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THE EFFECT OF ELECTROMAGNETIC INTERACTIONS ON THE PROTON SPECTRUM IN FREE NEUTRON β-DECAY

G.G.Bunatian

In the β decay of an unpolarized free neutron, the effect of electromagnetic interactions on the proton recoil spectrum is studied in the light of the experiments which are carried out and planned for now. The corrections to the energy distribution of protons prove to amount to the value of a few per cent. Nowadays, this is substantial for obtaining with a high accuracy, of $\sim 1\,\%$ or better, the characteristics of weak interactions by processing the data of the experiments on the proton distribution in the free neutron β-decay.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

Влияние электромагнитных взаимодействий на спектр протонов в β-распаде свободного нейтрона

Г.Г.Бунатян

Влияние электромагнитных взаимодействий на спектр протонов отдачи в β-распаде неполяризованного свободного нейтрона изучается в свете экспериментов, которые на сегодняшний день уже проводятся и планируются. Получено, что поправки к импульсному распределению протонов достигают величины нескольких процентов. В настоящее время это существенно для определения с высокой точностью, $\sim 1\,\%$ и лучше, характеристик слабых взаимодействий из обработки данных, получаемых в экспериментах, где исследуется импульсное распределение протонов в β-распаде свободного нейтрона.

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Nowadays, it has been well realized that the characteristics of weak interactions are to be acquired with a precision better than 1% in order to judge definitively the validity of the general principals of the modern elementary particle theory. The treatment of the β-decay of free neutrons has been rightly conceived to provide the straightforward way to inquire into weak interactions in general.

Despite the study of the neutron β-decay has been lasting long since, properly speaking, for all the time of neutron physics itself existing, it was restricted until a little while ago by the high-precision reliable inquiry into the neutron lifetime τ and the electron momentum distribution only, as a matter of fact (see, for instance, [1]). Nowadays, the situation is thought to alter towards the study of the proton distribution in the final state of neutron β-decay. The various new experiments as well as the substantial improvements of some previous ones [2,3] have recently been launched in order to obtain, with an accuracy better than 1 %, both the proton distribution itself solely and the distributions of electrons and protons simultaneously; the appropriate measurements are believed to come to fruition before long. What encourages us to set out our modest calculation is just the research [2,3].

Hence, the proton distribution after the β-decay of unpolarized neutrons,

$$dW(E_P) = dW(|\mathbf{P}|) = w(E_P)dE_P = w(|\mathbf{P}|)d|\mathbf{P}|, \tag{1}$$

currently measured in [2, 3], is calculated outright in terms of the received effective Lagrangian, descending from the general field theory (see, for instance, Refs. 4, 5),

$$L_{\rm int} = L_{pnw} + L_{e\gamma} + L_{p\gamma}\ , \tag{2}$$

$$L_{pnw}(x) = \frac{G_F\cdot|V_{ud}|}{\sqrt{2}}(\bar{\psi}_e(x)\gamma^\alpha(1+\gamma^5)\psi_\nu(x))\ \times$$
$$\times\bar{\Psi}_p(x)[\gamma_\alpha g_V(q) + g_{WM}\sigma_{\alpha\nu}q_\nu + (\gamma_\alpha g_A(q) + g_{IP}q_\alpha)\gamma^5]\Psi_n(x), \tag{3}$$

which includes the interaction of an electromagnetic field A with electrons

$$L_{e\gamma}(x) = -e\bar{\psi}_e(x)\gamma^\mu\psi_e(x)\cdot A_\mu(x), \tag{4}$$

and similarly with protons, $L_{p\gamma}$. In (3), (4), the notations are alike ones in Ref. 4, q is the four-momentum transferred in the β-decay process, the system of units $h = c = 1$ is adapted, $g_V^n(0) = 1$ is presumed for the neutron decay; $\Psi_n(x), \Psi_p^+(x)$ render the baryon fields in the initial and final states, and ψ_e, ψ_ν stand for the electron and antineutrino fields, respectively. The value $G_F = 1.16639(2)\cdot 10^{-5}$ GeV^{-2} has been fixed by the muon lifetime [6], and $|V_{ud}| \approx 0.9744 \pm 0.0010$ [1,6] is the Cabibbo–Kobayashi–Maskawa, CKM, [7] quark-mixing matrix element. The value $g_A(0) = 1.2662$ [1, 6] is also adopted in the further numerical evaluations.

In the common simplified treatment, when the electromagnetic interaction (4) is turned off, the nucleon mass is presumed to be infinite, $M_N \to \infty$, and, consequently, the terms with g_{WM}, g_{IP} disappear and the q-dependence of $g_V(q)$, $g_A(q)$ is negligible, only the very terms with $g_V(0)$, $g_A(0)$ in (3) cause the bulk of the yield of protons (1) at a given value of the momentum $P = |\mathbf{P}|$ [8–10]

$$dW_0(P; |V_{ud}|, g_V(0), g_A(0)) = dP\cdot w_0(P; |V_{ud}|, g_V(0), g_A(0)). \tag{5}$$

So far as an accuracy better than 1% has to be procured, all the peculiarities in describing β-decay became of value and must be properly allowed for. In the well-known work [11], all the corrections entailed by accounting for the finiteness of the nucleon mass have been thoroughly evaluated in studying the electron and antineutrino distribution [1]. The results of Ref. 11 can be adjusted to acquire the correspondent corrections to the distribution (1); for that case, the calculations [9] might be referred to. Here, we elaborate the effect of electromagnetic interactions on the distribution (5).

With the consistent allowance for electromagnetic interactions, calculating the distribution (1) runs in general way [4,5,12], and after a good deal of the plain and unsophisticated, but slightly cumbersome and time-consuming calculations (for details see Ref. 13), the distribution (1), to the first order in the fine structure constant α, sets out in the eventual form

$$
\begin{aligned}
& dW(P;|V_{ud}|, g_V, g_A, \alpha) = dP \cdot w(P;|V_{ud}|, g_V, g_A, \alpha) = \\
& = dP{\cdot}P\frac{G_F^2|V_{ud}|^2}{4\pi^3}\int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\,\varepsilon\,\omega_{\nu 0}\Bigg\{(1+3g_A^2)\Big\{(1+a_0\mathcal{N})\Bigg(1+ \\
& +\frac{\alpha}{2\pi}\Bigg(\frac{2}{v}\mathcal{J}+3\ln(\frac{M_p}{m})-\frac{9}{2}-4\ln(\frac{2\omega_{\nu 0}}{m})\big(1+\frac{1}{2v}\ln(x)\big)-\frac{2}{v}\mathcal{K}- \\
& -v^2\int\limits_{-1}^{1}dy\frac{1-y^2}{(1-vy)^2}\ln\Big(\frac{v\varepsilon+y\omega_{\nu 0}+\sqrt{{\omega_{\nu 0}}^2+v^2\varepsilon^2+2yv\omega_{\nu 0}\varepsilon}}{2v\varepsilon}\Big)\Bigg)\Bigg)+ \\
& +\frac{\alpha}{2\pi}\Bigg(-\frac{1}{v}\ln(x)\big(v^2+a_0\mathcal{N}\big)+\frac{v}{\omega_{\nu 0}}\times \\
& \times\Bigg\{\int\limits_{-1}^{1}dx[v^2\big((\omega_{\nu 0}-\varepsilon)I_0-I_1\big)\frac{1-x^2}{b^2}+\frac{1}{b\varepsilon}\big(\omega_{\nu 0}I_1-I_2\big)]+ \\
& +\int\limits_{\tilde{k}(\varepsilon)}^{k(\varepsilon)}\frac{dk}{k}(\omega_{\nu 0}-k)[v^2(k+\varepsilon)\big(i_0-i_2\big)+\frac{k^2}{\varepsilon}\big(i_0-vi_1\big)]\Bigg\}- \\
& -\frac{a_0 v}{2\varepsilon\omega_{\nu 0}}[\int\limits_{0}^{\omega_{\nu 0}}dk\big(A(1)+B(1)+C(1)\big)+\int\limits_{\tilde{k}(\varepsilon)}^{\omega_{\nu 0}}dk\big(A(x_2)+B(x_2)+D(x_2)\big)]\Bigg)\Big\}+ \\
& +\frac{\alpha}{2\pi}\Bigg(\frac{7}{4}+6g_A+\frac{33}{4}g_A^2+\ln(\frac{\Lambda}{M_p})\big(3\mathcal{N}(1-g_A^2)+9g_A^2+12g_A+3\big)\Bigg)\Bigg\}.
\end{aligned}
\tag{6}
$$

Here, the notations are introduced: $\omega_{\nu 0}=\Delta-\varepsilon$, $\Delta=M_n-M_p$, $p_e=v\varepsilon$,

$$\varepsilon_{1,2}=-\frac{1}{2}[\pm P-\Delta+\frac{m^2}{\pm P-\Delta}]\,,\quad a_0=\frac{1-g_A^2}{1+3g_A^2},$$

$$\mathcal{N}(\varepsilon,P)=-\frac{1}{2\omega_{\nu 0}\varepsilon}(p_e^2-P^2-\omega_{\nu 0}^2)\,,\quad \tilde{v}_P(P,\varepsilon)=\sqrt{v^2-\frac{m}{M_p\varepsilon^2}[\omega_{\nu 0}^2-P^2(\frac{m}{M_p}+1)]}\,,$$

$$\mathcal{J}=[\Big(\frac{1}{2}\ln(x)\Big)^2-\mathcal{F}(1/x-1)+\pi^2\frac{v}{\tilde{v}_P(P,\varepsilon)}]\,,\quad x=\frac{1-v}{1+v},\quad \mathcal{F}(z)=\int\limits_0^z\frac{dt}{t}\ln(1+t)\,,$$

$$\mathcal{K} = \frac{1}{2}(\mathcal{F}(x) - \mathcal{F}(1/x) - \ln(1/x)\cdot\ln(\frac{1-v^2}{4})) - \\ -v + \frac{1}{2}\ln(x) + \mathcal{F}(v) - \mathcal{F}(-v),$$

$$\tilde{k}(\varepsilon) = (\omega_{\nu 0} + P - \varepsilon v)/2\,, \quad k(\varepsilon) = (\omega_{\nu 0} - P + \varepsilon v)/2\,,$$

$$i_n(k,\varepsilon) = \int\limits_{-1}^{x_2(k,\varepsilon)} \frac{dx x^n}{r b^2}\,, \quad b = 1 - xv\,, \quad I_n(\omega_{\nu 0}) = \int\limits_{0}^{\omega_{\nu 0}} \frac{dk k^n}{r}\,,$$

$$A(x) = (\omega_\nu^2 - P^2)\int\limits_{-1}^{x} \frac{dx\cdot d(x)}{l^3}\,, \quad \omega_\nu = \omega_{\nu 0} - k\,, \quad B(x) = \int\limits_{-1}^{x} \frac{dx\cdot d(x)}{l}\,, \tag{7}$$

$$C(x) = \int\limits_{-1}^{x} \frac{dx 2\varepsilon v^2(1-x^2)}{l b^2}(xv\varepsilon - \omega_\nu)\,, \quad D(x) = \int\limits_{-1}^{x} \frac{dx}{l k b^2}(\omega_\nu^2 + l^2 - P^2)\varepsilon(1-x^2)v^2\,,$$

$$r(k,x) = \sqrt{v^2\varepsilon^2 + 2vkx\varepsilon + k^2}\,, \quad d(x) = [l^2 + \varepsilon^2(1-xv^2) + k\varepsilon b - \varepsilon(\varepsilon + k)]/b\,,$$

$$l = \sqrt{k^2 + {p_e}^2 + 2p_e k x}\,, \quad x_2(k,\varepsilon) = \frac{1}{2kp_e}[(\omega_\nu + P)^2 - (p_e + k)^2] + 1 \le 1\,,$$

and $m = 0.511$ MeV, $M_n = 939.57$ MeV, $M_p = 938.28$ MeV are the electron, neutron, and proton masses, respectively; ε and v stand for the electron energy and velocity. For briefness's sake, we don't pull out explicitly the vast expressions of the integrals in (7), though they all are amenable to straightforward analytical evaluation. Surely, on setting α=0 we arrive at the uncorrected distribution (5).

With the received interaction (2)–(4) underlying the inquiry, the ad hoc effective cut-off parameter Λ has emerged in order to prevent the ultraviolet divergencies which would come, as usually (see, for instance, [4]), from the integrals over four-momenta of virtual photons which occur in calculating (6). We are not on the point of treating the whole problem how to remove the ultraviolet divergency out of the radiative corrections to neutron β-decay. In the course of our upright calculation, we just take for granted the received recipe, first set forth in Refs. 14 and perfectly confirmed in the profound papers [15], which prescribes the Λ value to be equal to the mass of Z- or W-boson, $M_Z \approx 91$ GeV, $M_W \approx 80$ GeV. So, the numerical evaluations are performed presuming $\Lambda = M_Z$. While the dependence of (6) on the Λ value is slight enough as being due to the terms $\sim \ln(\Lambda/M_p)$, the contribution from these terms into (6) is of value, so far the accuracy about 1% or better goes. What is to emphasize here is that the contribution $\sim \ln(\Lambda/M_p)$ in (6) would vanish at all, as in the case of the decay of the μ meson [16], if the relation $g_A = -g_V$ (i.e., $g_A = -1$ in the notations adopted here) held true, in perfect agreement with the general theorem ascertained in Ref.17. It might be well to point out that, till now, this stringent constraint has not been adhered to in many a calculation (see, for instance, [18]).

It is of value that the electromagnetic corrections to (5) due to the terms multiple of a_0 in (6) is of the same order as the corrections which do not depend on the combination a_0 immediately. Let us recall that the term in (6) containing $1/\tilde{v}_P$ is usually associated with the «Coulomb correction» (see, for instance, [18, 19]). What is of value to emphasize here is that this term has been wrought up in our treatment, simultaneously with all the other electromagnetic corrections, in outright consistent evaluating dictated directly by the original effective interaction (2)–(4).

So, to the first α-order, we have acquired the complete effect of electromagnetic interactions on the proton distribution (1), without treating separately the «Coulomb correction» and the so-called «model-independent» and «model-dependent» parts of the radiative corrections as contrasted to what was done in the investigations [18–21]. Being immediately expressed just in terms of the quantities $g_A, |V_{ud}|, ...$ involved in (3), our final result (6) stands in one-to-one correspondence with the form of the original effective interaction (2)–(4), unlike the results set out in Refs. 18–21. Thus, confronting (6) with the experimental data of [2, 3] we get in position to judge the validity of the form of the original effective interaction (2)–(4), in particular, to ascertain the values of $g_A, |V_{ud}|, ...$

The effect of electromagnetic interactions on the proton distribution is described by the modification

$$\delta\big(w(P)/W\big) = \frac{w(P)/W - w_0(P)/W_0}{w_0(P)/W_0}\,, \tag{8}$$

of the relative distribution

$$w_0(P)/W_0\,, \tag{9}$$

where W_0 and W stand for the uncorrected and corrected total decay probability, respectively. The quantity (8) is set out in Fig. 1. Let us note the curious peculiarity on the curve at $P = \Delta - m = 0.7833$ MeV caused just by the very term with $2\pi^2/\tilde{v}_P$ in (6). Certainly, the modification of the total decay probability evaluated through (6), (5),

$$\delta W = \frac{\int\limits_0^{\sqrt{\Delta^2-m^2}} dP \cdot \Big(w(P) - w_0(P)\Big)}{\int\limits_0^{\sqrt{\Delta^2-m^2}} dP \cdot w_0(P)}\,, \tag{10}$$

comes out to be strictly equal to the value $\delta W = 8.05\%$ obtained earlier in Ref. 22.

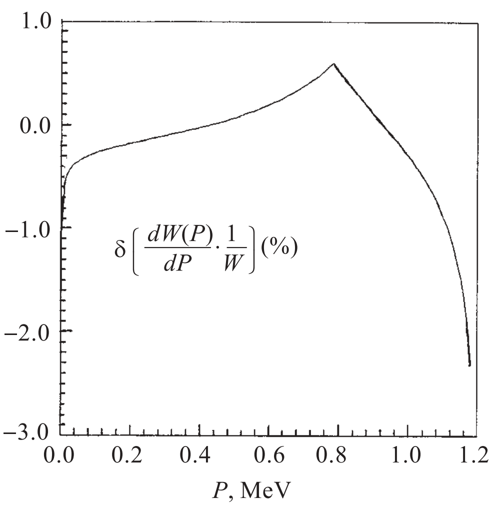

Fig. 1. The modification (8) (in per cent) of the relative distribution of protons (9) owing to electromagnetic interactions

The results acquired make us realize that the whole effect of electromagnetic interactions amounts to several percent. Nowadays, this is of value to ascertain with a high accuracy, of $\sim 1\%$ or better, the genuine form of the effective interaction (2)–(4), in particular to gain the strict values of $g_A, |V_{ud}|, ...$, from processing the high-precision measurements of the proton spectra [2, 3], which are liable to be set out before long.

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