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MONOCHROMATIC NEUTRINOS FROM MASSIVE FOURTH GENERATION NEUTRINO ANNIHILATION IN THE SUN AND EARTH

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Accumulation inside the Earth and Sun of heavy (with the mass of 50 GeV) primordial neutrinos and antineutrinos of the fourth generation and their successive annihilation is considered. The minimal estimations of annihilational fluxes of monochromatic e, μ, τ neutrinos (neutrinos and antineutrinos) with the energy of 50 GeV are $4.1 \cdot 10^{-6}$ cm$^{-2}\cdot$s^{-1} from the Earth core and $1.1 \cdot 10^{-7}$ cm$^{-2}\cdot$s^{-1} from the Sun core. That makes the analysis of underground neutrino observatory data the additional source of information on the existence of massive stable 4th generation neutrino. It is shown that due to the kinetic equilibrium between the influx of the neutrinos and their annihilation the existence of new $U(1)$-gauge interaction of the 4th generation neutrino does not virtually influence the estimations of annihilational e-, μ-, τ-neutrino fluxes.

Рассматривается накопление Землей и Солнцем реликтовых тяжелых (с массой 50 ГэВ) нейтрино четвертого поколения с последующей их аннигиляцией. Минимальные оценки потоков монохроматических электронных, мюонных и тау-нейтрино (нейтрино и антинейтрино) с энергией 50 ГэВ от аннигиляции тяжелых нейтрино составляют $4{,}1 \cdot 10^{-6}$ см$^{-2}\cdot$с$^{-1}$ из центра Земли и $1{,}1 \cdot 10^{-7}$ см$^{-2}\cdot$с$^{-1}$ из центра Солнца, что делает анализ данных подземных нейтринных обсерваторий дополнительным источником информации о существовании нейтрино 4-го поколения. Показано, что вследствие кинетического равновесия между прилетом космических нейтрино и их аннигиляцией существование нового $U(1)$-калибровочного взаимодействия у нейтрино 4-го поколения практически не влияет на оценку аннигиляционных потоков e-, μ-, τ-нейтрино.

INTRODUCTION

The search for a new (fourth) generation of quarks and leptons is one of the most important longstanding problems of high-energy physics. In addition to experiments on accelerators the effective instruments of such search can serve cosmological and astrophysical manifestations of a new generation, for which the existence of a new neutrino (ν_4) as the lightest, perhaps, stable weakly interacting neutral particle of the 4th generation has special significance. More than 20 years ago it was shown [1] that neutrino with the mass of a few GeV can be a candidate to the role of a cold dark matter particle in the Universe. The modern experimental constraint on the new neutrino mass follows from the experimental data on the Z-boson width: the mass of ν_4 must be greater than $m_Z/2 \approx 45$ GeV, where m_Z is the Z-boson mass. The analysis of possible virtual effects of the 4th family in the particle data constrains the mass of the 4th neutrino in the allowed range around 50 GeV [2]. The results of DAMA experiment on underground search for WIMPs do not exclude WIMP's mass about 50 GeV, that is consistent with the allowed window for the new neutrino mass derived from the particle data. If the neutrino with such mass existed then the respective neutrino and antineutrino pairs would have to be in equilibrium in the early Universe. After their freeze-out primordial neutrinos would have to retain in the Universe but their contribution into the modern density is estimated to be rather small ($\Omega \sim 10^{-4}$) [3]. So the relic 4th generation neutrino cannot play the role of the dominant dark matter component and cosmological test of their existence requires the analysis of more refined astrophysical effects. Such analysis should be carried out assuming the presence of the dominant dark matter of another nature.

In a series of papers [4–7] the possible experimental (on LEP) [4, 6] and astrophysical [4, 5, 7] manifestations of new heavy stable neutrino existence were revealed. As any other form of nonrelativistic dark matter, the heavy neutrinos must concentrate in galaxies. Under supposing charge symmetry, together with massive neutrinos the equal number of their antineutrinos concentrates in galaxies. That must lead to neutrino-antineutrino annihilation effects. In the work [7] it was shown that neutrino-antineutrino annihilation in halo of our Galaxy could account for galactic diffuse gamma-background recently discovered by EGRET. Such gamma-background can hardly arise from annihilation of a neutralino, the popular candidate to WIMP, predicted in supersymmetric models, due to the Majorana nature of a neutralino. The fluxes on the Earth of particles from heavy neutrino annihilation in the galactic halo were calculated in [4, 5]. The positron, antiproton, gamma fluxes were shown to be most sensitive for experimental probe of such annihilation. The study of other possible astrophysical effects of the 4th neutrino is of evident interest.

In many works [8–10] the accumulation of WIMPs (neutralino, neutrino) in the Earth and Sun with their successive annihilation giving the fluxes of known neutrinos were considered. The annihilational e, μ, τ neutrinos can be detected by underground neutrino telescopes. In the present work the analogous processes for concretely the 4th neutrino with the mass of 50 GeV are analyzed. The profound signature for such neutrino is the existence of the annihilation channel to the monochromatic neutrino and antineutrino. Also the possibility for heavy neutrino (together with all the other quarks and leptons of the 4th generation) to have new interaction is briefly considered. The additional interaction (or even several) appears in realistic variants of superstring theories in low-energy limit. The influence of new interaction on the effects of heavy neutrino annihilation is estimated.

1. CAPTURE OF GALACTIC MASSIVE NEUTRINOS BY THE EARTH AND SUN

One estimates massive neutrino (further simply neutrino) number density not perturbed by Earth's and Sun's (gravitating body) gravitational fields supposing the number density distribution in the Galaxy in the form

$$n = \xi(r)\langle n\rangle = \frac{\kappa}{1 + r^2/r_0^2}\langle n\rangle,$$

where $\langle n\rangle \approx 10^{-11}(\Omega/10^{-4})$ cm^{-3} is the mean number density in the Universe with Hubble constant $h = 0.65$; $\kappa = \dfrac{n(r=0)}{\langle n\rangle}$; r is the distance to galactic centre; r_0 is the characteristic distance of distribution. The ratio κ of the neutrino concentration in galactic centre to the average one in the Universe, as shown in [11] for weakly interacting matter, is connected with analogous ratio between baryonic (B) and dark matter (DM) densities (in the latter the heavy neutrinos contribute with a small weight)

$$\kappa = \frac{n(r=0)}{\langle n \rangle} = \left(\frac{\rho_{\rm B}(r=0)}{\langle \rho_{\rm B+DM} \rangle} \right)^{3/4} \approx 10^6.$$

Put $r_0 = 1.2$ kpc and for solar system take $r = r_{\rm S} = 8.5$ kpc, then for neutrino number density not perturbed by Sun's and Earth's gravitational fields, that we will indicate by subscript «∞», we obtain

$$n_\infty = \xi_{\rm S} \langle n \rangle \approx 2 \cdot 10^4 \langle n \rangle \approx 2 \cdot 10^{-7} \ \ {\rm cm}^{-3}.$$

The mean velocity of neutrinos in the Galaxy is taken equal to $v_\infty = 300$ km/s, which corresponds to the neutrino kinetic energy $T_\infty = 25$ keV. Near gravitating body (in potential field) one can show for a neutrino of fixed energy that the concentration and velocity are in the following relationship

$$n/v = {\rm const.} \tag{1}$$

Neutrino capture rate by gravitating body will be

$$\dot{N}_{\rm capt} = \sum_A \int n \sigma v w_1 n_A \, dV. \tag{2}$$

Here n and n_A are, respectively, the concentrations of incident neutrinos and nuclei[1] with atomic number A in Earth's or Sun's matter; σ is the cross section of neutrino-nucleus interaction; v is the neutrino velocity; w_1 is the probability for a neutrino to lose in one collision sufficient energy for capturing by gravitational field; integration is over gravitating body volume. The number density and velocity of incident neutrinos are considered to be «perturbed» by gravitational field of the Sun or Earth (according to (1)), but the change of these characteristics of neutrino flux owing to their interaction with the matter is not taken into account. With the use of Eq. (1), the Eq. (2) can be transformed to the form

$$\dot{N}_{\rm capt} = \sum_A \frac{n_\infty}{v_\infty} \int \sigma v^2 w_1 n_A \, dV. \tag{3}$$

The distribution density of Earth's matter will be assumed homogeneous, the other more realistic distributions with its much more complicated form lead to small change of the result. For this distribution the square escape (parabolic) velocity dependent on dimensionless radius $x = R/R_0$, where R_0 is the gravitating body radius, inside the Earth has the form

$$v^2_{\rm esc\,E}(x) = v^2_{\rm esc\,E} \frac{(3 - x^2)}{2}, \tag{4}$$

where $v_{\rm esc\,E} \equiv v_{\rm esc\,E}(1) = 11.2$ km/s is Earth's surface parabolic velocity. Obviously, Earth's gravitational field does not influence neutrino concentration and velocity. It is essential just

[1] Interaction of neutrino with electrons is negligible.

for factor w_1. Also Sun's field near the Earth is not significant. In the present work we will not take into account the possibility of existence of «slow» neutrino component revealed in [10]. According to [10] this population of neutrinos moving along stretched orbit around the Sun appears in the result of weak collisions of background neutrinos with nuclei of the Sun. The role of such population in the processes of neutrino accumulation by the Earth can be important due to large value of w_1, that is accounted for much lower velocity of these neutrinos (of the order of Earth's orbital velocity) as compared to velocity of neutrino in the Galaxy. The role of slow component in effects of accumulation and annihilation of 4th generation neutrino in the Earth will be considered in separate work.

Distribution of Sun's matter density with the solar radius x is supposed in accordance with standard solar model [12] and it can be approximated by function

$$\rho_{\rm S}(x) = 148(1-x^2)\,\exp\left(-\frac{9.04x^2}{0.17+x^{1.52}}\right)\ \text{g/cm}^3. \tag{5}$$

The x dependence for square velocity $v^2_{\rm esc\,S}(x)$ is well approximated by function

$$v^2_{\rm esc\,S}(x) = v^2_{\rm esc\,S}\left(4.011\frac{1-x^{1.868}}{1+8.05x^{1.868}}+1\right), \tag{6}$$

where $v_{\rm esc\,S} \equiv v_{\rm esc\,S}(1) = 618$ km/s. The values of parabolic and maximal total velocities for neutrinos in Sun's centre and of kinetic energies corresponding to them are respectively equal to $v_{\rm esc\,S}(0) = 1383$ km/s, $v_{\rm S}(0) = \sqrt{v^2_\infty + v_{\rm esc\,S}(0)} = 1416$ km/s, $T_{\rm esc\,S}(0) = 513$ keV, $T_{\rm S}(0) = T_\infty + T_{\rm esc\,S}(0) = 556$ keV.

Now let us consider cross sections of interaction of massive neutrino with nuclei.

The wavelength of neutrino with the mass 50 GeV and the velocities $300 \div 1416$ km/s are correspondingly $\lambda = (3.9 \div 0.84)\cdot 10^{-13}$ cm. That is comparable with nucleus sizes. Therefore it is necessary to take into account the effects of the finite size of nuclei, i.e., that nuclei are not point-like. For all nuclei presented in the considerable amount inside the Sun and Earth the lowest excited energies (> 0.8 MeV) exceed the kinetic neutrino energy (< 0.56 MeV). Therefore the scattering of a neutrino on a nucleus we will consider as the elastic one on the whole nucleus. The amplitude of scattering of a neutrino (antineutrino) on a nucleus in this case is determined by coherent isoscalar vector neutral weak coupling and in the nonrelativistic limit is given by expression [13]

$$M = \frac{G_{\rm F}}{2\sqrt{2}} F_V F(q^2),$$

where $G_{\rm F}$ is Fermi's constant; $F_V = 2Z - A - 4Z\sin^2\theta_{\rm W} = -(A - 1.074Z)$; A, Z are the atomic number and the charge of the nucleus; $\theta_{\rm W}$ is the Weinberg angle; $F(q^2)$ is the form factor, taking into account nucleus non-point-likeness. The cross section will be

$$\sigma = \frac{G^2_{\rm F}\mu^2}{8\pi} F^2_V \eta(T) = \sigma_0 \eta(T),$$

where $\mu = \dfrac{m_\nu m_A}{m_\nu + m_A}$ is the reduced mass of neutrino m_ν and nucleus m_A; (T) is a factor, taking into account nucleus non-point-likeness and dependent on kinetic energy (velocity) of a neutrino. For proton $F + V \approx 0$, and the axial coupling plays the main role in interaction with a neutrino. The cross section of the proton-neutrino interaction in nonrelativistic limit takes the form

$$\sigma = \frac{G_{\mathrm{F}}^2 \mu^2}{8\pi}(3F_A^2 + F_V^2)\eta(T) = \sigma_0 \eta(T). \tag{7}$$

For a proton (as well as for a neutron) $F_A \approx 1.25$.

The dependence of form factor on q^2 is assumed in accordance with the model of nucleon Fermi gas for all nuclei. That, generally speaking, is not quite correct for light nuclei. Since the neutrino energy is not enough for the excitation of nuclei, the nucleon Fermi gas is considered as strongly degenerated. In this case

$$F(q^2) = \frac{3(\sin\, y - y\, \cos\, y)}{y^3}, \quad y = qa,$$

where q is the modulus of neutrino 3-momentum transferred to nucleus. The factor $\eta(T)$ will have the form

$$\begin{gathered} \eta = \frac{9}{4}\frac{-1 + \cos\, 2y_{\max} - 2y_{\max}^2 + 2y_{\max}\, \sin\, 2y_{\max} + 2y_{\max}^4}{y_{\max}^6}, \\ y_{\max} = q_{\max} a = 2\mu\beta a \equiv \frac{v}{v_{\mathrm{point}}}, \end{gathered} \tag{8}$$

with limits

$$\eta \approx 1 \;\; \text{for} \;\; y_{\max}^2 \ll 1 \;\; (\text{or} \;\; v^2 \ll v_{\mathrm{point}}^2),$$

$$\eta \approx \frac{9}{2y_{\max}^2} \;\; \text{for} \;\; y_{\max}^2 \gg 1 \;\; (\text{or} \;\; v^2 \gg v_{\mathrm{point}}^2),$$

where $\beta = v/c$ is the neutrino velocity (nucleus velocity is neglected everywhere); c is the velocity of light; $v_{\mathrm{point}} = \dfrac{c}{2\mu a}$ is the characteristic minimal velocity at which the nucleus cannot be considered point-like. The nucleus size is $a = 1.25\ \mathrm{fm}\, A^{1/3} \dfrac{A^{1/3}}{158\ \mathrm{MeV}}$.

It is worth to note the important role of heavy nuclei in massive neutrino capture. Firstly, the cross section rises strongly with increasing atomic number, for $m_A \ll m_v$, $v^2 \ll v_{\mathrm{point}}^2$ it behaves as A^4. Secondly, for nuclei with the mass close to 50 GeV for kinematics reasons the probability of greater energy loss by neutrino in one collision increases. Correspondingly for them the probability of neutrino capture in one collision w_1 for that the neutrino has to lose the energy $T_\infty = 25$ keV will be greater. The value w_1 is defined by probability distribution in neutrino energy transferred to the nucleus. The form of such distribution depends on nucleus non-point-likeness: it has «table-like» form for point-like nucleus and in the opposite case, when the non-point-likeness is essential, falls down with the energy transfer. In the general case w_1 can be written as

$$w_1 = 1 - \frac{f(y_\infty)}{f(y_{\max})}, \tag{9}$$

where $y_\infty = \sqrt{m_v m_A}\beta_\infty a$, $\beta_\infty = v_\infty/c$. Function $f(y)$ is contained in η (see (8)) and has the form

$$f(y) = \frac{-1 + \cos 2y - 2y^2 + 2y \sin 2y + 2y^4}{2y^4} \tag{10}$$

with the limits

$f(y) \approx \dfrac{2y^2}{9}$ for $y \ll 1$ and
$f(y) \approx 1 - \dfrac{1}{y^2}$ for $y \ll 1$ (the small term $1/y^2$ is important in the case $y_{\max} \gg 1$ and $y_\infty \gg 1$);

w_1 is meaningful for $y_\infty > y_{\max}$ or

$$v < \gamma v_i nfty, \quad \gamma = \frac{m_v + m_A}{2\sqrt{m_v m_A}}, \tag{11}$$

$(v = \sqrt{v_\infty^2 + v_{\rm esc}^2(x)})$. In the case of the Earth the neutrino can be captured in collision with nuclei with the mass close to 50 GeV ($46 \div 54$ GeV) only. In this mass interval the nucleus of iron, which is abundant in the Earth, hits successfully. The probability w_1 reaches in this case $\sim 10^{-3}$. In the case of the Sun the inequality (11) holds for all nuclei except hydrogen. The probability w_1 for hydrogen (proton) is positive just in the region inside the Sun with $x \in [0; 0.29]$.

Using Eqs. (8), (9) for η and w_1, expression (3) can be transformed to

$$\dot{N}_{\rm capt} = \sum_A \frac{n_\infty}{v_\infty} \sigma_0 N_{A\,\rm tot} \frac{\int v^2 \eta w_1 n_A dV}{N_{A\,\rm tot}}. \tag{12}$$

Here $N_{A\,\rm tot} = \dfrac{\langle \alpha_A \rangle M}{A} N_A$ is the total number of nuclei A in gravitating body; $\langle \alpha_A \rangle$ is the mass fraction of element A in the whole gravitating body with mass M; N_A is Avogadro's number; $n_A = \dfrac{\alpha_A \rho}{A} N_A$ is the number density of nuclei A. In the limiting cases formula (12) can be simplified.

$$\begin{gathered} \dot{N}_{\rm capt} \cong \sum_A \tfrac{n_\infty}{v_\infty} \sigma_0 N_{A\,\rm tot} (\langle v_{\rm esc}^2 \rangle - \delta^2 v_\infty^2) \quad \text{for} \quad v^2 \ll v_{\rm point}^2, \\ \delta^2 \equiv \gamma^2 - 1 - \left(\tfrac{m_v - m_A}{m_v + m_A} \right)^2. \end{gathered} \tag{13}$$

In Eq. (13) $\langle v_{\rm esc}^2 \rangle$ means the square escape velocity averaged over the (number) density of nuclei A in the Earth or the Sun. In the case of homogenous relative abundance of element A over the volume of the gravitating body, i.e., when $\alpha_A = {\rm const} = \langle \alpha_A \rangle$, $\langle v_{\rm esc}^2 \rangle$ is reduced to square escape velocity averaged over total density

$$\langle v_{\rm esc}^2 \rangle = \frac{\int v_{\rm esc}^2(x) \rho \, dV}{M}.$$

In this case for chosen density distribution of the Earth we have $\langle v_{\rm esc\,E}^2 \rangle = \dfrac{6}{5} v_{\rm esc\,E}^2$. For the Sun one obtains $\langle v_{\rm esc\,S}^2 \rangle \approx 3.3 v_{\rm esc\,S}^2$.

In another limiting case Eq. (12) has the form

$$\dot{N}_{\rm capt} = \sum_A \frac{n_\infty}{v_\infty}\sigma_0 N_{A\,\rm tot}\frac{9}{2}v^2_{\rm point}(1 - f(y_\infty)) \quad \text{for} \quad v^2 \gg v^2_{\rm point}. \tag{14}$$

All notions being used in Eqs. (12)–(14) are introduced above.

The data on chemical composition of Earth's interiors, in which we are first of all interested in the iron abundance, are virtually absent. It is just known that the iron is likely to predominate in Earth's core and that it is contained in considerable amount in the mantle. We put iron mass fraction in the Earth equal to $\langle\alpha_{\rm Fe}\rangle = 20\,\%$ and we will consider its abundance homogeneous inside Earth's volume. The account of the increase of iron number density towards Earth's centre would lead to small increase of the result.

For simplicity we will assume the solar chemical composition homogeneous too and take it from the observation of solar atmosphere. The calculation based on the standard solar model shows, generally speaking, the increase of the helium fraction, decrease of the hydrogen fraction and radial variability of abundance of other elements. The account of such inhomogeneity would lead to small increase of the result mainly due to the increase of helium fraction.

2. ACCUMULATION AND ANNIHILATION OF MASSIVE NEUTRINOS IN THE EARTH AND SUN

Let us consider now the processes of accumulation and annihilation of neutrinos.

To calculate the rate of neutrino capture by the Earth we take into account $n \approx n_\infty$, $v \approx v_\infty$. It is convenient to use the expression (3) and to factor out the integral sign everywhere except w_1. Then this formula will be analogous to Eq. (13) with the additional factor η defined by (8) and we obtain

$$\eta \approx 0.86,$$

$$\langle w_1\rangle = \frac{6}{5}\frac{v^2_{\rm esc\,E}}{v^2_\infty} - \delta^2 \approx 1.47\cdot 10^{-3},$$

$$\dot{N}_{\rm capt} = n_\infty v_\infty \sigma_0 \eta N_{\rm Fe\,tot}\langle w_1\rangle \approx 1.0\cdot 10^{14}\ {\rm s}^{-1}.$$

The estimations show that the contribution of multiple collisions to neutrino capture does not exceed 10 %. Also the account of neutrino velocity distribution is not significant. Indeed, if the neutrinos have Maxwellian distribution with the mean velocity 300 km/s, then the result rises in 1.3 times. If we take into account in this distribution the motion of solar system (relative to galactic centre) with the velocity 200 km/s, then the result rises in less than 10 %. We do not expect a large difference in correction, connected with velocity distribution of neutrinos, for the case of the Sun.

The neutrinos, captured by Earth's field, during the time (of the order of a year) much less than Earth's age lose in collisions with the matter their energy down to thermal energy of nuclei and concentrate in Earth's core. We will further refer to this process as to the thermalization and to its characteristic time as to the time of thermalization. In order to estimate the amount of neutrino accumulated in the Earth, for simplicity we will assume that the concentration of thermalized neutrinos is distributed homogeneously inside the sphere of radius $x_{\rm therm}$. This radius is determined as the maximal distance at which a particle can be with total energy equal to $T_{\rm therm} + U(0)$, where $T_{\rm therm}$ and $U(0)$ are thermal energy of particles (molecules) and potential energy of neutrino in Earth's centre. The temperature in Earth's centre is not known exactly and it is supposed in the range $3000 \div 10000$ K. We will put $T_{\rm therm} = 1$ eV. The value of $x_{\rm therm}$ is obtained with the account of Eq. (4) from the equation

$$T_{\rm therm} - \frac{3T_{\rm esc\,E}}{2} = -T_{\rm esc\,E}\frac{(3 - x^2_{\rm therm})}{2},$$

$T_{\rm esc\,E} \equiv \dfrac{m_v v^2_{\rm esc\,E}}{2}$ eV, and is equal to $x_{\rm therm} \approx 0.24$. The accumulation of neutrinos inside the Earth (growth of thermalized neutrino concentration) is stopped when the rate of their annihilation becomes equal to capture rate $\dot{N}_{\rm ann.\,eq} = \dot{N}_{\rm capt}$ (the energy realization from annihilation in this case will be $0.84 \cdot 10^{13}$ erg/s that corresponds to $0.5 \cdot 10^{12}$ of the total energy flux incident on the Earth from the Sun). Note that the account of real thermodynamic distribution of thermalized neutrinos must lead to additional process of evaporation (escape out the Earth). But as it was shown in [9] for WIMP mass greater than about 10 GeV the annihilation rate significantly exceeds the evaporation one. So our approximation is in principle valid. The annihilation cross section of neutrinos is determined by weak neutrino interaction near Z-boson resonance. In nonrelativistic approximation the cross section of such annihilation into pair of neutrino and antineutrino of certain kind is

$$\sigma_{\rm ann.\,into\,v_e\bar{v}_e} \cong \frac{\bar{g}^4}{2^8\pi}\frac{m_v^2}{(4m_v^2 - m_Z^2)^2}\frac{1}{\beta^*} \approx \frac{1.28 \cdot 10^{-34}\ {\rm cm}^2}{\beta^*}.$$

And the total cross-section of annihilation is equal to

$$\sigma_{\rm ann} = \frac{\sigma_{\rm ann.\,into\,v_e\bar{v}_e}}{Br(Z \to v_e\bar{v}_e)} \approx \frac{1.93 \cdot 10^{-33}\ {\rm cm}^2}{\beta^*},$$

where $\bar{g}\sqrt{4\sqrt{2}G_{\rm F}m_Z^2}$ is the dimensionless constant of the weak interaction; $G_{\rm F}$ is the Fermi constant; m_Z is the Z-boson mass; $\beta^* = v^*/c$; v^* is the velocity of neutrino v_4 in the centre of mass system; $Br(Z \to v_e\bar{v}_e) \approx 6.67\,\%$ is the branching ratio of the Z-boson decay into neutrino and antineutrino pair of certain kind. The equilibrium concentration $n_{\rm eq}$ and the corresponding total amount $N_{\rm eq}$ of thermalized neutrinos will be ($\dot{N}_{\rm ann.\,eq} = (n^2_{\rm eq}/4)\sigma_{\rm ann}v_{\rm rel}V_{\rm therm}$, where $v_{\rm rel} = \sqrt{2}v^*$ is the mean relative velocity of neutrinos, $V_{\rm therm} = \dfrac{4}{3}\pi(R_{\rm E}x_{\rm therm})^3 \approx 1.5 \cdot 10^{25}\ {\rm cm}^3$)

$$n_{\rm eq} \approx 0.58 \cdot 10^6\ {\rm cm}^{-3},$$

$$N_{\rm eq} = n_{\rm eq}V_{\rm therm} \approx 0.88 \cdot 10^{31}\ \text{(corresponds to mass of 780 tons)}.$$

The time necessary to establish such equilibrium is equal to

$$t_{\rm eq} = \frac{N_{\rm eq}}{\dot{N}_{\rm capt}} \approx 2.6 \cdot 10^9\ {\rm y}.$$

The obtained time is comparable with Earth's age (it is only 2 times smaller), therefore the current (accumulated for Earth's lifetime) amount of neutrinos N can differ from N_{eq}. Extracting N from equation $\dot{N} = \dot{N}_{\text{capt}} - \dot{N}_{\text{ann}}$ we obtain

$$N = N_{\text{eq}} \tanh\left(\frac{t_{\text{E}}}{t_{\text{eq}}}\right) = 0.964 N_{\text{eq}},$$

where t_{E} is modern Earth's age. Correspondingly $\dot{N}_{\text{ann}}$ will be

$$\dot{N}_{\text{ann}} = 0.964^2 N_{\text{capt}} = 0.93 \dot{N}_{\text{capt}}.$$

In our approximation we can assume $N \approx N_{\text{eq}}$, $\dot{N}_{\text{ann}} \approx \dot{N}_{\text{capt}}$.

20 % of neutrinos v_4 annihilated via intermediate Z boson produce monochromatic neutrinos of known types ($v_e \bar{v}_e$, $v_\mu \bar{v}_\mu$ $v_\tau \bar{v}_\tau$) with the energy of 50 GeV. Their flux on Earth' surface will be

$$I = \frac{0.2 \dot{N}_{\text{ann}}}{4\pi R_{\text{E}}^2} \approx 4.1 \cdot 10^{-6}\ \text{cm}^{-2} \cdot \text{s}^{-1}.$$

The partial flux for each neutrino type («neutrino + antineutrino») is 3 times smaller.

For the Sun the neutrino capture rates on concrete nuclei $\dot{N}_{\text{capt. on}\,A}$ are given in Table 1 (the elements are arranged in the order of their concentration decrease in the Sun, the data about chemical composition were taken from [14]).

Table 1.

Element	$\dot{N}_{\text{capt. on}\,A}$, 10^{20} s^{-1}	Element	$\dot{N}_{\text{capt. on}\,A}$, 10^{20} s^{-1}	Element	$\dot{N}_{\text{capt. on}\,A}$, 10^{20} s^{-1}
$_1$H	0.94	$_{26}$Fe	3.43	$_{14}$Si	1.27
$_2$He	4.60	$_{16}$S	0.75	$_{24}$Cr	0.05
$_8$O	5.89	$_{18}$Ar	0.29	$_{17}$Cl	0.02
$_6$C	1.62	$_{13}$Al	0.11	$_{15}$P	0.01
$_{10}$Ne	1.94	$_{20}$Ca	0.15	$_{25}$Mn*	0.03
$_7$N	0.53	$_{11}$Na	0.05	$_{19}$K	0.01
$_{12}$Mg	0.97	$_{28}$Ni	0.18	$_{22}$Ti	0.01
$_{14}$Si	1.27	$_{24}$Cr	0.05	$_{27}$C0	0.01

*The lowest excited level of nucleus $_{25}$Mn^{55} is 126 keV. By interacting with neutrino with the energy (inside the Sun) of $131 \div 556$ keV the inelastic scattering is possible. Here we did not taken it into account. While calculating interaction cross-section the formula (7) was used.

The total rate of neutrino v_4 capture by the Sun is

$$\dot{N}_{\text{capt}} \approx 2.2 \cdot 10^{21}\ \text{s}^{-1}.$$

The total neutrino v_4 flux through the full solar surface is $n\sigma_{\text{S}} v = \dfrac{n_\infty}{v_\infty} v^2 \pi R_{\text{S}}^2 \approx 4.8 \cdot 10^{23}\ \text{s}^{-1}$. Here n and v are number density and velocity of neutrinos on the (level of) solar surface ($v = 637$ km/s), $\sigma_{\text{S}} = \pi R_{\text{S}}^2$ is Sun's cross section. One can see that the incident neutrino flux is much greater than $\dot{N}_{\rm capt}$. Therefore the approach, in which the change of incident neutrino concentration induced by interaction with a matter is neglected, is valid. Also it should be noted that the relation between $\dot{N}_{\rm capt}$ and incident flux together with the applicability of our approach does not depend on n_∞.

The thermalization time of neutrinos captured by the Sun is much smaller than Sun's age. It is of the order of a year as well as for the Earth. Neutrino kinetic energy in solar centre will be $T_{\rm therm} = 1.9$ keV, potential energy is obtained from expression (6) by multiplying it by the factor $-\mu_\nu/2$. Then $x_{\rm therm} \approx 0.017$, $V_{\rm therm} \approx 0.69 \cdot 10^{28}$ cm^3, $n_{\rm eq} \approx 1.3 \cdot 10^8$ cm^{-3}, $N_{\rm eq} \approx 0.87 \cdot 10^{36}$ (that corresponds to mass $0.78 \cdot 10^{14}$ g), $t_{\rm eq} \approx 1.2 \cdot 10^7$ y, that means $N = N_{\rm eq}$, $\dot{N}_{\rm ann} = \dot{N}_{\rm capt}$. The energy realization from v_4 annihilation is $1.8 \cdot 10^{20}$ erg/s $=$ $4.7 \cdot 10^{14} L_{\rm S}$, where $L_{\rm S} = 3.8 \cdot 10^{33}$ erg/s is the luminosity of the Sun.

The 50 GeV annihilational neutrinos of known types will be partially captured inside the Sun. The cross section of neutrino and antineutrino (with the energy of 50 GeV) interaction with nucleon differ from each other and are [15] $5.1 \cdot 10^{37}$ cm^2 and $2.6 \cdot 10^{37}$ cm^2, respectively. Note the difference of cross sections for different kinds of annihilational neutrinos is negligible. The fractions of neutrinos and antineutrinos escaping from the Sun will be respectively 0.63 and 0.79. The fluxes on the Earth of monochromatic neutrinos, antineutrinos and summary («neutrinos + antineutrinos») from v_4 annihilation in the Sun will be

$$I_v = 0.50 \cdot 10^7 \ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1},$$

$$I_{\bar{v}} = 0.63 \cdot 10^7 \ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1},$$

$$I_{(v\bar{v})} = 1.1 \cdot 10^7 \ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1}.$$

For each neutrino kind these fluxes are correspondingly 3 times smaller.

The most uncertain parameter in our model is the neutrino number density n_∞, which is proportional to $\xi_{\rm S}\Omega$. The values $\dot{N}_{\rm capt}$, I and equilibrium time $t_{\rm eq}$ depend on the number density n_∞. The results are generalized for the case of arbitrary $\xi_{\rm S}\Omega$ by the following way

$$I \approx 1.1 \cdot 10^{-7} \left(\frac{\xi_{\rm S}\Omega}{2}\right) \tanh^2\left(\sqrt{\frac{\xi_{\rm S}\Omega}{1.2 \cdot 10^{-5}}}\right) \ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1} \ - \ \text{for the Sun.}$$

For the Earth, generalizing also the results for the case of arbitrary mass fraction of iron, we have

$$I \approx 4.1 \cdot 10^{-6} \left(\frac{\xi_{\rm S}\Omega}{2}\right) \left(\frac{\alpha_{\rm Fe}}{20\,\%}\right) \tanh^2\left(\sqrt{\frac{\xi_{\rm S}\Omega}{0.5}\frac{\alpha_{\rm Fe}}{20\,\%}}\right) \ \mathrm{cm}^{-2} \cdot \mathrm{s}^{-1} \ - \ \text{for the Sun.}$$

Here $\xi_{\rm S}\Omega = 2$ corresponds to the magnitude used in this work ($\xi_{\rm S}\Omega = 2 \cdot 10^4 \cdot 10^{-4} = 2$, $n_\infty = 2 \cdot 10^{-7}$ cm^{-3}), $\tanh x \approx 1$ for $x > 1$.

The solid angle of the annihilational neutrino flux from Earth's core is about 1sr, the one for the Sun will be determined by angle resolution of detector. The fluxes of atmospheric neutrinos depend on direction especially for high energy electronic neutrinos [16]: for horizontal direction the fluxes are greater than for vertical one. The comparison of fluxes of monochromatic annihilational neutrinos with the atmospheric ones in vertical direction for different energies is presented in Table 2. In the first column the ratios of considered fluxes

Table 2.

	50 GeV	>50 GeV	>1 GeV
$v_\mu \bar{v}_\mu$	3	0.14	$0.6 \cdot 10^{-4}$
$v_e \bar{v}_e$	50	2.4	$2 \cdot 10^{-4}$

are given for 50 GeV with 1 GeV (conventional) resolution, in other columns the integral atmospheric neutrino fluxes were taken.

Note that the analogous ratios for the averaged over angles atmosphreric fluxes are a few times smaller. It should be reminded that these relations were obtained for $\xi_\mathrm{S}\Omega = 2$, $\alpha_\mathrm{Fe} = 20\,\%$.

It is important to note that the monochromatic e-, μ-, τ-neutrino (in equal amounts) fluxes are characteristic signature of the v_4 annihilation as compared to analogous case of accumulation and annihilation in Sun's and Earth's cores of another popular candidate to WIMP — neutralino. Neutralino due to its Majorana nature cannot annihilate directly into neutrinos. Neutrinos are produced from decays of neutralino annihilation products and as a result of interaction of these products with the environment. Because of that their spectrum becomes more soft. Much more muonic, and very few taonic neutrinos are produced. The same soft part of e-, μ-, τ-neutrino spectrum is present in the v_4 annihilation case, too.

3. THE EFFECTS OF NEW LONG-RANGE INTERACTION IN THE 4th NEUTRINO ANNIHILATION

The existence of the 4th generation of quarks and leptons, to which the 4th neutrino belongs, can be theoretically grounded in the framework of superstring theory. In its realistic variants in the low-energy limit the additional to standard model $U(1)$-gauge symmetry (or even several) appears [17], which can be ascribed to the 4th fermion generation only and does not involve quarks and leptons of the other 3 known generations. The new $U(1)$-gauge interaction can be similar to the electromagnetic one (e/m). For definiteness let us denote its «fine structure» constant as α_y, and its gauge massless boson call y photon. The influence of new interaction on the effects of the 4th generation neutrino will be considered in separate work. Here we will briefly consider how new interaction influences the effects of accumulation and annihilation in the Earth and Sun. New interaction leads to new annihilation channel $v_4\bar{v}_4 \to yy$. The cross section of this process is analogous to cross section of 2-photon annihilation of electrons and in the nonrelativistic limit $\alpha_y \ll \beta^* \ll 1$ is given by [18]

$$\sigma_{y-\mathrm{ann}} \cong \frac{\pi\alpha^2}{2m_v^2}\frac{1}{\beta^*} \approx \frac{1.30\cdot 10^{-35}\ \mathrm{cm}^2}{\beta^*}\left(\frac{\alpha_y}{\alpha}\right)^2,$$

where $\alpha = \dfrac{1}{137}$ is the e/m constant. This cross section is by more than two orders of magnitude smaller than via Z boson one (15) with $\alpha_y = \alpha$. Therefore when $\alpha_y \sim \alpha$ the dominant neutrino annihilation channel will still proceed via Z boson. The new interaction constant can be larger ($\alpha_y > \alpha$), but for all reasonable values of α_y, following from GUT models, even in this case Z-boson mediated process dominates.

Another manifestation of new interaction plays more important role. The existence of analogous to Coulomb long range interaction between neutrinos for their velocities $\beta^* < 2\pi\alpha_y$ leads to the increase of the interaction cross section («Sakharov enhancement»), including annihilation via Z boson, being expressed by «Coulomb factor» [19]

$$C(\beta^*) = \frac{2\pi\alpha_y/\beta^*}{1 - \exp\left(-2\pi\alpha_y/\beta^*\right)}.$$

For $\alpha_y = \alpha$ and velocity of neutrinos in the Galaxy 300 km/s, velocities (maximal) of the thermalized neutrinos corresponding to $T_{\rm therm}$ in the Earth and Sun this factor is equal to, respectively, 46, 7300, 166, that correspondingly increases the annihilation cross sections. In the picture considered above the values $n_{\rm eq}$, $N_{\rm eq}$, $t_{\rm eq}$ decrease inversely proportional to the square root of Coulomb factor, that is 85 times for the Earth and 13 times for the Sun. The fluxes of e, μ, tau neutrinos from annihilation v_4 correspond to equilibrium case $\dot{N}_{\rm ann.\,eq} = \dot{N}_{\rm capt}$ and do not change, and since as earlier $t_{\rm eq} \gg t_{\rm therm}$ the neutrinos v_4 have time to concentrate in the core before they annihilate. So, owing to kinetic equilibrium the existence of new interaction does not influence the estimated neutrino fluxes from annihilation.

CONCLUSION

This work continues the studies of effects of the existence of the 4th generation neutrino [$3 \div 7$]. A number of effects of new neutrinos allows one to separate their signatures on the background of other candidates to WIMP and can serve as an instrument to discriminate the components of multicomponent galactic dark matter. For example, the annihilation v_4 in halo leads to appearance in cosmic positron spectrum of characteristic feature near energy m_v [4, 5]. For annihilation v_4 in the Earth and Sun the signature is the prediction of fluxes of monochromatic (with the energy $= m_v$) neutrinos of known kinds including τ neutrinos. It is more difficult to distinguish v_4 from other WIMPs with the use of soft continuous part of spectrum of annihilational neutrinos connected with the other channels of annihilation v_4. Moreover this soft part of spectrum has comparably low spectral density because of comparably small density of relic v_4. For the search of the 4th neutrinos effects from Sun, could be of interest the difference of about 25 % in fluxes of monochromatic neutrinos and antineutrinos, appearing due to more strong neutrino capture by solar matter in comparison with antineutrino one. Also it can be noted that the flux from v_4 annihilation in halo of the Galaxy depends stronger on their number density in the Galaxy n, as n^2, than that from Earth's or Sun's cores, which is proportional to n (provided that the equilibrium is established). So the uncertainty in obtained fluxes connected with sufficiently large uncertainty of neutrino number density in the Galaxy is less significant in the latter case.

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