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ON VERY HIGH MULTIPLICITY DISTRIBUTIONS

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We show within a geometrical model developed in earlier papers that multiplicity distributions are cut off at large multiplicities. The sharpness of this cut-off is related to the sharpness (narrowness) of the assumed impact-parameter distribution of the multiplicities. The position and motion of the cut-off point are related to geometrical and KNO scaling and their violation, in particular, by the rise of the ratio $\sigma_{\rm el}/\sigma_{\rm t}$. At the LHC energies a change of the regime, connected with the transition from shadowing to antishadowing, is expected.

В рамках развитой ранее геометрической модели показано, что при очень больших множественностях распределения частиц обрываются. Резкость края связана с узостью предполагаемого распределения множественностей по прицельному параметру. Положение и движение точки обрыва (края распределения) связаны с геометрическим и KHO-скейлингом, а также с их нарушением, в частности с ростом отношения упругого к полному сечению. При энергиях ускорителя LHC ожидается смена режима, связанная с переходом от экранировки к антиэкранировке.

Our knowledge about high-energy multiplicity distributions comes from the data collected at the ISR, $Sp\bar{p}S$ collider (UA1, UA2 and UA5 experiments) and the Tevatron collider (CDF and E735 experiments). It should be noticed that the recent results from the E335 Collaboration taken at the Tevatron [1] do not completely agree with those obtained by the UA5 Collaboration at comparable energies at the $Sp\bar{p}S$ collider [2]. Notice that the delicate features of $\Psi(z)$ at very large multiplicities, near the large-z edge can be better seen if the variable z is used instead of n.

The properties of multiplicity distribution of secondaries $P(n)$ at high values of n remain among the topical problems in high-energy physics. As pointed out in a series of recent papers [3], the underlying dynamics behind these rare processes may be quite different from the bulk of events.

It became common [4–7] to approximate the observed distributions by the convolution of two binomial distributions, accounting for the general «bell-like» shape of $P(n)$ with the observed structures («knee» and possible oscillations) superimposed.

One of the hottest issues in this field is the dynamics of very high multiplicities (VHM) [8], close to the kinematical limit imposed by the phase space. The VHM events are very rare, making up only about 10^{-7} of the total cross-sections at the LHC energy, which makes their experimental identification very difficult. An intriguing question is the possible existence of a cut-off in the VHM region, beyond $z = n/\langle n\rangle \approx 5$, where $\langle n\rangle$ is the mean multiplicity. In our opinion, a better understanding of the underlying physics can be inferred only in a model

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involving both elastic and inelastic scattering, related by unitarity. Such an approach has been advocated in a series of papers [9–11], summarized in Ref. [12].

After a brief summary of the main ideas behind this approach, we analyze the relation of the distribution of secondaries at very high multiplicities and the possible transition from shadowing to antishadowing manifests itself in the behavior of the elastic and total cross-sections [13].

The basic idea of the geometrical approach to multiple production, used in the present paper, is that the number of the secondaries at a given impact parameter ρ, $n(\rho, s)$ is proportional to the amount of the hadronic matter in the collision or the overlap function $G(\rho, s)$

$$\langle n(\rho, s)\rangle = N(s)G(\rho, s), \tag{1}$$

where $N(s)$ is related to mean multiplicity, not specified in this approach, and $G(\rho, s)$ is the overlap function, related by unitarity to elastic scattering

$$\mathrm{Im}\, h(\rho, s) = |h(\rho, s)|^2 + G(\rho, s), \tag{2}$$

where $h(\rho, s)$ is the elastic amplitude in the impact parameter representation.

We show that the existence of a cut-off at high multiplicities in the distribution $\Psi(z)$ is related to the validity of GS and KNO scaling. The «cut-off» in question results from the assumed impact-parameter dependence of the multiplicities. For the simple case of a narrow (δ function) distribution the resulting cut-off is sharp (a Θ function). More realistic, smoothed distribution in the impact parameter will produce a «shoulder» — in n (or in z) — a gradual change in the slope $B(n) = d/dn\, \ln P(n)$ at highest n (or z).

Unitarity, a key issue in this approach, enters the definition of both the elastic amplitude and the inelastic one (the overlap function).

In the u-matrix unitarization (see [12] and references therein)

$$G_{\mathrm{in}} = \frac{\mathrm{Im}\, u}{1 + 2\mathrm{Im}\, u + |u|^2}, \tag{3}$$

where u is the elastic amplitude (input, or the «Born term»).

We use a dipole (DP) model for the elastic scattering amplitude, exhibiting geometrical features and fitting the data. After u-matrix unitarization, the elastic amplitude reads (see [12])

$$h(\rho, s) = \frac{u}{1 - iu}, \tag{4}$$

where $u(y, s) = ig\, \mathrm{e}^{-y}$, $y = \rho^2/(4\alpha' L)$, and $L \equiv \ln s$.

Remarkably, the ratio of the elastic to total cross-sections in this model,

$$\frac{\sigma_{\mathrm{el}}}{\sigma_{\mathrm{t}}} = 1 - \frac{g}{(1+g)\ln(1+g)} \tag{5}$$

fixes the (energy-dependent) values of the parameter g. Typical values of g for several representative energies are quoted in [12].

Rescattering corrections to $G_{\mathrm{in}}(\rho, s)$ here will be accounted for phenomenologically according to the following prescription (see [12] and earlier reference therein):

$$G_{\mathrm{in}}(\rho, s) = |S(\rho, s)|\tilde{G}_{\mathrm{in}}(\rho, s), \tag{6}$$

where $S(\rho, s)$ is the elastic scattering matrix, related to the u matrix by

$$S(\rho, s) = \frac{1 + iu(\rho, s)}{1 - iu(\rho, s)}. \tag{7}$$

This procedure is not unique. For example, it allows the following generalization (see [12] and earlier reference therein):

$$G_{\text{in}}(\rho, s) = |S(\rho, s)|^{\alpha} \tilde{G}^{\alpha}(\rho, s), \tag{8}$$

where α is a parameter, varying between 0 and 1.

We assume

$$\langle n(\rho, s) \rangle = N(s) \tilde{G}^{\alpha}_{\text{in}}(\rho, s). \tag{9}$$

The moments are defined by (see [12] and earlier references therein)

$$\langle n^k(s) \rangle = \frac{N^k(s) \int G_{\text{in}}(\rho, s)(G^{\alpha}_{\text{in}}(\rho, s))^k d^2\rho}{\int G_{\text{in}}(\rho, s) d^2\rho}. \tag{10}$$

Now we insert the expression for the DP with the u-matrix unitarization (4) into (10) to get

$$\langle n^k(s) \rangle = \frac{N^k(s)(1+g)}{g} \int\limits_0^g \frac{dx}{(1+x)^2} \left(\left(\frac{1+x}{1-x} \right)^{\alpha} \frac{x}{(1+x)^2} \right)^k. \tag{11}$$

The mean multiplicity $\langle n(s) \rangle$ is defined as

$$\langle n(s) \rangle = \frac{N(s)(1+g)}{g} \int\limits_0^g \frac{x dx}{(1+x)^4} \left(\frac{1+x}{1-x} \right)^{\alpha} = \frac{N(s)}{a}. \tag{12}$$

For the distributions, we have

$$P(n) = \frac{1+g}{g} \int\limits_0^g \frac{dx}{(1+x)^2} \delta \left(n - N \left(\frac{1+x}{1-x} \right)^{\alpha} \frac{x}{(1+x)^2} \right). \tag{13}$$

Integration in (13) gives

$$\psi(z) = \langle n \rangle P(n) = \frac{1+g}{g} \frac{x(1-x)}{z(1+x)[(1-x)^2 + 2\alpha x]},$$

where $z = n/\langle n \rangle$.

Since the above integral is nonzero only when the argument of the δ function vanishes,

$$n = N \left(\frac{1+x}{1-x} \right)^{\alpha} \frac{x}{(1+x)^2},$$

one gets a remarkable relation

$$z = \frac{ax}{(1+x)^{2-\alpha}(1-x)^{\alpha}}. \tag{14}$$

To calculate the distribution $\Psi(z)$, one needs the solution of equation (14). It can be found explicitly for two extreme cases, namely, $\alpha = 0$ and $\alpha = 1$. Otherwise, it can be calculated numerically.

The maximal value of z, corresponding to $x = g$ (x varies between 0 and g), can be found as

$$z_{\max} = \frac{ag}{(1+g)^{2-\alpha}|(1-g)|^{\alpha}}. \tag{15}$$

It can be seen from (15) that $z_{\max}$ is a constant if g is energy-independent. The experimentally observed ratio $\sigma_{\rm el}/\sigma_{\rm t}$ varies between 53 and 900 GeV from 0.174 to 0.225, implying the variation of g from 0.489 to 0.702, uniquely determined by the above ratio. This monotonic increase of $g(s)$ in its turn pushes $z_{\max}(s)$ outwards, terminating when g reaches unity (according to [12] this will happen around 10 TeV, i. e., at the future LHC), when after the term $|1-g|$ in (15) will start rising again, pulling $z_{\max}(s)$ back to smaller values. That is, $z_{\max}(s)$ has its own maximum in s at $g = 1$.

The unusual behavior of $z_{\max}(s)$ is not the only interesting feature of the present approach. This effect can be related to the behavior of the ratio $\sigma_{\rm el}/\sigma_{\rm t}$. As argued by Troshin and Tyurin (see [13]), $\sigma_{\rm el}/\sigma_{\rm t}$ may pass the so-called black disc limit and continue rising in a new, «antishadowing» mode of the u-matrix unitarity approach (multiplicity distributions were not considered in that paper). According to the recent calculations [14] the transition from shadowing to antishadowing will also occur in the LHC energy region.

To summarize, we found a regularity connecting the geometrical properties in high-energy dynamics (GS and KNO scaling) with the dynamics of the high-multiplicity processes. We showed, in particular, that exact geometrical, or KNO scaling, implying constant g in our model, results in a cut-off at large z of the distribution function $\Psi(z)$. Any departure from scaling (energy dependence of g in our model) shifts the point z_m according to Eq. (15). Within the present accelerator energy domain (ISR, SPS, Tevatron), g varies from about 0.5 to about 0.8. It will reach the critical value $g = 1$ at LHC, where we predict a change of the regime: $z_{\max}(s)$ will start decreasing and the black disc limit will be passed (which, as shown in [13, 14], is not equivalent to the violation of the unitarity limit, but means passage from shadowing to antishadowing [13, 14]).

It should be noted that the model assumptions reduce the predictive power of model. These assumptions concern mostly the way absorption corrections are introduced and the assumption of the local (δ function) dependence of multiplicities on the impact parameter. Both assumptions, as well as others, can be modified. As a result, we have quantitative rather than qualitative changes in the results.

To test our prediction, the local slope $B(n)$ should be calculated directly from the data. A rapid change of $B(n)$ near highest values of n (or z) will already indicate the presence of a cut-off under discussion.

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