mu^+\mu^+\pi^-$ decay, and then in section 4 discuss expected rates of this and other $\not{L}$ processes in the light of the present neutrino observations. In section 5 we study the possible contribution of massive neutrinos in the resonant domain of the $K^+ \to \mu^+\mu^+\pi^-$ decay and derive the constraints on the mixing of these neutrinos with ν_μ. Astrophysical and cosmological implications of these massive hundred-MeV neutrinos are shortly addressed.

1. MAJORANA NEUTRINO MASS MATRIX AND NEUTRINO-COUNTING EXPERIMENTS

Consider an extension of the SM with the three left-handed weak doublet neutrinos $\nu'_{Li} = (\nu'_{Le}, \nu'_{L\mu}, \nu'_{L\tau})$ and n species of the SM singlet right-handed neutrinos $\nu'_{Ri} = (\nu'_{R1}, ..., \nu'_{Rn})$. The general mass term for this set of fields can be written as

$$-\frac{1}{2}\overline{\nu'}\mathcal{M}^{(\nu)}\nu'^{c} + \text{H.c.} = -\frac{1}{2}(\bar{\nu}'_{_L}, \overline{\nu'^{c}_{_R}})\begin{pmatrix} \mathcal{M}_L & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{pmatrix}\begin{pmatrix} \nu'^{c}_{_L} \\ \nu'_{_R} \end{pmatrix} + \text{H.c.} = $$
$$= -\frac{1}{2}\sum_{i=1}^{3+n} m_{\nu i}\overline{\nu^c}_i\nu_i + \text{H.c.} \tag{1}$$

Here $\mathcal{M}_L$ and $\mathcal{M}_R$ are 3×3 and $n\times n$ symmetric Majorana mass matrices, M_D is $3\times n$ Dirac-type matrix. Rotating the neutrino mass matrix by the unitary transformation

$$U^T\mathcal{M}^{(\nu)}U = \text{diag}\,\{m_{\nu i}\} \tag{2}$$

to the diagonal form, we end up with $n+3$ Majorana neutrinos $\nu_i = U^*_{ki}\nu'_k$ with the masses $m_{\nu i}$. In special cases there may appear among them pairs with masses degenerate in absolute values. Each of these pairs can be collected into a Dirac neutrino field. This situation corresponds to conservation of certain lepton numbers assigned to these Dirac fields.

The considered generic model must contain at least three observable light neutrinos while the other states may be of arbitrary mass. In particular, they may include hundred-MeV neutrinos, which we will consider in section 5. Presence or absence of these neutrino states is a question for experimental searches.

Let us point out that the presence of more than three light neutrinos is not excluded by the neutrino-counting LEP experiments measuring the invisible Z-boson width Γ_{inv}. Actually it counts not the number of light neutrinos but the number of active flavours. To see this let us write down the $Z\nu\nu$ interaction term as

$$Z^\mu\sum_{\alpha=e,\mu,\tau}\overline{\nu'_\alpha}\gamma_\mu\nu'_\alpha = Z^\mu\sum_{\alpha=e,\mu,\tau}\sum_{m,n=1}^{n+3}U_{\alpha m}U^*_{\alpha n}\overline{\nu_n}\gamma_\mu\nu_m \equiv \sum_{m,n=1}^{n+3}\mathcal{P}_{mn}\overline{\nu_n}\gamma_\mu\nu_m, \tag{3}$$

where the last two expressions are written in the mass eigenstate basis. For the case of only three massive neutrinos one has $\mathcal{P}_{mn} = \delta_{mn}$ as a consequence of unitarity of $U_{\alpha n}$. In general $\mathcal{P}_{mn}$ is not a diagonal matrix and flavor-changing neutral currents in the neutrino sector become possible at tree level. However, if all the neutrinos are significantly lighter than Z-boson with the mass M_Z, their contribution to the invisible Z-boson width is

$$\Gamma_{\text{inv}} = \sum_{m,n=1}^{n+3}|\mathcal{P}_{mn}|^2\,\Gamma_\nu^{\text{SM}} = \Gamma_\nu^{\text{SM}}\sum_{\alpha,\beta=e,\mu,\tau}\delta_{\alpha\beta}\delta_{\alpha\beta} = 3\Gamma_\nu^{\text{SM}}, \tag{4}$$

where Γ_ν^{SM} is the SM prediction for the partial Z-decay width to one pair of light neutrinos. This chain of equalities follows again from the unitarity of $U_{\alpha n}$. Thus, irresspective of the number of light neutrinos with masses $m_\nu \ll M_Z/2$ the factor 3 in the last step counts the number of weak doublet neutrinos. This conclusion changes in the presence of heavy neutrinos N with masses $M_N > M_Z/2$ which do not contribute to $\Gamma_{\rm inv}$. In this case the unitarity condition is no longer valid and the factor 3 changes to a smaller value.

Having these arguments in mind, we introduce in section 5 neutrino states with masses in the hundred MeV region. These states can be composed of sterile and active neutrino flavors as described in the present section.

2. CONSTRAINTS FROM $\not{L}$ PROCESSES. GENERAL PATTERN

Let us examine some generic features of those constraints on the Majorana neutrino mass matrix which can be derived from $\not{L}$ processes.

Majorana neutrino mass terms in (1) violate lepton number conservation by two units $\Delta L = 2$ and thus can induce $\not{L}$ processes with $\Delta L = 2n$. W-boson loops can be used to convert $\Delta L \neq 0$ from the neutrino to the charged lepton sector. Therefore $\not{L}$ processes offer one of the most straightforward ways to test the Majorana nature of neutrinos and extract information on the neutrino mass matrix $\mathcal{M}^{(\nu)}$.

Note that at energies below the new physics thresholds only $\Delta L = 2n$ can be realized provided that the baryon number is conserved ($\Delta B = 0$). This is a simple consequence of the Lorentz invariance and the fact that the spinor SM fields are represented only by leptons and quarks. To prevent $\Delta B \neq 0$ one has to contract the Lorentz indices of the external lepton fields only with each other without involving quark fields. Thus only $\not{L}$ processes with even number of external leptons, i.e., $\Delta L = 2n$ processes, can proceed at these energies. This means that any $\not{L}$ process in this energy domain is related to the Majorana neutrino mass receiving contributions from virtual Majorana neutrino exchange. Certainly only $\Delta L = 2$ processes are of practical interest. The Majorana neutrino exchange contribution to the rate of a $\Delta L = 2$ process with two external leptons $l_i l_j$ can be written schematically as

$$\Gamma_{ij} = c \int_{s_1^-}^{s_1^+} ds \sum_k \left| \frac{U_{ik} U_{jk} m_{\nu k}}{s \pm m_{\nu k}^2} \right|^2 G(s/m_0^2) \; + ... \; . \tag{5}$$

The function in the absolute value brackets originates from the $\not{L}$ Majorana neutrino propagator $\langle 0|T(\nu(x)\nu^T(y))|0\rangle$. Since the only source of $\not{L}$ in the neutrino sector is given by the Majorana neutrino masses $m_{\nu k}$ the decay rate (5) vanishes when $m_{\nu k} = 0$. The ellipsis in this equation denotes terms whose explicit form is irrelevant for the present general discussion. In (5) $G(z)$ is a smooth positively definite smearing function which depends on a particular process, $s_1^{\pm}$ are the limits of integration determined by the masses of the external particles involved in the process. The sign $+(-)$ in the denominator corresponds to the t-(s-)channel neutrino exchange. For the s-channel contribution the total neutrino width Γ_ν has to be taken into account if masses in the resonant region of the s-channel exchange $s_1^- \leq m_\nu \leq s_1^+$ are considered. The non-zero neutrino decay widths $\Gamma_{\nu k}$ can be introduced via the substitution $m_{\nu k} \to m_{\nu k} - (i/2)\Gamma_{\nu k}$.

From the form of Γ_{ij} one can infer that, as a function of neutrino masses $m_{\nu k}$, it has a maximal value $\Gamma_{ij}^{\max}$ for certain configuration of neutrino masses. This observation leads to the conclusion that the sensitivity $\Gamma^{\exp}$ of a concrete experiment searching for $\not{L}$ must satisfy the condition $\Gamma^{\exp} \leq \Gamma_{ij}^{\max}$, otherwise no information on neutrino contribution is derivable.

The experiment, having passed this condition, provides certain constraints. Assume that neutrino fields can be divided into light ν_i and heavy N_i states with masses $m_{\nu i} \ll \sqrt{s_1^-}$ and $\sqrt{s_1^+} \ll M_N$ respectively. Then in this «Light-Heavy» neutrino scenario, equation (5) can be approximately rewritten as

$$\Gamma_{ij} = |\langle m_\nu \rangle_{ij}|^2 m_0^{-1} \mathcal{A}_\nu + \left| \left\langle \frac{1}{M_N} \right\rangle_{ij} \right|^2 m_0^3 \mathcal{A}_N \pm \mathrm{Re} \left[\langle m_\nu \rangle_{ij} \left\langle \frac{1}{M_N} \right\rangle_{ij} \right] m_0 \mathcal{A}_{\nu N}, \qquad (6)$$

where the dimensionless coefficients $\mathcal{A}_i$ can be obtained for a concrete process from the equation such as (5). In the above equations $m_0 \sim \sqrt{s_1^\pm}$ is a typical scale of the $\not{L}$ process under consideration.

The average masses in (6) are determined in the standard way:

$$\langle m_\nu \rangle_{ij} = \sum_{k=\text{light}} U_{ik} U_{jk} m_{\nu k}, \quad \left\langle \frac{1}{M_N} \right\rangle_{ij} = \sum_{k=\text{heavy}} \frac{U_{ik} U_{jk}}{M_{Nk}}. \qquad (7)$$

Summation over light and heavy neutrinos implies masses $m_{\nu k} \ll \sqrt{s_1^-}$ and $M_{Nk} \gg \sqrt{s_1^+}$ respectively.

An experimental constraint on the rate of a certain $\not{L}$ process derived from its non-observation at the experimental sensitivity $\Gamma^{\exp}$ can be translated with the aid of (6) into the bounds:

$$\Gamma_{ij} \leq \Gamma^{\exp} \longrightarrow \begin{cases} |\langle m_\nu \rangle_{ij}| \leq \exp \nu \equiv \sqrt{m_0 \Gamma^{\exp} / \mathcal{A}_\nu} \\ \left| \left\langle \frac{1}{M_N} \right\rangle_{ij} \right| \leq \exp N \equiv \sqrt{\Gamma^{\exp} / (m_0^3 \mathcal{A}_\nu)} \end{cases}. \qquad (8)$$

However this is only possible if the experimental sensitivity satisfies the consistency conditions

$$\exp \nu \ll \sqrt{s_1^-} \sim m_0, \quad \exp N^{-1} \gg \sqrt{s_1^+} \sim m_0. \qquad (9)$$

Otherwise experimental data cannot be translated into the constraints (8) as was done, for instance, in [10, 15, 22]. If the consistency conditions (9) are not satisfied, one has to use the initial equation (5).

The following remark is in order. If all the neutrino states are light, satisfying $m_{\nu k} \ll \sqrt{s_1}$, then the following relation takes place:

$$\langle m_\nu \rangle_{ij} = \mathcal{M}_{ij}^{(\nu)}. \qquad (10)$$

This relation is not true if there are heavy neutrino states with masses not satisfying the condition $m_\nu \ll \sqrt{s_1^-}$. According to (7), they do not contribute to $\langle m_\nu \rangle_{ij}$ measured in some $\not{L}$ process. Therefore a concrete $\not{L}$ process can give direct information on the entry $\mathcal{M}_{ij}^{(\nu)}$ of the Majorana neutrino mass matrix only under the assumption that all the neutrino masses are small compared to a typical scale of this process $m_{\nu k} \ll m_0 \sim \sqrt{s_1^\pm}$, or assuming that the heavy states are sterile. From this point of view $\not{L}$ processes with larger typical scales m_0 are preferable.

In the subsequent sections we will study concrete $\not{L}$ processes from the viewpoint of their ability to probe neutrino properties. We will consider the conventional neutrino spectrum with three light neutrinos plus a number of kinematically unattainable heavy states as well as a model with additional intermediate-mass neutrinos in the hundred MeV domain.

3. K-MESON NEUTRINOLESS DOUBLE MUON DECAY

Here we shortly outline the theoretical framework for the $K^+ \to \mu^+\mu^+\pi^-$ decay. Recently this $\not{L}$ process attracted attention [9, 11] as a possible probe of the neutrino sector complimentary to other known processes.

In the SM extension with Majorana neutrinos there are two lowest order diagrams, shown in Fig. 1, which contribute to the $K^+ \to \mu^+\mu^+\pi^-$ decay. We concentrate on the s-channel neutrino exchange diagram in Fig. 1,*a*, which plays a central role in our analysis. The t-channel diagram in Fig. 1,*b* requires in general a detailed hadronic structure calculation. In [7] this diagram was evaluated in the Bethe–Salpeter approach and shown to be an order of magnitude smaller than the diagram in Fig. 1,*a*, for light and intermediate-mass neutrinos.

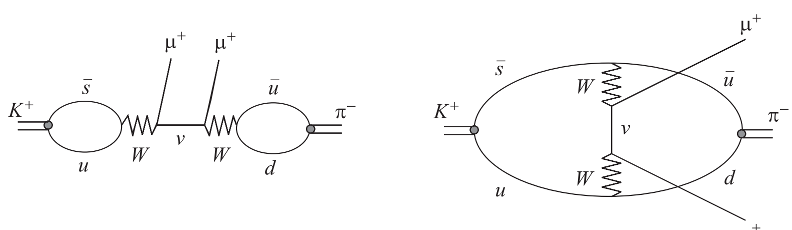

Fig. 1. The lowest-order diagrams contributing to the $K^+ \to \mu^+\mu^+\pi^-$ decay

The contribution from the factorizable s-channel diagram in Fig. 1,*a* can be calculated in a straightforward way, without referring to any hadronic structure model. A final result for the $K^+ \to \mu^+\mu^+\pi^-$ decay rate is given by [11]

$$\Gamma(K^+ \to \mu^+\mu^+\pi^-) = c \int_{s_1^-}^{s_1^+} ds_1 \left| \sum_k \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \right|^2 G\left(\frac{s_1}{m_K^2}\right) +$$

$$+ 2\frac{c}{m_K^2} \mathrm{Re} \sum_{k,n} \left[\int_{s_1^-}^{s_1^+} ds_1 \frac{U_{\mu k}^2 m_{\nu k}}{s_1 - m_{\nu k}^2} \int_{s_2^-}^{s_2^+} ds_2 \left(\frac{U_{\mu n}^2 m_{\nu n}}{s_2 - m_{\nu n}^2} \right)^* H\left(\frac{s_1}{m_K^2}, \frac{s_2}{m_K^2}\right) \right]. \tag{11}$$

The unitary mixing matrix U_{ij} relates weak ν' and mass ν neutrino eigenstates ($\nu_i' = U_{ij}\nu_j$). The numerical constant in (11) is

$$c = (G_F^4/32)(\pi)^{-3} f_\pi^2 f_K^2 m_K^5 |V_{ud}|^2 |V_{us}|^2, \tag{12}$$

where $f_{K} = 1.28\ f_{\pi}$, $f_{\pi} = 0.668\ m_\pi$ and $m_K = 494$ MeV is the K-meson mass. The functions $G(z)$ and $H(z_1, z_2)$ in (11) after the phase space integration can be written in an explicit algebraic form

$$G(z) = \frac{\phi(z)}{z^2}\left[h_{+-}(z)h_{--}(z) - x_\pi^2 h_{-+}(z)\right]\left[x_\mu^2 + z - (x_\mu^2 - z)^2\right],$$
$$H(z_1, z_2) = h_{--}(z_1)h_{--}(z_2) + x_\pi^2[r_+(z_1z_2) - \\ - x_\mu^2 t(z_1, z_2, 1)] - r_-(z_1z_2)t(z_1, z_2, x_\mu). \tag{13}$$

Here we defined $x_i = m_i/m_{K}$ and introduced the functions

$$h_{\pm\pm}(z) = z \pm x_\pi^2 \pm x_\mu^2, \quad r_\pm(z_1z_2) = z_1z_2 - x_\pi^2 \pm x_\mu^4,$$
$$t(z_1, z_2, z_3) = z_1 + z_2 - 2z_3^2, \quad \phi(z) = \lambda^{1/2}(1, x_\mu^2, z)\lambda^{1/2}(z, x_\mu^2, x_\pi^2) \tag{14}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$.

With $y = s_1/m_{K}^2$ the integration limits in (11) are

$$s_1^- = m_{K}^2(x_\pi + x_\mu)^2, \quad s_1^+ = m_{K}^2(1 - x_\mu)^2,$$
$$s_2^\pm = \frac{m_{K}^2}{2y}\left[2y(1 + x_\mu^2) - (1 + y - x_\mu^2)h_{-+}(y) \pm \phi(y)\right]. \tag{15}$$

Assuming that neutrinos can be separated into light ν_k and heavy N_k states, with masses $m_{\nu i} \ll \sqrt{s_1^-}$ and $\sqrt{s_1^+} \ll M_{Nk}$, we can rewrite (11) in the approximate form (6) with the dimensionless coefficients

$$\mathcal{A}_\nu = 4.0 \cdot 10^{-31}, \quad \mathcal{A}_N = 7.0 \cdot 10^{-32}, \quad \mathcal{A}_{\nu N} = 1.7 \cdot 10^{-31}. \tag{16}$$

With these numbers we can estimate the current upper bound on the $K^+ \to \mu^+\mu^+\pi^-$ decay rate from the experimental data on other processes.

4. «LIGHT-HEAVY» NEUTRINO SCENARIO

Here we assume that all the neutrino mass eigenstates can be divided into very light ν_i and very heavy N_i states with masses, respectively, much smaller and much larger than the characteristic energy scale m_0 of the studied $\not{L}$ process. Let us consider in this «Light-Heavy» neutrino scenario several typical examples of $\not{L}$ processes and estimate their ability to constrain the average masses $\langle m_\nu\rangle_{ij}$, $\langle 1/M_N\rangle_{ij}$ as well as their possible rates.

At present the highest experimental sensitivity to the Majorana neutrino contribution has been achieved in the neutrinoless double beta decay ($0\nu\beta\beta$) $(A, Z) \to (A, Z+2) + 2e^-$. A typical scale of this process is set by the nucleon Fermi momentum $m_0 \sim p_{F} \approx 100\,$MeV. The current constraints from the $0\nu\beta\beta$ decay are [4, 5]

$$|\langle m_\nu\rangle_{ee}| \le 0.2 \div 0.6\ \text{ eV}, \quad \left|\left\langle \frac{1}{M_N}\right\rangle_{ee}\right| \lesssim \left(9.0 \cdot 10^7\ \text{GeV}\right)^{-1}. \tag{17}$$

The uncertainty relates to the uncertainties in the nuclear matrix elements and treatment of the background conditions. The first constraint in (17), assuming that all the neutrinos are much lighter than 100 MeV, provides a direct constraint on the $\mathcal{M}_{ee}^{(\nu)}$ entry of the Majorana neutrino mass matrix. Evidently these constraints satisfy the consistency conditions (9).

Experiments searching for the other $\not{L}$ processes have not yet reached enough sensitivity to establish meaningful constraints directly on the neutrino mass matrix elements.

For instance, experiments on the muon-to-positron nuclear conversion $\mu^- + (A, Z) \to e^+ + (A, Z-2)$ in ^{48}Ti give at current sensitivity the following upper bound on the branching ratio [21]

$$\mathcal{R}(\mu^- \to e^+) = \frac{\Gamma(\mathrm{Ti} + \mu^- \to \mathrm{Ca} + e^+)}{\Gamma(\mathrm{Ti} + \mu^- \to \mathrm{Sc} + \nu_\mu)} \le 1.7 \cdot 10^{-12} \quad (90\,\%\,\mathrm{C.L.}). \tag{18}$$

Assuming that all the neutrinos are much lighter than the typical energy scale $m_0 \sim m_\mu = 105\,\mathrm{MeV}$ of this reaction, one finds the bound

$$|\langle m_\nu \rangle_{\mu e}| \le 17\,(80)\ \mathrm{MeV} \tag{19}$$

for the proton pairs of the final nucleus in the singlet (triplet) state [12]. These constraints are marginal from the viewpoint of the consistency condition (9).

Direct searches for the $K^+ \to \mu^+\mu^+\pi^-$ decay by the E865 experiment at BNL [23] give

$$\mathcal{R}_{\mu\mu} = \frac{\Gamma(K^+ \to \mu^+\mu^+\pi^-)}{\Gamma(K^+ \to \mathrm{all})} \le 3.0 \cdot 10^{-9} \quad (90\,\%\,\mathrm{C.L.})\,. \tag{20}$$

Applying the approximate equation (6) to this case, one gets the limit

$$|\langle m_\nu \rangle_{\mu\mu}| \le 500\ \mathrm{GeV}, \tag{21}$$

which makes no sense because it does not satisfy the condition (9) with the typical energy scale of this process $m_0 \sim m_K = 494\,\mathrm{MeV}$. Thus the approximate equation (6) is not applicable to the present experimental situation for the $K^+ \to \mu^+\mu^+\pi^-$ searches. A similar picture holds for all the known $\not{L}$ processes searched for in various experiments (for other examples see [17]).

Viewing these processes from the standpoint of neutrino observations, one finds that, except for the $0\nu\beta\beta$ decay, they have very small rates in the «Light-Heavy» neutrino scenario.

Atmospheric and solar neutrino oscillation data demonstrate $\delta m^2 \ll (1\,\mathrm{eV})^2$, suggesting that all the neutrino mass eigenstates are approximately degenerate at the 1 eV scale [24]. This observation in combination with the tritium beta decay endpoint allows one to set upper bounds on masses of all the three neutrinos [24] $m_{e,\mu,\tau} \le 3\,\mathrm{eV}$. Thus in the three-neutrino scenario one derives

$$|\langle m_\nu \rangle_{ij}| \le 9\ \mathrm{eV} \qquad \text{for} \qquad i, j = e, \mu, \tau. \tag{22}$$

This is much lower than the existing constraints on this quantity from $\not{L}$ processes, except for the $0\nu\beta\beta$ decay, which gives a significantly more stringent upper bound (17). With the constraint (22) we can predict the rate of various $\not{L}$ processes.

Let us substitute the upper bound (22) into equation (6) written for the $K^+ \to \mu^+\mu^+\pi^-$ decay with coefficients given in (16). This gives rise to the following extremely small branching ratio:

$$\mathcal{R}_{\mu\mu} = \frac{\Gamma(K^+ \to \mu^+\mu^+\pi^-)}{\Gamma(K^+ \to \text{all})} \leq 3.0 \cdot 10^{-30} \quad \text{(3 light neutrino scenario)}. \tag{23}$$

Assume there exist, in addition, heavy neutrinos N with their masses in the GeV region. Using the current LEP limit on heavy unstable neutral leptons $M_N \geq 54.4$ GeV [25], we get

$$|\langle M_N^{-1} \rangle_{\mu\mu}| \leq n\,(54.4\ \text{GeV})^{-1}\,, \tag{24}$$

where n is the number of heavy neutrinos.

This limit, being substituted in (6) together with the limit (22), results in the upper bound

$$\mathcal{R}_{\mu\mu} \leq 1.0 \cdot 10^{-19} \quad \text{(3 light} + 1 \text{ heavy neutrino scenario)}. \tag{25}$$

Comparison of the theoretical predictions (23) and (25) with the experimental bound in (20) clearly shows that both cases are far from being ever detected. A similar conclusion is true for the other $\not{L}$ processes except the $0\nu\beta\beta$ decay.

On the other hand, experimental observation of these processes at larger rates would suggest some new physics beyond the SM, or, as we will see for the case of the $K^+ \to \mu^+\mu^+\pi^-$ decay, the presence of an extra neutrino state ν_j with mass in the hundred MeV domain. As we discussed in section 1, the extra massive neutrino states ν_j can appear as a result of mixing of the three active neutrinos with certain number of sterile neutrinos. These massive neutrinos are at present searched for in many experiments [27]. The ν_j states would manifest themselves as peaks in differential rates of various processes, and can give rise to significant enhancement of the total rate if their masses lie in an appropriate region.

5. HUNDRED-MeV NEUTRINOS IN THE $K^+ \to \mu^+\mu^+\pi^-$ DECAY

Assume there exists a massive Majorana neutrino ν_j with the mass m_j

$$\sqrt{s_1^-} \approx 245\ \text{MeV} \leq m_j \leq \sqrt{s_1^+} \approx 389\ \text{MeV}. \tag{26}$$

In this mass range the s-channel neutrino exchange diagram in Fig. 1,*a* absolutely dominates over the t-channel diagram in Fig. 1,*b*, irrespective of hadronic structure. Here the diagram in Fig. 1,*a* blows up because the integrand of the first term in (11) has a non-integrable singularity at $s = m_j^2$. Therefore, in this resonant domain the total ν_j-neutrino decay width $\Gamma_{\nu j}$ has to be taken into account. This can be done by the substitution $m_j \to m_j - (i/2)\Gamma_{\nu j}$.

The total decay width $\Gamma_{\nu j}$ of the Majorana neutrino ν_j with mass in the resonant domain (26) receives contributions from the following decay modes:

$$\nu_j \longrightarrow \left\{ \begin{array}{l} e^+\pi^-, \quad e^-\pi^+, \quad \mu^+\pi^-, \quad \mu^-\pi^+ \\ e^+e^-\nu_e^c, \quad e^+\mu^-\nu_\mu^c, \quad \mu^+e^-\nu_e^c, \quad \mu^+\mu^-\nu_\mu^c \\ e^-e^+\nu_e, \quad e^-\mu^+\nu_\mu, \quad \mu^-e^+\nu_e, \quad \mu^-\mu^+\nu_\mu \end{array} \right. . \tag{27}$$

Since $\nu_j \equiv \nu_j^c$ it can decay in both $\nu_j \to l^- X(\Delta L = 0)$ and $\nu_j \to l^+ X^c(\Delta L = 2)$ channels. Calculating partial decay rates, we obtain [11]

$$\Gamma(\nu_j \to l\pi) = |U_{lj}|^2 \frac{G_F^2}{4\pi} f_\pi^2 m_j^3 F(y_l, y_\pi) \equiv |U_{lj}|^2 \Gamma_2^{(l)}, \tag{28}$$

$$\Gamma(\nu_j \to l_1 l_2 \nu) = |U_{l_1 j}|^2 \frac{G_F^2}{192\pi^3} m_j^5 H(y_{l1}, y_{l2}) \equiv |U_{l_1 j}|^2 \Gamma_3^{l_1 l_2}, \tag{29}$$

where $y_i = m_i/m_j$ and

$$F(x,y) = \lambda^{1/2}(1, x^2, y^2)[(1+x^2)(1+x^2-y^2) - 4x^2], \tag{30}$$

$$H(x,y) = 12 \int_{z_1}^{z_2} \frac{dz}{z}(z-y^2)(1+x^2-z)\lambda^{1/2}(1,z,x^2)\lambda^{1/2}(0,y^2,z). \tag{31}$$

The integration limits are $z_1 = y_{l_2}^2,\ z_2 = (1-y_{l_1})^2$ and $F(0,0) = H(0,0) = 1$. Summing up all the decay modes in (27), one gets for the total ν_j width

$$\begin{aligned} \Gamma_{\nu j} &= 2|U_{\mu j}|^2(\Gamma_2^{(\mu)} + \Gamma_3^{(\mu e)} + \Gamma_3^{(\mu\mu)}) + 2|U_{ej}|^2(\Gamma_2^{(e)} + \Gamma_3^{(ee)} + \Gamma_3^{(e\mu)}) \equiv \\ &\equiv |U_{\mu j}|^2 \Gamma_\nu^{(\mu)} + |U_{ej}|^2 \Gamma_\nu^{(e)}. \end{aligned} \tag{32}$$

In the resonant domain (26) $\Gamma_{\nu j}$ reaches its maximum value at $m_j = \sqrt{s_1^+}$. Assuming for the moment $|U_{\mu j}| = |U_{ej}| = 1$, we estimate this maximum value to be $\Gamma_{\nu j} \approx 4.7 \cdot 10^{-10}$ MeV. Since $\Gamma_{\nu j}$ is so small in the resonant domain (26), the neutrino propagator in the first term of (11) has a very sharp maximum at $s = m_j^2$. The second term, being finite in the limit $\Gamma_{\nu j} = 0$, can be neglected in the considered case. Thus, with a good precision we obtain from (11)

$$\Gamma^{\rm res}(K^+ \to \mu^+\mu^+\pi^-) \approx c\pi G(z_0) \frac{m_j |U_{\mu j}|^4}{|U_{\mu j}|^2 \Gamma_\nu^{(\mu)} + |U_{ej}|^2 \Gamma_\nu^{(e)}} \tag{33}$$

with $z_0 = (m_j/m_K)^2$. This equation allows one to derive, from the experimental bound of (20), constraints on the ν_j neutrino mass m_j and the mixing matrix elements $U_{\mu j}, U_{ej}$ in a form of a 3-dimensional exclusion plot. However one may reasonably assume that $|U_{\mu j}| \sim |U_{ej}|$. Then from the experimental bound (20) we derive a 2-dimensional $m_j - |U_{\mu j}|^2$ exclusion plot given in Fig. 2. For comparison we also present in Fig. 2 the existing bounds taken from [26]. As shown in the figure, the experimental data on the $K^+ \to \mu^+\mu^+\pi^-$ decay exclude a region unrestricted by the other experiments. The constraints can be summarized as

$$|U_{\mu j}|^2 \leq (5.6 \pm 1) \cdot 10^{-9} \quad \text{for} \quad 245 \leq m_j \leq 385 \text{ MeV}. \tag{34}$$

The best limit $|U_{\mu j}|^2 \leq 4.6 \cdot 10^{-9}$ is achieved at $m_j \approx 300$ MeV. Note that these limits are compatible with our assumption that $|U_{\mu j}| \sim |U_{ej}|$ since in this mass domain typically $|U_{ej}|^2 \leq 10^{-9}$ [27].

The constraints from $K^+ \to \mu^+\mu^+\pi^-$ in Fig. 2 and (34) can be significantly improved in the near future by the experiments E949 at BNL and E950 at FNAL [28]. It is important

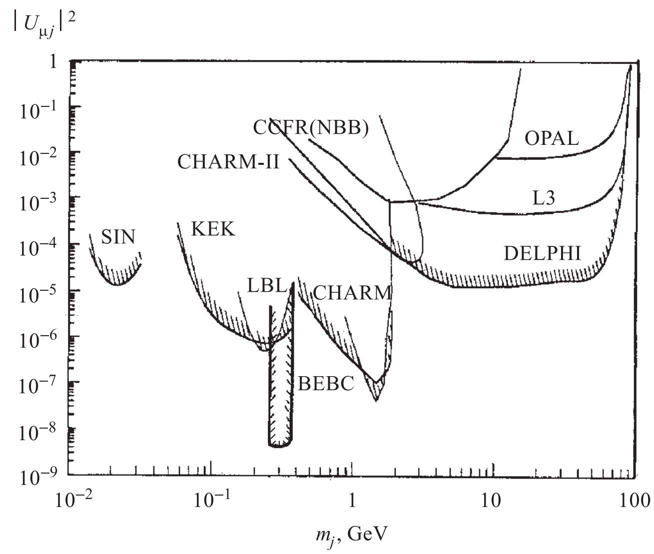

Fig. 2. Exclusion plots in the plane $|U_{\mu j}|^2 - m_j$. Here $U_{\mu j}$ and m_j are the heavy neutrino ν_j mixing matrix element to ν_μ and its mass respectively. The domains above the curves are excluded by various experiments according to the recent update in [25]. The region excluded by the $K^+ \to \mu^+\mu^+\pi^-$ decay [11] covers the interval $249 \le m_j \le 385$ MeV and extends down to $|U_{\mu j}|^2 \le 4.6 \cdot 10^{-9}$

to notice that in the resonant domain we have $\Gamma^{\rm res}(K^+ \to \mu^+\mu^+\pi^-) \sim |U|^2$, while outside $\Gamma(K^+ \to \mu^+\mu^+\pi^-) \sim |U|^4$. Thus in the resonant mass domain the $K^+ \to \mu^+\mu^+\pi^-$ decay has a significantly better sensitivity to the neutrino mixing matrix element. In forthcoming experiments the upper bound on the ratio in (20) can be improved by two orders of magnitude or even more. Then this experimental bound could be translated to the limit $|U_{\mu j}|^2 < 10^{-11}$ and stronger.

6. HUNDRED-MeV NEUTRINOS IN ASTROPHYSICS AND COSMOLOGY

It is well known that massive neutrinos may have important cosmological and astrophysical implications. They are expected to contribute to the mass density of the universe, participate in cosmic structure formation, big-bang nucleosynthesis, supernova explosions, imprint themselves in the cosmic microwave background, etc. (for a review see, for instance, [29]). This implies certain constraints on the neutrino masses and mixings. Currently, for massive neutrinos in the mass region (26), the only available cosmological constraints arise from the mass density of the universe and cosmic structure formation.

The contribution of stable massive neutrinos to the mass density of the universe is described by the «Lee–Weinberg» $\Omega_\nu h^2 - m_\nu$ curve. From the requirement that the universe is not «overclosed» this leads to the two well-known solutions $m_\nu \le 40$ eV and $m_\nu \gtrsim 10$ GeV which seem to exclude the domain (26). For the unstable neutrinos, however, the situation is different. They may decay early to light particles and, therefore, their total energy can be significantly «redshifted» down to the «overclosing» limit. Constraints on the neutrino lifetimes τ_{ν_j} and masses m_j in this scenario were found in [30]. In the mass region (26) we have an order of magnitude estimate

$$\tau_{\nu_j} < (\sim 10^{14})\ \mathrm{s} \qquad \text{(mass density limit).} \tag{35}$$

Decaying massive neutrinos may also have specific impact on the cosmic structure formation, introducing new stages in the evolution of the universe. After they decay into light relativistic particles the universe returns for a while from the matter to the radiation domination phase. This may change the resulting density fluctuation spectrum since the primordial fluctuations grow due to gravitation instability during the matter-dominated stages. Comparison with observations leads to an upper bound on the neutrino lifetime [31]. In the mass region (26) one finds roughly

$$\tau_{\nu_j} < (\sim 10^{7})\ \mathrm{s} \qquad \text{(structure formation limit).} \tag{36}$$

On the other hand, on the basis of (32), assuming $|U_{\mu j}|^2 \sim |U_{ej}|^2 \leq 4.6 \cdot 10^{-9}$ as in (34), we find conservatively

$$10^{-2}\ \mathrm{s} < \tau_{\nu_j} \qquad \text{(theoretical limit).} \tag{37}$$

Thus, massive neutrinos with masses in the interval (26) are not yet excluded by the known cosmological constraints (35) and (36) and there remains a wide open interval of allowed mixing matrix elements:

$$(\sim 10^{-18}) < |U_{\mu j}|^2,\ |U_{ej}|^2 < (\sim 10^{-9}). \tag{38}$$

Big-bang nucleosynthesis and the SN 1987A neutrino signal may presumably lead to much more restrictive constraints [18]. Unfortunately, as yet the analysis [18] of these constraints does not involve the mass region (26). It may happen that these constraints, in combination with our constraints in (34), close the window for neutrinos with masses in the interval (26). Then the only physics left to be studied using the $K^+ \to \mu^+\mu^+\pi^-$ searches would be physics beyond the SM other than neutrino issues. Nevertheless, significant model dependence of all the cosmological constraints should be carefully considered before such a determining conclusion is finally drawn.

CONCLUSION

We analyzed some generic properties of $\Delta L = 2$ lepton number violating processes and the constraints derivable from them on neutrino masses and mixing matrix elements. We discussed consistency conditions for experimental bounds when these bounds can be translated into the upper limits on the average neutrino mass $\langle m_\nu \rangle$ or the inverse average mass $\langle 1/M_N \rangle$. We found that, excepting the neutrinoless double decay, the other $\Delta L = 2$ processes are unable to provide us with sensible constraints on these quantities. Using the neutrino oscillation data, the tritium beta decay and the LEP searches for the heavy neutral lepton, we estimated constraints on their rates in the scenario with three light and several heavy neutrinos. Typical values of these rates are far from being reached experimentally in the near future.

We studied the potential of the K-meson neutrinoless double muon decay as a probe of Majorana neutrino masses and mixings. We found that this process is very sensitive to the hundred-MeV neutrinos ν_j in the resonant mass range (26). We analyzed the contribution of these neutrinos to the $K^+ \to \mu^+\mu^+\pi^-$ decay rate and derived stringent upper limits on the Majorana neutrino mixing matrix element $|U_{\mu j}|^2$ from current experimental data. We presented these limits in the form of a 2-dimensional exclusion plot, and compared them with existing limits. The $K^+ \to \mu^+\mu^+\pi^-$ decay excludes a domain previously unrestricted experimentally. We stressed that the known astrophysical and cosmological constraints do not yet exclude hundred-MeV neutrinos satisfying these $K^+ \to \mu^+\mu^+\pi^-$ constraints.

Finally, we notice that the decay $K^+ \to \mu^+\mu^+\pi^-$ can in principle probe lepton number violating interactions beyond the Standard Model. However, according to recent studies [9, 32], supersymmetric interactions both with and without R-parity conservation seem to be beyond the reach of the K-decay experiments.

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