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THERMAL MULTIFRAGMENTATION, NUCLEAR FOG AND CRITICAL TEMPERATURE FOR THE LIQUID–GAS PHASE TRANSITION

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Thermal multifragmentation of hot nuclei is interpreted as the nuclear liquid–fog phase transition. The charge distribution of the intermediate mass fragments produced in p (8.1 GeV) + Au collisions is analyzed in the framework of the statistical multifragmentation model with the nuclear critical temperature for the liquid–gas phase transition T_c as a free parameter. It is found (from the best fit of the calculations and data) that $T_c = (20 \pm 3)$ MeV (90 % CL).

Тепловая мультифрагментация горячих ядер интерпретируется как фазовый переход жидкость–туман в ядерном веществе. Зарядовое распределение фрагментов промежуточной массы, возникающих в соударениях p (8,1 ГэВ) + Au, анализируется в рамках статистической модели мультифрагментации. При этом критическая температура для фазового перехода жидкость–газ T_c используется в качестве свободного параметра. Из наилучшего согласия расчетов с экспериментом найдено, что $T_c = (20 \pm 3)$ МэВ (при уровне достоверности 90 %).

1. THE NUCLEAR EQUATION OF STATE AND THERMAL MULTIFRAGMENTATION

The investigation of the decay properties of the very hot nuclei is one of the most challenging topics of nowadays nuclear physics. The excitation energy of the hot nuclei (500–700 MeV) is comparable with the total binding energy. They disintegrate via a new multibody decay mode — thermal multifragmentation. This process is characterized by the copious emission of intermediate mass fragments (IMF, $2 < Z \leq 20$) which are heavier than alpha particles but lighter than fission fragments. Such multibody disintegration is not an exotic but the main decay channel of a very hot nuclear system.

The development of this field for the last two decades has been strongly stimulated by an idea that this process is related to the nuclear liquid–gas phase transition. One of the

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first nuclear models, suggested by N. Bohr, K. Weizsäcker and Ya. I. Frenkel 65 years ago, is the liquid-drop model, which is alive now. The liquid–gas phase transition in the nuclear matter was predicted much later [1–3] on the basis of the similarity between van der Waals and nucleon–nucleon interactions. In both cases the attraction between particles is replaced by repulsion at a small interaction range. As a result, the equations of the state are similar

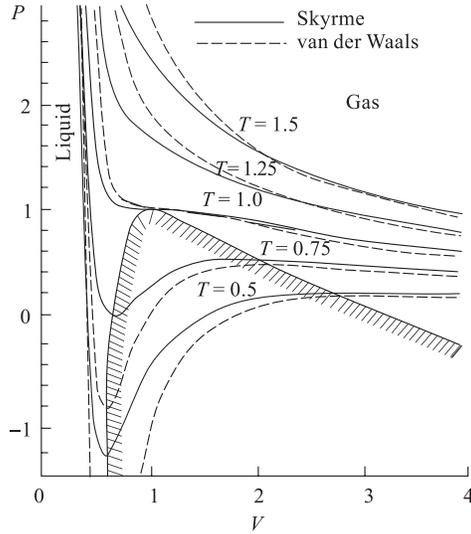


Fig. 1. Comparison of the equation of state for a van der Waals gas and for a nuclear system interacting through a Skyrme force (relative units are used)

for so different systems. It is well seen in the phase diagram (Fig. 1) taken from [2]. The figure shows the isotherms for pressure as a function of volume calculated for the van der Waals system and the Fermi gas of nucleons interacting through Skyrme forces. The scales are the same for both cases due to the use of dimensionless variables: pressure, volume and temperature are given as ratios to the critical values P_c , $V_c = 1/\rho_c$ (ρ_c is the critical density) and T_c . The very steep part of the isotherms (on the left) corresponds to the liquid phase. The gas phase is presented by the right parts of the isotherms, where pressure is changing smoothly with increasing volume. A point of peculiar interest is the part of the diagram below the hatched line, where the isotherms correspond to negative compressibility. The density here is significantly reduced as compared to the liquid phase. This is a spinodal region characterized by the phase instability. One can imagine that a hot nucleus expands due to thermal pressure and enters into unstable region. Due to density fluctuations, a homogeneous system converts into

the phase mixed state, consisting of droplets (IMFs) surrounded by nuclear gas (nucleons and light composite particles). In fact, the final state of this transition is a nuclear fog [3]. The neutrons fly away with the energies corresponding to the system temperature, while the charged particles are additionally accelerated in the Coulomb field of the system.

An effective way to produce hot nuclei is collision of heavy ions with energies of up to hundreds of MeV per nucleon. But in this case heating of nuclei is accompanied by compression, strong rotation and shape distortion, which may essentially influence the decay properties of hot nuclei. One gains simplicity, and the picture becomes clearer when light relativistic projectiles (first of all, protons, antiprotons, pions) are used. In contrast to heavy ion collisions, fragments are emitted by the only source — the slowly moving target spectator. Its excitation energy is almost entirely thermal. Light relativistic projectiles provide therefore a unique possibility of investigating «thermal multifragmentation», which is governed by the thermodynamic properties of a hot nuclear system.

The disintegration time is determined by the time scale of the density fluctuations and is expected to be very short (≈ 30 fm/c). This is a scenario of the thermal nuclear multifragmentation as a spinodal decomposition, considered in a number of theoretical and experimental papers (see, for example, [4–12] and review papers [13, 14]). It was proved experimentally

that thermal fragmentation takes place at reduced (3–4 times) density [15–17] and the decomposition time is short (less than 100 fm/c) [18–20]. The spinodal decomposition is, in fact, the liquid–fog phase transition in nuclear system.

2. THE CRITICAL TEMPERATURE FOR THE LIQUID–GAS PHASE TRANSITION

An important model parameter of this scenario is the critical temperature for the nuclear liquid–gas phase transition T_c at which the isotherm in the phase diagram has an inflection point. The surface tension vanishes at T_c , and only the gas phase is possible above this temperature. There are many calculations of T_c for finite nuclei. In Refs. [1, 2, 21, 22], for example, it is done by using a Skyrme effective interaction and the thermal Hartree–Fock theory. The values of T_c were found to be in the range 10–20 MeV depending on the chosen Skyrme interaction parameters and the details of the model. There are still no reliable experimental data for T_c , though this is claimed in a number of papers (see table).

The experimental data on the critical temperature for nuclei

Ref.	[23]	[24]	[25]	[26]	[27]
T_c , MeV	~ 5	11–12	6.7 ± 0.2	7.8 ± 0.2	> 10
Method	$Y(A_{\text{IMF}})$, Fisher’s model				$\sigma_{\text{fission}}(T)$

The main source of the experimental information for T_c is the fragment yield. In some statistical models of nuclear multifragmentation the shape of the IMF charge (or mass) distribution $Y(Z)$ is sensitive to the ratio T/T_c . The charge distribution is well described by the power law $Y(Z) \sim Z^{-\tau}$ for a wide range of the colliding systems [28]. In earlier studies on multifragmentation [3, 23] the power-law behavior of the IMF yield was interpreted as an indication of the proximity of the excited system to the critical point for the liquid–gas phase transition. This was stimulated by the application of Fisher’s classical droplet model [29], which predicted a pure power-law droplet-size distribution with the minimal value of $\tau = 2-3$ at the critical point.

In Ref. [23] Hirsch et al. estimate T_c to be ~ 5 MeV simply from the fact that the mass distribution is well described by a power law for IMFs produced in the collision of p (80–350 GeV) with Kr and Xe targets. In fact, the fragment mass distribution is not exactly described by the power law, therefore Panagiotou et al. [24] suggested the use of the term τ_{app} , an apparent exponent, to stress that the exact power-law description takes place only at the critical temperature. In paper [24] the experimental data were gathered for different colliding systems to get the temperature dependence of τ_{app} . As a temperature, the inverse slope of the fragment energy spectra was taken in the range of the high-energy tail. The minimal value of τ_{app} was obtained at $T = 11-12$ MeV, which was claimed as T_c . The later data smeared out this minimum. Moreover, it became clear that the «slope» temperature for fragments does not coincide with the thermodynamical one, which is several times smaller.

A more sophisticated use of Fisher’s droplet model for the estimation of T_c has been recently made by Elliott, Moretto et al. [25, 26]. The model was modified by including

the Coulomb energy release when a particle moves from the liquid to the vapor. The multifragmentation data from the Indiana Silicon Sphere collaboration for $\pi(8 \text{ GeV}/c) + \text{Au}$ collisions were analyzed with this refined model [25]. The extracted critical temperature is $T_c = (6.7 \pm 0.2) \text{ MeV}$. In the recent paper [26] the same analysis technique is applied to the data for the multifragmentation in collisions of Au, La, Kr (at 1.0 GeV per nucleon) with a carbon target (EOS collaboration). The extracted values of T_c are (7.6 ± 0.2) , (7.8 ± 0.2) and $(8.1 \pm 0.2) \text{ MeV}$ respectively.

There is only one paper in which T_c is estimated by using data other than the fragmentation ones. In Ref. [27] it is done by the analysis of the temperature dependence of the fission probability for ${}^4\text{He} + {}^{184}\text{W}$ collisions [30]. It was concluded that $T_c > 10 \text{ MeV}$ in contrast to the result of Refs. [25, 26].

It should be noted that in some papers the term «critical temperature» is not used in the strict thermodynamical sense given above. In Ref. [31] multifragmentation in Au + Au collisions at 35 A·MeV was analyzed with the so-called Campi plots to prove that the phase transition takes place in the spinodal region. The characteristic temperature for that process was denoted as T_{crit} and found to be equal to $(6.0 \pm 0.4) \text{ MeV}$. In the recent paper [32] the bond percolation model is used to interpret 10.2 GeV/c $p + \text{Au}$ multifragmentation data. The critical value of the percolation parameter $p_c = 0.65$ was found from the analysis of the IMF charge distribution. The corresponding «critical temperature» of $(8.3 \pm 0.2) \text{ MeV}$ is estimated by using the model relation between the percolation control parameter « p » and the excitation energy. The more appropriate term for this particular temperature is «break-up» or «crack» temperature, as suggested in Ref. [33]. This temperature corresponds to onset of the fragmentation of the nucleus entering the phase coexistence region.

Having in mind the shortcomings of Fisher's model [34, 35], we have made an attempt to estimate the critical temperature in the framework of the statistical multifragmentation model (SMM) [36].

3. ESTIMATION OF T_c USING SMM

Within this model one considers a microcanonical ensemble of all break-up channels composed of nucleons and excited fragments of different masses. It is assumed that an excited nucleus expands to a certain volume and then breaks up into nucleons and hot fragments. It is also assumed that at the break-up time the nucleus is in thermal equilibrium characterized by the channel temperature T determined from the energy balance. The probability W_j of a decay channel j is proportional to its statistical weight:

$$W_j \sim \exp S_j(E_x, A_0, Z_0), \quad (1)$$

where S_j is the entropy of the system in a state corresponding to the decay channel j . The excitation energy, mass and charge of the decaying system are denoted by E_x , A_0 and Z_0 , respectively. The fragments with mass number $A > 4$ are treated as heated nuclear liquid drops.

Channels are characterized by the multiplicities, N_{AZ} , of fragments AZ . The channels

entropy is obtained by summing the entropies of all the particles in a given channel:

$$S_j = \sum N_{AZ} S_{AZ}, \quad S_{AZ} = - \left(\frac{\partial F_{AZ}}{\partial T} \right)_V. \quad (2)$$

The fragment free energy F_{AZ} is a sum of volume, surface, symmetry, Coulomb and translational terms:

$$F_{AZ} = F_{AZ}^V + F_{AZ}^S + F_{AZ}^{\text{sym}} + F_{AZ}^C + F_{AZ}^t. \quad (3)$$

The surface energy term, F_{AZ}^S , depends on the critical temperature, so the fragment charge distribution is sensitive to the value of T_c . The following expression is used in the SMM for F_{AZ}^S :

$$F_{AZ}^S = a_s(T) A^{2/3}, \quad a_s(T) = a_s(0) \left(\frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4}, \quad (4)$$

with $a_s(T) = 4\pi r_0^2 \sigma(T)$, where $\sigma(T)$ — temperature-dependent coefficient of the surface tension. This equation was obtained in Ref. [38], devoted to the theoretical study of thermodynamical properties of a plane interface between two phases of nuclear matter (liquid and gas) in equilibrium. This parameterization is successfully used by the SMM for describing the multifragment decay of hot finite nuclei.

The comparison of the measured and calculated fragment charge distributions is the way to estimate the critical temperature T_c .

Statistical model describes well the properties of the thermal fragmentation of the target spectators produced in the collision of the light relativistic ions. As an example, Fig. 2, *a* shows the measured by the FASA collaboration and calculated fragment charge distributions for collisions of p (8.1 GeV), ${}^4\text{He}$ (4 and 14.6 GeV) and ${}^{12}\text{C}$ (22.4 GeV) with Au target. Experiments have been done using the 4π -setup FASA installed at Dubna Synchro-phasotron [12].

The reaction mechanism for the light relativistic projectiles is usually divided into two stages. The first one is a fast energy-depositing stage, during which very energetic light particles are emitted and a nuclear remnant is excited. We use the intranuclear cascade model (INC) [37] for describing the first stage. The second stage is described by the SMM, which considers multibody decay of a hot and expanded nucleus. But such a two-stage approach fails to explain the observed IMF multiplicities. An expansion stage is inserted between the two parts of the calculation. The excitation energies and the residual masses are then fine tuned [12] to get agreement with the measured IMF multiplicities, i. e., the values for the residual (after INC) masses and their excitation energies are scaled on an event-by-event basis. The lines in Fig. 2, *a* give the charge distributions calculated in the framework of this combined model, INC + Expansion + SMM, assuming $T_c = 18$ MeV. The agreement between the data and the model prediction is very good.

Figure 2, *b* shows the power-law fit of the distributions with the τ parameter given in the insert as a function of the beam energy. The corresponding thermal excitation energy range is 3–6 MeV/nucleon. The power-law parameter exhibits the so-called critical behavior showing a minimum at the excitation energy corresponding to the temperature three times lower than

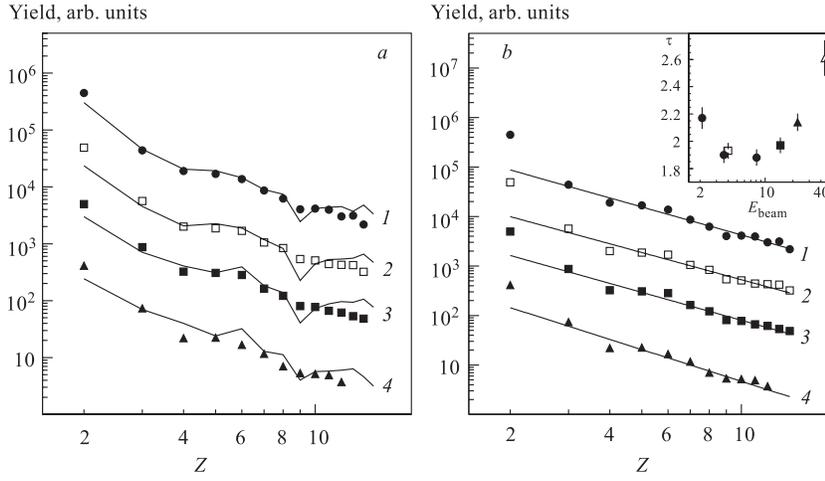


Fig. 2. Fragment charge distributions for $p + \text{Au}$ at 8.1 GeV (\bullet , 1), ${}^4\text{He} + \text{Au}$ at 4 GeV (\square , 2), ${}^4\text{He} + \text{Au}$ at 14.6 GeV (\blacksquare , 3) and ${}^{12}\text{C} + \text{Au}$ at 22.4 GeV (\blacktriangle , 4): a) the lines are calculated by the INC + Exp. + SMM model (normalized at $Z = 3$); b) the power-law fits with τ parameters given in the insert as a function of beam energy (in GeV)

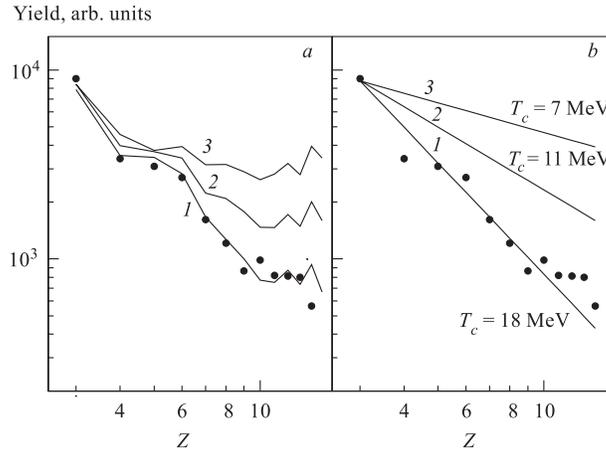


Fig. 3. Fragment charge distribution for $p + \text{Au}$ at 8.1 GeV (dots): a) the lines are calculated by the INC + Exp. + SMM model, assuming $T_c = 18$ MeV (1), 11 MeV (2) and 7 MeV (3); b) the power-law fits

the assumed T_c . A conventional explanation of that is given in Ref. [12], so this minimum for τ has no relation to any criticality [28].

In the present paper the calculations are performed for p (8.1 GeV) + Au collisions with T_c as a free parameter. For all values of T_c the calculations with the INC + Exp. + SMM model have been properly adjusted [12] to get the mean IMF multiplicity close to the measured

one. Figure 3, *a* shows the comparison of the measured fragment charge distribution and the model predictions for $T_c = 7, 11$ and 18 MeV. The statistical errors of the measurements do not exceed the size of the dots. The data are corrected for the counting rate loss caused by the cutoff (~ 1.2 MeV/nucleon) in the low-energy part of the IMF spectra. This correction is the largest ($\sim 15\%$) for the heavier IMFs. The calculations are close to the data for $T_c = 18$ MeV. The estimated mean temperature of the fragmenting system is around 6 MeV, the mean charge and mass numbers are 67 and 158 respectively. The theoretical curves deviate from the data with decreasing T_c .

Figure 3, *b* gives the results of the power-law fits for the data and model calculations (in the range $Z = 3-11$). The Be yield was corrected in the fitting procedure for the loss of unstable ${}^8\text{Be}$. The final results are shown in Fig. 4. The measured power-law exponent is given as a band with a width determined by the statistical error. The size of the symbols for the calculated values of τ_{app} is of the order of the error bar. From the best fit of the data and calculations one concludes that $T_c = (20 \pm 3)$ MeV at 90% confidence level.

Figure 4 shows also the results of the calculations with $a_s(T)$ linearly dependent on T/T_c [25, 26]:

$$a_s(T) = a_s(0) \left(1 - \frac{T}{T_c} \right). \quad (5)$$

The calculated values of τ_{app} in this case are remarkably lower than the measured one.

CONCLUSION

Thermal multifragmentation of hot nuclei is interpreted as the liquid–fog phase transition. The critical temperature for the nuclear liquid–gas phase transition T_c (at which surface tension vanishes) is estimated by using statistical multifragmentation model. For that purpose, the IMF charge distribution for $p + \text{Au}$ collisions at 8.1 GeV has been analyzed within the SMM with T_c as a free parameter. The value $T_c = (20 \pm 3)$ MeV (90% CL) obtained from the best fit to the data should be considered as some effective value of the critical temperature averaged over all the fragments produced in the collision. This value is significantly larger than those found in Refs. [25, 26] by the analysis of the multifragmentation data in terms of Fisher’s droplet formalism. Although our value for T_c is model-dependent, as is any other estimate of the critical temperature, the analysis presented here provides strong support for a value of $T_c > 15$ MeV.

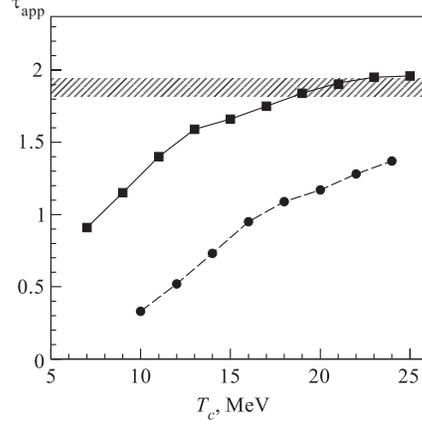


Fig. 4. The power-law exponent for p (8.1 GeV) + Au collision. The band corresponds to the measured value and its error bar. The symbols are obtained by the power-law fits of IMF charge distributions calculated assuming different values of T_c and different parametrization of the surface tension: squares are for Eq. (4), solid circles are for Eq. (5)

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