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INTEGRAL REPRESENTATION FOR STRUCTURE FUNCTIONS AND TARGET MASS EFFECTS

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A method of studying target mass effects based on the Jost–Lehmann–Dyson integral representation for structure functions of the inelastic lepton-hadron scattering is developed; it accumulates general principles of local quantum field theory. It is shown that the expression obtained for the structure function that depends on the target mass has a correct spectral property.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Интегральное представление для структурных функций и эффекты, связанные с массой мишени

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Развит метод изучения эффектов, обусловленных учетом массы мишени, для структурных функций неупругого лептон-адронного рассеяния, основанный на интегральном представлении Йоста–Лемана–Дайсона, которое аккумулирует общие принципы локальной квантовой теории поля. Показано, что выражение для структурной функции, полученное таким образом и содержащее зависимость от массы мишени, находится в согласии со спектральным свойством.

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1. The inclusive cross section for inelastic lepton-hadron scattering is expressed as the Fourier transform of the expectation value of the current product $J(z)J(0)$ in the target state. The operator product expansion (OPE) is a powerful tool to study inelastic scattering processes. This method has been applied to define the contribution of target mass terms to the structure functions in paper [1]. The scheme that has been elaborated is the following. The first step is to organize the OPE by using the operators with definite twist and to take the leading twist contribution to get the free-field OPE. The second step is to collect the terms in the OPE of the form $(q \cdot P)^n$ and relate corresponding coefficients to the moments of the structure function. Then, one can restore physical structure functions by inverting the moments through the Mellin transformation. These functions are parameterized by the quark distribution function $F(x)$ that appears with the argument

$$\xi = \frac{2x}{1 + \sqrt{1 + 4x^2\epsilon}}\,, \tag{1}$$

where x is the Bjorken scaling variable $x = Q^2/2\nu = Q^2/2(q \cdot P)$, and ϵ is expressed through the target mass M and the transfer momentum Q as $\epsilon = M^2/Q^2$. The scaling variable (1) is usually called the Nachtmann variable [2]. The trouble with the ξ scaling has widely been discussed in the literature (see, for example, [3–6]). For example, the structure function $W_2(x, Q^2)$ within this method reads [5]

$$W_2(x, Q^2) = x^2 \frac{\partial^2}{\partial x^2} \left[\frac{xG(\xi)}{\xi(1 + \xi^2 4M^2/Q^2)} \right] , \qquad G(x) = \int_x^1 dy \, (y - x) \, F(y) \, ,$$

where $F(x)$ is the quark distribution function. The defect of this equation is that there is a clear mismatch at $x = 1$. The physical structure function $W_2(Q^2, x)$ in the left-hand side vanishes at $x = 1$, whereas in the right-hand side does not.

In this paper, it is proposed to use the Jost–Lehmann–Dyson (JLD) integral representation for the structure function accumulating general principles of the theory. We argue that in this case it is possible to get an expression for the structure function in terms of the quark distribution incorporating the target mass effects and having the correct spectral property.

The fact that an approximation can conflict with general principles of a theory is not rare event in quantum physics. For example, it is well known that when the renormalization group equation for the running coupling is solved directly, there arise unphysical singularities, for example, the ghost pole in the one-loop approximation, and they subsequently appear in physical quantities. This trouble can be resolved within the analytic approach proposed in [7, 8] and elaborated in [9–17]. This method combines the renormalization invariance and the Q^2 analyticity of the Källén–Lehmann type has revealed new important properties of the analytic coupling [7, 8, 15]. The invariant analytic formulation essentially modifies the behavior of the analytic running coupling in the infrared region by making it stable with respect to higher-loop corrections. This is radically different from the situation encountered in the standard renormalization-group perturbation theory, which is characterized by strong instability with respect to the next-loop corrections in the domain of small energy scale. The analytic perturbation theory leads to new nonpower-series expansions with new nonsingular functions [16]. Applying this algorithm to analyze the amplitudes of processes like the e^+e^--annihilation into hadrons [13], the inclusive τ decay [11, 17], and the sum rules for the inelastic lepton-hadron scattering [14], it has been demonstrated that, in addition to loop stability, the analytic perturbation theory results are much less sensitive to the choice of the renormalization scheme than those in the standard approach. The three-loop level practically insures both the loop saturation and the scheme invariance of the relevant physical quantities in the entire energy or momentum range.

The method that will be considered here is a generalization of the idea used in the analytic approach to quantum chromodynamics. We base our consideration on the JLD representation for structure functions of the inelastic lepton-hadron process that has been suggested in [18, 19]. The structure functions depend on two arguments, and the corresponding representation that accumulates the fundamental properties of the theory (such as relativistic invariance, spectrality, and causality) has a more complicated form in our analysis than in the representation of the Källén–Lehmann type for functions of one variable. We use the 4-dimensional integral representation proposed by Jost and Lehmann [18] for the so-called symmetric case. A more general case has been considered by Dyson [19], and similar representation is therefore often called the Jost–Lehmann–Dyson representation. Applications of this representation to automodel asymptotic structure functions were considered by Bogoliubov, Vladimirov, and Tavkhelidze [20]; some of these results and notation will be used in what follows.

2. The proof of the JLD representation is based on the most general properties of the theory, such as covariance, Hermiticity, spectrality, and causality [21]. For the function $W(\nu, Q^2)$ satisfying all these conditions, there exists a real moderately growing distribution $\psi(\mathbf{u}, \lambda^2)$ such that the JLD integral representation holds; in the nucleon rest frame, this can be written as [20]

$$W(\nu, Q^2) \;=\; \varepsilon(q_0) \int d\mathbf{u}\, d\lambda^2\, \delta\big[q_0^2 - (M\mathbf{u} - \mathbf{q})^2 - \lambda^2\big]\, \psi(\mathbf{u}, \lambda^2), \tag{2}$$

where the function $\psi(\mathbf{u}, \lambda^2)$ has a support for

$$\rho = |\mathbf{u}| \le 1, \qquad \lambda^2 \ge \lambda^2_{\min} = M^2\big(1 - \sqrt{1-\rho^2}\,\big)^2 .$$

For the process under consideration, the physical values of ν and Q^2 are positive. We, thus, can neglect the factor $\varepsilon(q_0) = \varepsilon(\nu)$ and keep the same notation for $W(\nu, Q^2)$. Taking into account that the weight function $\psi(\mathbf{u}, \lambda^2) = \psi(\rho, \lambda^2)$ is radial-symmetric, as follows from covariance, we write down the JLD representation for W in the covariant form,

$$W(\nu, Q^2) = \\ = \int_0^1 d\rho\, \rho^2 \int_{\lambda^2_{\min}}^{\infty} d\lambda^2 \int_{-1}^{1} dz\, \delta\big(Q^2 + M^2\rho^2 + \lambda^2 - 2z\rho\sqrt{\nu^2 + M^2 Q^2}\,\big)\psi(\rho, \lambda^2)\,. \tag{3}$$

As follows from representation (3), a natural scaling variable is given by

$$s \;=\; x\sqrt{\frac{1+4\epsilon}{1+4x^2\epsilon}}\,, \tag{4}$$

which accumulates the root structure determined by the δ-function argument. At the same time, in the physical region of the process, the s variable changes in the same way as the Bjorken variable x does, i.e., from zero to one (cf. [22]). The variable s depends on the mass of the target (the nucleon) and is different from both the Bjorken variable and the Nachtmann variable. However, only the s variable leads to the moments that have the analytic properties in Q^2 that we need.

Defining the modified s moments of the structure functions [15], $\mathcal{M}_n(Q^2)$, and introducing the weight function

$$U_n(\sigma) = \frac{1}{n}\int_0^1 d\rho\, \rho^{n+1} \theta(\sigma - \sigma_{\min})\, \psi(\rho, \sigma - M^2\rho^2)\,,$$

we obtain the representation

$$\mathcal{M}_n(Q^2) = (Q^2)^{n-1}\int_0^{\infty} d\sigma \frac{U_n(\sigma)}{(\sigma + Q^2)^n}, \tag{5}$$

which implies the analyticity of $\mathcal{M}_n(Q^2)$ in the complex Q^2 plane cut along the negative semiaxis, i.e., the Källén–Lehmann type analyticity.[1] The relation between analytic moments and the x moments can be found in [15].[2]

The JLD representation (3) can be rewritten in the form

$$W(\nu,Q^2) = \\ = \int_0^1 d\beta \int_0^\infty d\sigma\, \delta\left[\sigma + Q^2 + 2M^2\left(1-\sqrt{1-\beta^2}\right) - \frac{\beta}{s}\, Q^2\sqrt{1+4\epsilon}\right] H(\beta,\sigma) \quad (6)$$

convenient for our further consideration, where we introduced the new weight function $H(\beta,\sigma)$ connected with the initial weight function $\psi(\rho,\lambda^2)$ via an integral expression and supported in: $\{0<\beta<1;\, \sigma>0\}$.

Introducing the function $\mathcal{F}(x,Q^2)$ that corresponds to the structure function $W(\nu,Q^2)$, when the target mass M is neglected, one finds the representation

$$\mathcal{F}(x,Q^2) = \int_x^1 dy\, H\left[y, \left(\frac{y}{x}-1\right)Q^2\right]\,. \quad (7)$$

Define a parton distribution function $F(x)$ as the limit of $\mathcal{F}(x,Q^2)$ as $Q^2\to\infty$. The limit of the weight function $H(x,\sigma)$, when the second argument goes to infinity, is determined by $H(x)$. From Eq. (7) we find the simple relation

$$F(x) = \int_x^1 dy\, H\,(y)\,, \quad (8)$$

and, therefore, the weight function $H(x)$ connects with the parton distribution $F(x)$ as follows $H(x) = -dF(x)/dx$. Thus, in the Bjorken limit, the weight function H in the JLD representation is associated with the derivative of the parton distribution.

3. Now we consider the method of incorporating the target mass corrections. To make our explanation more transparent and not to obscure an essence of the approach with details of technical character we here consider the case of scalar currents. Following the approach suggested in [1], consider the twist-two symmetrical local operators $\bar\psi\,\partial^{\mu_1}\cdots\partial^{\mu_{2N}}\,\psi$. For massless quarks $< P|O^{\mu_1\cdots\mu_{2N}}|P >= O_{2N}\,\{P^{\mu_1}\cdots P^{\mu_{2N}}\}$, where $\{P^{\mu_1}\cdots P^{\mu_{2N}}\}$ is a traceless combination of the products of vectors P^{μ_i}. By using the expression for the scalar combination of $\{P^{\mu_1}\cdots P^{\mu_{2N}}\}$ with the tensor $q_{\mu_1}\cdots q_{\mu_{2N}}$ and relating the parameters O_k according to [1] to the moments of the quark distribution function $F(x)$ of the parton language

$$O_k = \int_0^1 dx\, x^{k-2}\, F(x)\,, \quad (9)$$

[1] In [23], the Deser–Gilbert–Sudarshan integral representation [24] was used to arrive at a similar statement regarding the analyticity of the Källén–Lehmann type for x moments. However, the status of this representation in quantum field theory is less clear, since it cannot be obtained starting only with the fundamental principles of the theory (see the discussion in [25]).

[2] Note here that in paper [15], a dispersion relation with respect to the s variable has been obtained, and a relation with the OPE has been established.

for the moments of the 'physical' structure function $W(x,Q^2)$, we find

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2}\, W(x,Q^2) = \frac{1}{n!} \sum_{m=0}^{\infty} \frac{(n+m)!}{m!}\, \epsilon^m\, O_{n+2m}\,. \tag{10}$$

The formal Mellin transformation of (10) gives

$$W(x,Q^2) = \frac{x}{\xi}\, \frac{F(\xi)}{1+\epsilon\,\xi^2}\,. \tag{11}$$

This relation has an obvious trouble with the spectrality at $x = 1$ that has been mentioned above.

4. This difficulty can be overcome by applying the JLD representation in a manner as the momentum analyticity is used for resolving the ghost pole problem.

The analytic moments can be written as follows

$$\mathcal{M}_n(Q^2) = \int_0^1 dx\, \frac{x^{n-2}}{(1+\epsilon x^2)^{n+1}}\, F(x)\,.$$

The first step of our procedure is to find the weight function $U_n(\sigma)$ in the representation (5) for the analytic moments. As a result, we have

$$U_n(\sigma) = U_n(\infty) + \frac{\sigma^2}{n}\Phi_n^{'}(\sigma) - 2\sigma\frac{n-1}{n}\Phi_n(\sigma) - (n-1)\int_\sigma^\infty ds\, \Phi_n(s)\,. \tag{12}$$

Here $U_n(\infty)$ is defined by the relation $\mathcal{M}_n(\infty) = U_n(\infty)/(n-1)$ and $\Phi_n(\sigma) = = \left(\sigma/M^2\right)^{(n-3)/2}\, F(\sqrt{\sigma/M^2})$.

The weight functions $H(\beta,\sigma)$ in (6) and $U_n(\sigma)$ in the integral representation for the analytic moments (5) are related as follows

$$U_n(\sigma) = \int_0^1 d\beta\, \beta^{n-1}\, \tilde{H}(\beta,\sigma)\,, \tag{13}$$

where $\tilde{H}(\beta,\sigma) = H(\beta, \sigma - 2M^2(1-\sqrt{1-\beta^2}))$. Thus, the functions $U_n(\sigma)$ are the moments of the weight function $H(\beta,\sigma)$ and, therefore, $U_n(\sigma)$ can be restored by the Mellin transformation.

Then, we represent the function $H(\beta,\sigma)$ in the form $H(\beta,\sigma) = H_0(\beta) + h(\beta,\sigma)$, where the function H_0 is connected with the parton distribution function, and define the function $\tilde{h}(\beta,\sigma) = h(\beta, \sigma - 2M^2(1-\sqrt{1-\beta^2}))$, for which one can write

$$\tilde{h}(\beta,\sigma) = \frac{1}{2\pi i} \int\limits_{\gamma-i\infty}^{\gamma+i\infty} dn\, \beta^{-n}\, \left[U_n(\sigma) - U_n(\infty)\right]\,,$$

where the difference $U_n(\sigma) - U_n(\infty)$ is expressed via the parton distribution function as follows

$$U_n(\sigma) - U_n(\infty) = \frac{1}{2M^2}\frac{\sigma^2}{n}\frac{\partial}{\partial\sigma}\left[\left(\frac{\sigma}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{\sigma}{M^2}}\right)\right] - \\ - \frac{\sigma}{M^2}\frac{n-1}{n}\left[\left(\frac{\sigma}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{\sigma}{M^2}}\right)\right] - \\ - \frac{n-1}{2M^2}\int_\sigma^\infty ds \left[\left(\frac{s}{M^2}\right)^{(n-3)/2} F\left(\sqrt{\frac{s}{M^2}}\right)\right] . \quad (14)$$

Next, we represent the structure function as $W(x, Q^2) = W_0(x, Q^2) + w(x, Q^2)$, where $W_0(x, Q^2)$ corresponds to the weight function $H_0(\beta)$; and $w(x, Q^2)$, to $h(\beta, \sigma)$, and express $W_0(x, Q^2)$ in the form

$$W_0(x, Q^2) = \int_0^1 d\beta\, \theta[f(\beta; x, \epsilon)]\, H_0(\beta), \quad f(\beta; x, \epsilon) = \frac{\beta}{s}\sqrt{1+4\epsilon} - 1 - 2\epsilon(1 - \sqrt{1-\beta^2})\,. \quad (15)$$

The variables β_- and β_+, if $x > \tilde{x} \equiv 1/\sqrt{1+4\epsilon x^2}$,

$$\beta_\pm = \frac{x\sqrt{1+4\epsilon x^2}}{1+4\epsilon x^2 + 4\epsilon^2 x^2}\left[1 + 2\epsilon \pm 2\epsilon\sqrt{\frac{1-x^2}{1+4\epsilon x^2}}\right], \quad (16)$$

are the roots of the equation $f(\beta; x, \epsilon) = 0$. Thus, we have

$$W_0(x, Q^2) = \begin{cases} F(\beta_-) - F(1), & 0 \le x < \tilde{x}, \\ F(\beta_-) - F(\beta_+), & \tilde{x} \le x \le 1. \end{cases} \quad (17)$$

The spectral property of $W_0(x, Q^2)$, its vanishing at $x = 1$, comes from the relation $\beta_-(x = 1) = \beta_+(x = 1)$. The function $W_0(x, Q^2)$ is a continuous function at $x = \tilde{x}$ because $\beta_+(\tilde{x}) = 1$.

For the function $w(x, Q^2)$, one finds

$$w(x, Q^2) = \int_0^1 d\beta\, \theta[f(\beta; x, \epsilon)]\, \theta[g(\beta; x, \epsilon)]\, \phi(\beta; x, \epsilon)\,, \quad (18)$$

where $f(\beta; x, \epsilon)$ is defined by Eq. (15), $g(\beta; x, \epsilon) = \left[(\beta/s)\sqrt{1+4\epsilon} - 1\right]/\epsilon - \beta^2$, and

$$\phi(\beta; x, \epsilon) = \frac{1}{4\sqrt{\tau}}\theta(\tau)\,\theta(1-\tau)\,\frac{\partial}{\partial(\sqrt{\tau})}\left[\sqrt{\tau}F(\sqrt{\tau})\right],$$

with $\tau \equiv \tau(\beta; x, \epsilon) = \left[(\beta/s)\sqrt{1+4\epsilon} - 1\right]/\epsilon$. The equation $\tau(\beta; x, \epsilon) = 1$ has the root $\beta_\tau = (1+\epsilon)s/\sqrt{1+4\epsilon}$. The solutions of the equation $g(\beta; \eta, \epsilon) = 0$ are connected with the ξ variable ($\xi_- = \xi$) and are of the form $\xi_\pm = (\sqrt{1+4\epsilon x^2} \pm 1)/2\epsilon x$.

The relative behavior of the functions $\beta_\pm$, β_τ, ξ, and $\eta = s/\sqrt{1+4\epsilon}$ as a function of x for $\epsilon = 0.5$ is shown in Fig. 1. This figure demonstrates that the ξ does not appear in the expression for the structure function, because the range of integration in Eq. (18) includes the interval from β_- to β_τ.

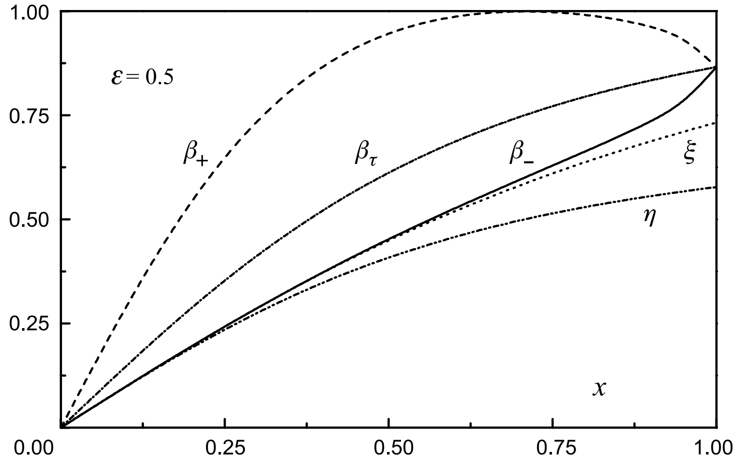

Fig. 1. The relative behavior of functions $\beta_{\pm}$, β_{τ}, ξ, and η as function of x for $\epsilon = 0.5$

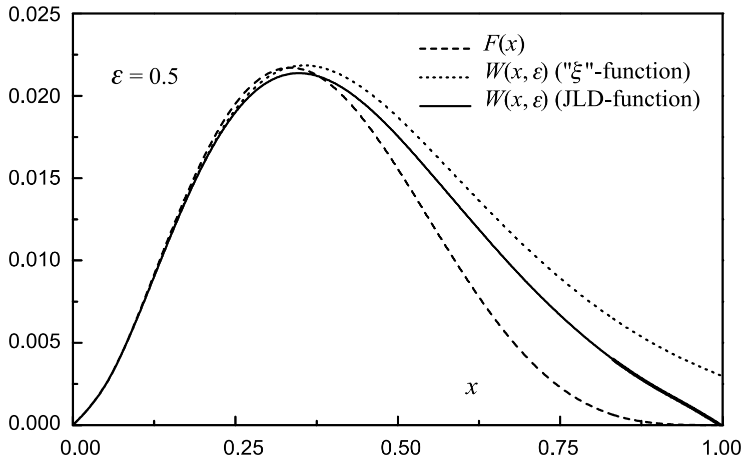

Fig. 2. Structure functions for $\epsilon = 0.5$

In Fig. 2, we plot the structure functions as functions of x for $\epsilon = 0.5$. The parton distribution is taken in the form $F(x) = x^2(1-x)^4$ (dashed curve). The physical structure functions, $W(x, \epsilon)$, that depend on the target mass, are obtained in two ways: the dotted curve was constructed by the "ξ"-scaling expression (11), and the solid line was constructed by using the JLD representation. This figure demonstrates the difference between these methods. The structure function obtained by the JLD representation has the correct spectral behavior at $x = 1$ as compared with the "ξ"-scaling prediction.

5. The JLD representation reflecting the general principles of the local quantum field theory (covariance, Hermiticity, spectrality, and causality) has been applied for studying the inelastic lepton-hadron process. Here we have concentrated on the well-known trouble that is a characteristic feature of the so-called "ξ"-scaling approach. We have argued that the approach proposed here gives the self-consistent method of incorporating the target mass dependence into the structure function and does not lead to the conflict with the spectral condition.

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