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NEUTRINO OSCILLATIONS, MASSES AND MIXING*

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PONTECORVO IDEA OF NEUTRINO OSCILLATIONS	545
INTRODUCTION	550
NEUTRINO MASSES AND MIXING	551
NEUTRINO OSCILLATIONS IN VACUUM	556
OSCILLATIONS BETWEEN TWO TYPES OF NEUTRINOS IN VACUUM	560
NEUTRINO OSCILLATIONS IN MATTER	562
NEUTRINO OSCILLATION DATA	568
NEUTRINO OSCILLATIONS IN THE FRAMEWORK OF THREE-NEUTRINO MIXING	580
β -DECAY EXPERIMENTS ON THE MEASUREMENT OF NEUTRINO MASS	585
NEUTRINOLESS DOUBLE β DECAY	587
CONCLUSIONS	590
REFERENCES	591

*Dedicated to B. Pontecorvo in 90th anniversary of his birth.

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The original B. Pontecorvo idea of neutrino oscillations is discussed. Neutrino mixing and basics of neutrino oscillations in vacuum and in matter are considered. Recent evidences in favour of neutrino oscillations, obtained in the Super-Kamiokande, SNO, KamLAND and other neutrino experiments are discussed. Neutrino oscillations in the solar and atmospheric ranges of the neutrino mass-squared differences are considered in the framework of the minimal scheme with the mixing of three massive neutrinos. Results of the tritium β -decay experiments and experiments on the search for neutrinoless double β decay are briefly discussed.

Обсуждается оригинальная идея нейтринных осцилляций Бруно Понтекорво. Рассматривается нейтринное смешивание и основные аспекты осцилляций нейтрино в вакууме и в веществе. Обсуждаются последние свидетельства нейтринных осцилляций, полученные в SNO, KamLAND и других экспериментах на нейтрино. Нейтринные осцилляции в солнечном и атмосферном диапазонах квадратов масс нейтрино рассмотрены в рамках минимальной схемы смешивания трех массивных нейтрино. Кратко обсуждаются результаты экспериментов по β -распаду трития и экспериментов по поиску безнейтринного двойного β -распада.

1. PONTECORVO IDEA OF NEUTRINO OSCILLATIONS

B. Pontecorvo came to the idea of neutrino oscillations in 1957, soon after parity violation in β decay was discovered by Wu et al. [1] and the two-component theory of massless neutrino was proposed by Landau [2], Lee and Yang [3], and Salam [4].

For the first time B. Pontecorvo mentioned the possibility of neutrino \rightleftharpoons antineutrino transitions in vacuum in his paper [5] on muonium \rightleftharpoons antimuonium transition ($\mu^- e^+ \rightleftharpoons \mu^+ e^-$). He believed in the analogy between the weak interaction of leptons and hadrons and looked in the lepton world for a phenomenon that would be analogous to $K^0 \rightleftharpoons \bar{K}^0$ oscillations. In his work of 1957 [5]

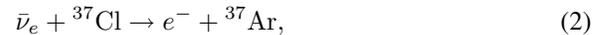
*Dedicated to B. Pontecorvo in 90th anniversary of his birth.

B. Pontecorvo wrote: «If the two-component neutrino theory should turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino \leftrightarrow antineutrino transitions could take place in vacuum».

B. Pontecorvo wrote his first paper on neutrino oscillations later in 1958 [6] at the time when F. Reines and C. Cowan [7] had just finished the famous reactor experiment, which led them to discover the electron antineutrino through the observation of the inverse β process



At that time R. Davis was doing an experiment with reactor antineutrinos [8]. R. Davis searched for production of ^{37}Ar in the process



which is allowed only if the lepton number is not conserved. A rumor reached B. Pontecorvo that R. Davis had seen some events (2). B. Pontecorvo, who had earlier been thinking about possible antineutrino \rightleftharpoons neutrino transitions, decided to study this possibility in detail. «Recently the question was discussed [5] whether there exist other *mixed* neutral particles beside the K^0 mesons, i. e., particles that differ from the corresponding antiparticles, with the transitions between particle and antiparticle states not being strictly forbidden. It was noted that the neutrino might be such a mixed particle, and consequently there exists the possibility of real neutrino \leftrightarrow antineutrino transitions in vacuum, provided that lepton (neutrino) charge is not conserved. This means that the neutrino and antineutrino are *mixed* particles, i. e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity».

In Ref. 6, B. Pontecorvo wrote that he has considered this possibility «. . . since it leads to consequences which, in principle, can be tested experimentally. Thus, for example, a beam of neutral leptons consisting mainly of antineutrinos when emitted from a nuclear reactor, will consist at some distance R from the reactor of half-neutrinos and half-antineutrinos».

In 1958 only the electron neutrino was known. In Ref. 6, B. Pontecorvo considered oscillations of the active right-handed antineutrino to a right-handed neutrino, the only possible oscillations in the case of one type of neutrino. In this paper, that was written at a time when the two-component neutrino theory had just appeared and the Davis reactor experiment was not yet finished, B. Pontecorvo discussed a possible explanation of the Davis «events». Later B. Pontecorvo understood that in the framework of the two-component neutrino theory, established after M. Goldhaber et al. experiment [9], right-handed neutrinos are practically

sterile particles*. B. Pontecorvo was in fact the first, who introduced the notion of sterile neutrinos in Ref. 10.

B. Pontecorvo stressed [6] that if the oscillation length R is large, it will be difficult to observe the effect of the neutrino oscillations in reactor experiments. He wrote: «Effects of transformation of neutrino into antineutrino and vice versa may be unobservable in the laboratory because of the large values of R , but will certainly occur, at least, on an astronomical scale».

B. Pontecorvo came back to the consideration of neutrino oscillations in 1967 [10]. At that time the phenomenological V-A theory of Feynman and Gell-Mann [12], Marshak and Sudarshan [13] was established and in the Brookhaven experiment [14], which was proposed by B. Pontecorvo in 1959 [15], it was proved that (at least) two types of neutrinos ν_e and ν_μ exist in nature. B. Pontecorvo confidence in nonzero neutrino masses and neutrino oscillations became stronger after these findings.

In Ref. 10, B. Pontecorvo discussed experiments in which conservation of lepton numbers was tested and came to the conclusion that there is «plenty of room for violation of lepton numbers, especially in pure leptonic interactions». He formulated [6, 10] the conditions at which neutrino oscillations are possible:

1. Lepton numbers, conserved by the usual weak interaction, are violated by an additional interaction between neutrinos.
2. Neutrino masses are different from zero.

In modern terminology these conditions are equivalent to the assumption that in the total Lagrangian there is a neutrino mass term, nondiagonal in flavour neutrino fields.

In Ref. 10, B. Pontecorvo considered different lepton numbers in the case of two types of neutrinos: additive electron and muon lepton numbers L_e and L_μ , ZKM lepton number [16] equal to 1 for e^- and μ^+ and -1 for e^+ and μ^- , etc. He stressed that if violation of L_e and L_μ occurs, in addition to the oscillations between active flavour neutrinos $\nu_\mu \rightleftharpoons \nu_e$ oscillations between active and sterile neutrinos ($\nu_\mu \leftrightarrow \bar{\nu}_{\mu L}$, $\nu_e \rightleftharpoons \bar{\nu}_{\mu L}$, etc.) become possible.

In the case of ZKM lepton number L for one four-component neutrino ν we have the following correspondence: $\nu_L \equiv \nu_e$, $\nu_R \equiv \bar{\nu}_\mu$, $\bar{\nu}_L \equiv \nu_\mu$, $\bar{\nu}_R \equiv \bar{\nu}_e$. If the interaction between neutrinos does not conserve L , only transitions between active neutrinos ($\nu_L \rightleftharpoons \bar{\nu}_L$ and $\nu_R \rightleftharpoons \bar{\nu}_R$) are then possible.

In his work of 1967 [10] B. Pontecorvo discussed the oscillations of solar neutrinos. At that time R. Davis started his famous experiment on the detection of the solar neutrinos. The experiment was based on the radiochemical method,

*If the lepton number is violated and neutrinos with definite masses are Majorana particles, the process (2) in principle is allowed. However, the cross section of this process is strongly suppressed by the factor $(m/E)^2$ (m is the neutrino mass; and E , the neutrino energy).

proposed by B. Pontecorvo in 1946 [11]. Solar neutrinos were detected via the observation of the Pontecorvo–Davis reaction



Quite remarkably, in [10] B. Pontecorvo envisaged the solar neutrino problem. Indeed, before the first results of Davis experiment were obtained, he wrote: «From an observational point of view the ideal object is the Sun. If the oscillation length is smaller than the radius of the Sun region effectively producing neutrinos (let us say one tenth of the Sun radius R_\odot or 0.1 million km for ${}^8\text{B}$ neutrinos, which will give the main contribution in the experiments being planned now), direct oscillations will be smeared out and unobservable. The only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total (active and sterile) neutrino flux».

It was shown by Gribov and Pontecorvo in 1969 [17] that in the neutrino mass term only active left-handed fields ν_{eL} and $\nu_{\mu L}$ can enter. The corresponding mass term is called Majorana mass term. Particles with definite masses are in this case Majorana neutrinos, and flavour fields ν_{eL} and $\nu_{\mu L}$ are connected with the Majorana fields ν_{1L} and ν_{2L} by the mixing relation

$$\begin{aligned} \nu_e &= \cos\theta \nu_{1L} + \sin\theta \nu_{2L}, \\ \nu_\mu &= -\sin\theta \nu_{1L} + \cos\theta \nu_{2L}, \end{aligned} \quad (4)$$

where θ is the mixing angle.

In Ref. 17, the expression for the survival probability of ν_e was obtained and oscillations of solar neutrinos in vacuum were considered.

In the seventies the Cabibbo–GIM mixing of d and s quarks was established. In Ref. 18, neutrino oscillations in the scheme of the mixing of two Dirac neutrinos ν_1 and ν_2 were considered. This scheme was based on the analogy between quarks and leptons. In the same work possible values of the mixing angle θ were discussed: «... it seems to us that the special values of mixing angle $\theta = 0$ (the usual scheme in which muonic charge is strictly conserved) and $\theta = \pi/4$ are of the greatest interest*».

Neutrino oscillations and in particular solar neutrino oscillations were considered in Ref. 19 in the general case of the Dirac and Majorana mass term: the latter does not conserve any lepton numbers. It is built from active left-handed fields ν_{lL} and sterile right-handed fields ν_{lR} .

*We know from today's data that small and $\pi/4$ values of neutrino mixing angles are really the preferable ones. Indeed, as we will see later, in the framework of the mixing of three massive neutrinos with three mixing angles the angle θ_{12} is small and the angles θ_{13} and θ_{23} are close to $\pi/4$.

In 1978 the first review on neutrino oscillations was published [20]. At that time the list of papers on neutrino oscillations was very short: except for the cited papers of B. Pontecorvo and his collaborators, it included only references [21–25] and [26, 27]. The review attracted the attention of many physicists to the problem of neutrino mass and neutrino oscillations*. In particular, it was stressed in this review that due to the interference character of neutrino oscillations the investigation of this phenomenon is the most sensitive method to search for small neutrino mass-squared differences.

During the preparation of the review [20] the authors became familiar with the work of Maki et al. [21], which was unknown to many physicists for many years. In this paper, in the framework of the Nagoya model, in which p , n , and Λ were considered as bound states of some vector particle B^+ and leptons, the mixing of two neutrinos was introduced. In addition to the usual neutrinos ν_e and ν_μ , which they called weak neutrinos, the authors introduced massive neutrinos ν_1 and ν_2 , which they called true neutrinos. In the Nagoya model, the proton was considered as a bound state of the B^+ and the neutrino ν_1 .

Maki, Nakagawa, and Sakata assumed that the fields of weak neutrinos and of true neutrinos are connected by an orthogonal transformation: «It should be stressed at this stage that the definition of the particle state of the neutrino is quite arbitrary; we can speak of *neutrinos* which are different from weak neutrinos but are expressed by a linear combination of the latter. We assume that there exists a representation which defines the true neutrinos through some orthogonal transformation applied to the representation of weak neutrinos:

$$\begin{aligned}\nu_1 &= +\nu_e \cos \delta + \nu_\mu \sin \delta, \\ \nu_2 &= -\nu_e \sin \delta + \nu_\mu \cos \delta.\end{aligned}\tag{5}$$

In Ref. 21, neutrino oscillations, as a phenomenon based on the quantum mechanics of a mixed system, were not considered. In connection with the Brookhaven experiment [14], which was going on at that time, the authors wrote: «In the present case, however, weak neutrinos are *not stable* due to occurrence of virtual transmutation $\nu_\mu \rightarrow \nu_e$ induced by the interaction $\mathcal{L} = g\bar{\nu}_2\nu_2 X^+ X$ (X is a field of heavy particles). If the mass difference between ν_1 and ν_2 , i. e., $|m_{\nu_2} - m_{\nu_1}| = m_{\nu_2}$, is assumed to be a few MeV, the transmutation time $T(\nu_e \leftrightarrow \nu_\mu)$ becomes $\sim 10^{-18}$ s for fast neutrinos with momentum \sim BeV/ c .

Therefore a chain of reactions such as

$$\pi^+ \rightarrow \mu^+ \nu_\mu, \quad \nu_\mu + Z(\text{nucleus}) \rightarrow Z' + (\mu^- / e^-)\tag{6}$$

is useful to check the two-neutrino hypothesis only when $|m_{\nu_2} - m_{\nu_1}| \leq 10^{-6}$ MeV under the conventional geometry of the experiments. Conversely,

*It has now about 500 citations.

the absence of e^- in the reaction (6) will be able not only to verify two-neutrino hypothesis but also to provide an upper limit of the mass of the second neutrino ν_2 if the present scheme should be accepted».

Recently very strong, convincing evidence in favour of neutrino oscillations, which we will discuss later, was obtained. It required many years of work and heroic efforts of many experimental groups to reveal the effects of tiny neutrino masses and neutrino mixing. From our point of view it is a proper time to give a tribute to the great intuition of B. Pontecorvo, who pursued the idea of neutrino oscillations for many years at a time when the general opinion, mainly based on the success of the two-component neutrino theory, favoured massless neutrinos*. The history of neutrino oscillations is an illustration of the importance of analogy in physics. It is also an illustration of the importance of new courageous ideas, not always in agreement with the general opinion.

2. INTRODUCTION

Convincing evidence of neutrino oscillations was obtained in the last 4–5 years in experiments with neutrinos from natural sources: in the atmospheric [29–31] and solar neutrino experiments [32–38]. Recently, strong evidence in favour of neutrino oscillations was obtained in the reactor KamLAND experiment [39].

The observation of neutrino oscillations implies that neutrino masses are different from zero and that the states of flavour neutrinos ν_e, ν_μ, ν_τ are coherent superpositions (mixtures) of the states of neutrinos with definite masses. All existing data, including astrophysical ones, suggest that neutrino masses are much smaller than the masses of leptons and quarks. The smallness of neutrino masses is a signature of a new, beyond the Standard Model, physics.

In this review we present the phenomenological theory of neutrino masses and mixing and the see-saw mechanism of neutrino mass generation [40]. We consider neutrino oscillations in vacuum and transitions between different flavour neutrinos in matter. Then we will discuss the recent experimental evidence in favour of neutrino oscillations: the results of the Super-Kamiokande atmospheric neutrino experiment [29], in which a significant up-down asymmetry of the high-energy muon events was observed, the results of the SNO solar neutrino experiment [36–38], in which a direct evidence for the transition of the solar ν_e into ν_μ and ν_τ was obtained, and the results of the KamLAND experiment [39], in which the disappearance of the reactor $\bar{\nu}_e$ was observed. We will also discuss the long baseline CHOOZ [41] and Palo Verde [42] reactor experiments in which no

*For a collection of the papers of B. Pontecorvo see Ref. 28.

indications in favour of neutrino oscillations were obtained. Yet the results of these experiments are very important for neutrino mixing.

Neutrino oscillations in the atmospheric and solar ranges of neutrino mass-squared differences will be considered in the framework of three-neutrino mixing. We will show that neutrino oscillations in these two regions are practically decoupled and are described in the leading approximation by the two-neutrino formulas.

The investigation of neutrino oscillations is based on the following assumptions, which are supported by all existing experimental data.

1. The interaction of neutrinos with other particles is described by the Standard Model of the electroweak interaction.

The Standard Charged Current (CC) and Neutral Current (NC) Lagrangians are given by

$$\mathcal{L}_I^{\text{CC}} = -\frac{g}{2\sqrt{2}} j_\alpha^{\text{CC}} W^\alpha + \text{h. c.}; \quad \mathcal{L}_I^{\text{NC}} = -\frac{g}{2 \cos \theta_W} j_\alpha^{\text{NC}} Z^\alpha. \quad (7)$$

Here g is the $SU(2)$ gauge coupling constant; θ_W is the weak angle; W^α and Z^α are the fields of the charged (W^\pm) and neutral (Z^0) vector bosons, and the leptonic charged current j_α^{CC} and neutrino neutral current j_α^{NC} are given by the expressions:

$$j_\alpha^{\text{CC}} = 2 \sum_l \bar{\nu}_{lL} \gamma_\alpha l_L, \quad j_\alpha^{\text{NC}} = \sum_l \bar{\nu}_{lL} \gamma_\alpha \nu_{lL}. \quad (8)$$

2. Three flavour neutrinos ν_e , ν_μ , and ν_τ exist in nature.

From the LEP experiments on the measurement of the width of the decay $Z \rightarrow \nu + \bar{\nu}$ the following value was obtained [43] for the number of flavour neutrinos n_{ν_f} :

$$n_{\nu_f} = 3.00 \pm 0.06, \quad (9)$$

and from the global fit of the LEP data the value

$$n_{\nu_f} = 2.984 \pm 0.008 \quad (10)$$

was found.

3. NEUTRINO MASSES AND MIXING

If neutrino fields enter only in the Standard Model (SM) Lagrangians (7), neutrinos are massless particles and the three flavour lepton numbers: electron (L_e), muon (L_μ), and tau (L_τ) are separately conserved. The hypothesis of neutrino mixing is based on the assumption that in the total Lagrangian there is a *neutrino mass term*, which does not conserve flavour lepton numbers. Several

mechanisms of the generation of the neutrino mass term were proposed. In this section we will discuss the most plausible see-saw mechanism [40].

Let us start with the phenomenological theory of the neutrino masses and mixing. There are two types of possible neutrino mass terms (see [44,45]).

1. *Dirac mass term*

$$\mathcal{L}^D = -\bar{\nu}'_R M^D \nu'_L + \text{h. c.} \quad (11)$$

Here

$$\nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \vdots \end{pmatrix}, \quad \nu'_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \\ \vdots \end{pmatrix} \quad (12)$$

and M^D is a complex nondiagonal matrix. We assume that in the column ν'_L not only left-handed flavour neutrino fields ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$ but also sterile fields could enter. Sterile fields do not enter into the standard charged and neutral currents (see discussion later).

The matrix M^D can be diagonalized by a biunitary transformation. We have (see [44,45]):

$$M^D = V m U^+, \quad (13)$$

where V and U are unitary matrices and $m_{ik} = m_i \delta_{ik}$ $m_i > 0$. From Eqs. (11) and (13) for the neutrino mass term we obtain the standard expression

$$\mathcal{L}^D = - \sum_i m_i \bar{\nu}_i \nu_i, \quad (14)$$

where ν_i is the field of neutrino with mass m_i .

The flavour field ν_{lL} is connected with left-handed field ν_{iL} by the mixing relation

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}. \quad (15)$$

It is obvious that the total Lagrangian is invariant under the global gauge transformation

$$\nu'_L \rightarrow e^{i\alpha} \nu'_L, \quad \nu'_R \rightarrow e^{i\alpha} \nu'_R; \quad l \rightarrow e^{i\alpha} l$$

where α is an arbitrary constant phase. From this invariance it follows that the total lepton number $L = \sum_l L_l$ is conserved and ν_i is the field of the Dirac neutrinos and antineutrinos with $L = 1$ for neutrinos and $L = -1$ for antineutrinos.

2. Majorana mass term

$$\mathcal{L}^M = -\frac{1}{2} (\bar{\nu}'_L)^c M^M \nu'_L + \text{h. c.} \quad (16)$$

Here

$$(\nu'_L)^c = C(\bar{\nu}')^T_L, \quad (\bar{\nu}'_L)^c = -(\nu')^T_L C^{-1},$$

where C is the charge conjugation matrix, which satisfies the conditions

$$C \gamma_\alpha^T C^{-1} = -\gamma_\alpha, \quad C^T = -C.$$

The matrix M^M is a *symmetrical* matrix. In fact, taking into account the Fermi-Dirac statistics of the field ν'_L , we have

$$(\bar{\nu}'_L)^c M^M \nu'_L = (\nu'_L)^T (C^{-1})^T (M^M)^T \nu'_L = (\bar{\nu}'_L)^c (M^M)^T \nu'_L.$$

From this relation we obtain

$$M^M = (M^M)^T.$$

For a symmetrical matrix we have (see [44,45])

$$M^M = (U^+)^T m U^+, \quad (17)$$

where U is a unitary matrix and $m_{ik} = m_i \delta_{ik}$; $m_i > 0$.

From Eqs.(16) and (17) it follows that the Majorana mass term takes the form

$$\mathcal{L}^M = -\frac{1}{2} \sum_i m_i \bar{\nu}_i \nu_i, \quad (18)$$

where ν_i is the field of neutrino with mass m_i , which satisfies *the Majorana condition*

$$\nu_i = \nu_i^c = C(\bar{\nu})^T_i. \quad (19)$$

The flavour field ν_{lL} is connected with the Majorana fields ν_{iL} by the mixing relation

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}. \quad (20)$$

In the case of the Majorana mass term there is no global gauge invariance of the total Lagrangian. Hence, Majorana neutrinos are truly neutral particles: they do not carry neither electric charge nor lepton numbers. In other words Majorana neutrinos and antineutrinos are identical particles.

If there are only flavour fields ν_{lL} in the column ν'_L , the number of the massive neutrinos ν_i is equal to the number of flavour neutrinos (three) and U is

a 3×3 Pontecorvo–Maki–Nakagawa–Sakata (PMNS) [6, 10, 21] unitary mixing matrix.

If, in the column ν'_L , there are also sterile fields ν_{sL} , the number of massive neutrinos ν_i will be larger than three. In this case the mixing relation takes the form

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li} \nu_{iL}, \quad \nu_{sL} = \sum_{i=1}^{3+n_s} U_{si} \nu_{iL}, \quad (21)$$

where n_s is the number of the sterile fields; U is a unitary $(3+n_s) \times (3+n_s)$ matrix; ν_i is the field of neutrino with mass m_i ($i = 1, 2, \dots, 3+n_s$).

Sterile fields can be right-handed neutrino fields, SUSY fields, etc. If more than three neutrino masses are small, the transition of the flavour neutrinos ν_e, ν_μ, ν_τ into sterile states becomes possible.

Of special interest is the so-called Dirac and Majorana mass term. Let us assume that, in addition to the flavour fields ν_{lL} , in the column ν'_L also the fields $(\nu_{lR})^c$ ($l = e, \mu, \tau$) enter*. The Majorana mass term (16) can then be presented in the form of the sum of the left-handed Majorana, Dirac and right-handed Majorana mass terms:

$$\begin{aligned} \mathcal{L}^{D+M} &= -\frac{1}{2} (\overline{\nu}_L)^c M_L^M \nu_L - \bar{\nu}_R M^D \nu_{lL} - \frac{1}{2} \bar{\nu}_R M_R^M (\nu_R)^c + \text{h. c.} = \\ &= -\frac{1}{2} (\overline{\nu}'_L)^c M^{D+M} \nu'_L + \text{h. c.} \end{aligned} \quad (22)$$

Here

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}, \quad (23)$$

M_L^M and M_R^M are complex nondiagonal symmetrical 3×3 Majorana matrices and M^D is a complex nondiagonal 3×3 Dirac matrix. After the diagonalization of the mass term (22) one gets

$$\nu_{lL} = \sum_{i=1}^6 U_{li} \nu_{iL}, \quad (\nu_{lR})^c = \sum_{i=1}^6 U_{\bar{l}i} \nu_{iL}, \quad (24)$$

where U is the unitary 6×6 mixing matrix and ν_i is the field of the Majorana neutrino with mass m_i .

* $(\nu_R)^c$ is the left-handed component of the conjugated field ν^c . In fact, we have

$$(\nu_R)^c = \frac{1-\gamma_5}{2} C \bar{\nu}^T = \nu_L^c.$$

The standard see-saw mechanism of the neutrino mass generation [40] is based on the assumption that the neutrino mass term is the Dirac and Majorana one, with $M_L^M = 0$. To illustrate this mechanism we will consider the simplest case of one type of neutrino. Let us assume that the standard Higgs mechanism with one Higgs doublet, which is the mechanism of generation of the masses of quarks and leptons, also generates the Dirac neutrino mass term

$$\mathcal{L}^D = -m \bar{\nu}_R \nu_L + \text{h. c.} \quad (25)$$

It is natural to expect that the mass m is of the same order of magnitude of the lepton or quark masses in the same family. However, we know from experimental data that neutrino masses are much smaller than the masses of leptons and quarks. In order to «suppress» the neutrino mass we will assume that there is a beyond the SM lepton number violating mechanism of generation of the right-handed Majorana mass term

$$\mathcal{L}_R^M = -\frac{1}{2} M \bar{\nu}_R (\nu_R)^c + \text{h. c.}, \quad (26)$$

with $M \gg m$ (usually it is assumed that $M \simeq M_{\text{GUT}} \simeq 10^{16}$ GeV). The total mass term is then the Dirac and Majorana one, with

$$M^{D+M} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}, \quad \nu'_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}. \quad (27)$$

After the diagonalization of the mass term (26) one gets

$$\begin{aligned} \nu_L &= i \cos \theta \nu_{1L} + \sin \theta \nu_{2L}, \\ (\nu_R)^c &= -i \sin \theta \nu_{1L} + \cos \theta \nu_{2L}, \end{aligned} \quad (28)$$

where ν_1 and ν_2 are the fields of the Majorana particles, with masses

$$\begin{aligned} m_1 &= -\frac{1}{2} M + \frac{1}{2} \sqrt{M^2 + 4m^2} \simeq \frac{m^2}{M} \ll m, \\ m_2 &= \frac{1}{2} M + \frac{1}{2} \sqrt{M^2 + 4m^2} \simeq M. \end{aligned} \quad (29)$$

The mixing angle θ is given by the relation

$$\tan 2\theta = \frac{2m}{M} \ll 1. \quad (30)$$

Thus, the see-saw mechanism is based on the assumption that, in addition to the standard Higgs mechanism of generation of the Dirac mass term, there exists a beyond the SM mechanism of generation of the right-handed Majorana mass term, which changes the lepton number by two and is characterized by the mass

$M \gg m^*$. The Dirac mass term mixes the left-handed field ν_L , the component of a doublet, with a singlet field $(\nu_R)^c$. As a result of this mixing the neutrino acquires Majorana mass, which is much smaller than the masses of leptons or quarks.

In the case of three generations, in the mass spectrum there will be three light Majorana masses, much smaller than the masses of quarks and leptons, and three very heavy Majorana masses of the order of magnitude of the lepton number violation scale M (see [46–48]).

Let us stress that if neutrino masses are of see-saw origin, then:

- Neutrinos with definite masses are Majorana particles.
- There are three light neutrinos.
- Heavy Majorana particles must exist.

The existence of the heavy Majorana particles, see-saw partners of neutrinos, could be a source of the baryon asymmetry of the Universe (see [49]).

4. NEUTRINO OSCILLATIONS IN VACUUM

In this section we will discuss the phenomenon of neutrino oscillations (see, for example, [44, 45]). If in the total Lagrangian there is a neutrino mass term, the flavour lepton numbers L_e, L_μ, L_τ are not conserved and transitions between different flavour neutrinos become possible.

Let us first define the states of flavour neutrinos ν_e, ν_μ, ν_τ in the case of neutrino mixing. The flavour neutrinos are particles which take part in the standard weak processes. For example, the neutrino that is produced together with μ^+ in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ is the muon neutrino ν_μ ; the electron antineutrino $\bar{\nu}_e$ produces e^+ in the process $\bar{\nu}_e + p \rightarrow e^+ + n$, etc.

In order to determine the states of the flavour neutrinos let us consider the decay

$$a \rightarrow b + l^+ + \nu_l. \quad (31)$$

If there is neutrino mixing, the state of the final particles is given by

$$|f\rangle = \sum_i |b\rangle |l^+\rangle |\nu_i\rangle \langle il^+b|S|a\rangle, \quad (32)$$

where $|\nu_i\rangle$ is the state of neutrino with momentum \mathbf{p} and energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}, \quad (p^2 \gg m_i^2),$$

and $\langle il^+b|S|a\rangle$ is the element of the S matrix.

*It is obvious that for charged particles such mechanism does not exist.

We will assume that the neutrino mass-squared differences are so small that the emission of neutrinos with different masses cannot be resolved in the neutrino production (and detection) experiments. In this case we have

$$\langle il^+b|S|a\rangle \simeq U_{li}^* \langle \nu_l l^+b|S|a\rangle_{\text{SM}}, \quad (33)$$

where $\langle \nu_l l^+b|S|a\rangle_{\text{SM}}$ is the SM matrix element of the process (31), calculated under the assumption that all neutrinos ν_i have the same mass. From (32) and (33) we obtain the following expression for the normalized state of the flavour neutrino ν_l :

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle. \quad (34)$$

Thus, in the case of mixing of neutrino fields with small mass-squared differences, the state of a flavour neutrino is a *coherent superposition (mixture)* of the states of neutrinos with definite masses*.

In the general case of active and sterile neutrinos we have

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle, \quad (35)$$

where the index α takes the values e, μ, τ, s_1, \dots . From the unitarity of the mixing matrix it follows that

$$\langle \nu_{\alpha'} | \nu_\alpha \rangle = \delta_{\alpha' \alpha}. \quad (36)$$

The phenomenon of neutrino oscillations is based on the relation (35). Let us consider the evolution of the mixed neutrino states in the vacuum. If at the initial time $t = 0$ a flavour neutrino ν_α is produced, the neutrino state at a time t will be:

$$|\nu_\alpha\rangle_t = e^{-iH_0 t} |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* e^{-iE_i t} |i\rangle, \quad (37)$$

where H_0 is the free Hamiltonian.

It is important that the phase factors in (37) for different mass components are different. Thus, the flavour content of the final state $|\nu_\alpha\rangle_t$ will differ from the initial one. As we shall see later, in spite of the small neutrino mass-squared differences, the effect of the transition of the initial neutrino into another flavour (or sterile) neutrino can be large.

Neutrinos are detected through the observation of CC and NC weak processes. By developing the state $|\nu_\alpha\rangle_t$ over the total system of neutrino states $|\nu_\alpha\rangle$, we

*The relation (34) is analogous to the relation which connects the states of K^0 and \bar{K}^0 mesons, particles with definite strangeness, with the states of K_S^0 and K_L^0 mesons, particles with definite masses and widths.

have

$$|\nu_\alpha\rangle_t = \sum_{\alpha'} A(\nu_\alpha \rightarrow \nu_{\alpha'}) |\nu_{\alpha'}\rangle, \quad (38)$$

where

$$A(\nu_\alpha \rightarrow \nu_{\alpha'}) = \langle \nu_{\alpha'} | e^{-iH_0 t} | \nu_\alpha \rangle = \sum_i U_{\alpha' i} e^{-iE_i t} U_{\alpha i}^* \quad (39)$$

is the amplitude of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ during the time t .

Taking into account the unitarity of the mixing matrix, from (39) we obtain the following expression for the probability of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ *:

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \left| \delta_{\alpha' \alpha} + \sum_{i \geq 2} U_{\alpha' i} U_{\alpha i}^* \left(\exp \left(-i \Delta m_{i1}^2 \frac{L}{2E} \right) - 1 \right) \right|^2, \quad (40)$$

where $L \simeq t$ is the distance between a neutrino source and a neutrino detector; E is the neutrino energy and $\Delta m_{i1}^2 = m_i^2 - m_1^2$.

Analogously, for the probability of the transition $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$ we have:

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) = \left| \delta_{\alpha' \alpha} + \sum_i U_{\alpha' i}^* U_{\alpha i} \left(\exp \left(-i \Delta m_{i1}^2 \frac{L}{2E} \right) - 1 \right) \right|^2. \quad (41)$$

Let us stress the following general properties of the transition probabilities:

- Transition probabilities depend on L/E .
- Neutrino oscillations can be observed if the condition

$$\Delta m_{i1}^2 \frac{L}{E} \gtrsim 1$$

is satisfied for at least one value of i .

- From the comparison of Eqs. (40) and (41) we conclude that the following relation holds

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\bar{\nu}_{\alpha'} \rightarrow \bar{\nu}_\alpha).$$

This relation is the consequence of the CPT invariance, intrinsic for any local field theory.

- If CP invariance in the lepton sector holds, the mixing matrix U is real in the Dirac case. In the Majorana case the mixing matrix satisfies the condition [50]

$$U_{\alpha i} = U_{\alpha i}^* \eta_i, \quad (42)$$

*We label neutrino masses in such a way that $m_1 < m_2 < m_3 < \dots$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino ν_i . From Eqs. (40), (41), and (42) we can conclude that in the case of CP invariance in the lepton sector we have the following relation

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}).$$

In conclusion let us introduce the parameters which characterize the 3×3 neutrino mixing matrix. In the general $n \times n$ case the unitary matrix is characterized by $n(n-1)/2$ angles and $n(n+1)/2$ phases. The phases of lepton fields are arbitrary. If neutrinos with definite masses are Dirac particles, the phases of neutrino fields ν_i are arbitrary as well. In this case the mixing matrix is characterized by

$$\frac{n(n+1)}{2} - 2(n-1) - 1(\text{common phase}) = \frac{(n-1)(n-2)}{2}$$

physical phases.

Thus, the 3×3 Dirac mixing matrix is characterized by three angles and one phase. Let us parameterize such matrix. Taking into account the unitary condition $\sum_i |U_{ei}|^2 = 1$, we can choose the first two elements of the first row in the following way

$$U_{e1} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{12}, \quad U_{e2} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{12}, \quad (43)$$

where θ_{12} is the mixing angle. The third element of the first row can be parameterized as follows

$$U_{e3} = \sin \theta_{13} e^{-i\delta}, \quad (44)$$

where δ is the CP phase; and θ_{13} , the second mixing angle. Thus, for $|\nu_e\rangle$ we have

$$|\nu_e\rangle = \sqrt{1 - |U_{e3}|^2} |\nu_{12}\rangle + U_{e3} |\nu_3\rangle, \quad (45)$$

where

$$|\nu_{12}\rangle = \cos \theta_{12} |\nu_1\rangle + \sin \theta_{12} |\nu_2\rangle. \quad (46)$$

The elements of the second and third line of the mixing matrix must be chosen in such a way that the condition

$$\langle \nu_{l'} | \nu_l \rangle = \delta_{l'l} \quad (47)$$

is satisfied.

It is obvious that the two orthogonal vectors

$$|\nu_{12}^\perp\rangle = -\sin \theta_{12} |\nu_1\rangle + \cos \theta_{12} |\nu_2\rangle, \quad (48)$$

$$|\nu_e^\perp\rangle = -U_{e3}^* |\nu_{12}\rangle + \sqrt{1 - |U_{e3}|^2} |\nu_3\rangle \quad (49)$$

are orthogonal to the vector $|\nu_e\rangle$.

We have introduced two angles θ_{12} , θ_{13} and the phase δ . The third mixing angle θ_{23} will enter into play as follows

$$|\nu_\mu\rangle = \cos \theta_{23} |\nu_{12}^\perp\rangle + \sin \theta_{23} |\nu_e^\perp\rangle, \quad (50)$$

$$|\nu_\tau\rangle = -\sin \theta_{23} |\nu_{12}^\perp\rangle + \cos \theta_{23} |\nu_e^\perp\rangle. \quad (51)$$

From Eqs. (45), (50), and (51) we can obtain all the mixing matrix elements. We shall be interested in the elements of the first line and third column. The former are given by Eqs. (43) and (44). For the elements $U_{\mu 3}$ and $U_{\tau 3}$ of the third column we have

$$U_{\mu 3} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{23}, \quad U_{\tau 3} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{23}. \quad (52)$$

If the ν_i are Majorana fields, their phases are fixed by the Majorana condition, Eq. (19). The Majorana mixing matrix can be presented in the form

$$U^M = U^D S(\alpha), \quad (53)$$

where U^D is the Dirac mixing matrix and $S_{ik}(\alpha) = e^{i(\alpha_i - \alpha_k)} \delta_{ik}$ is the phase matrix.

The number of physical phases in the Majorana mixing matrix is equal to

$$\frac{(n-1)(n-2)}{2} + (n-1) = \frac{n(n-1)}{2}. \quad (54)$$

Thus, the 3×3 unitary Majorana mixing matrix is characterized by three mixing angles and three CP phases.

From Eqs. (39) and (53) it is easy to derive the following relation between the amplitudes of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ for the Majorana and Dirac neutrinos:

$$\begin{aligned} A^M(\nu_\alpha \rightarrow \nu_{\alpha'}) &= \sum_i U_{\alpha'i}^M e^{-iE_i t} U_{\alpha i}^{M*} = \\ &= \sum_i U_{\alpha'i}^D e^{-iE_i t} U_{\alpha i}^{D*} = A^D(\nu_\alpha \rightarrow \nu_{\alpha'}). \end{aligned} \quad (55)$$

Hence, the investigation of neutrino oscillations does not allow one to distinguish between the cases of Dirac or Majorana neutrinos [51].

5. OSCILLATIONS BETWEEN TWO TYPES OF NEUTRINOS IN VACUUM

We will consider here the simplest case of the transition between two types of neutrinos. In this case the index i in Eq. (40) takes only one value ($i = 2$) and

for the transition probability we have

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \left| \delta_{\alpha'\alpha} + U_{\alpha'2} U_{\alpha 2}^* \left(\exp \left(-i \Delta m^2 \frac{L}{2E} \right) - 1 \right) \right|^2, \quad (56)$$

where $\Delta m^2 = m_2^2 - m_1^2$.

From (56) we obtain the following expression for the appearance probability:

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} A_{\alpha';\alpha} \left(1 - \cos \Delta m^2 \frac{L}{2E} \right) \quad (\alpha' \neq \alpha), \quad (57)$$

where

$$A_{\alpha';\alpha} = 4 |U_{\alpha'2}|^2 |U_{\alpha 2}|^2 = A_{\alpha;\alpha'}. \quad (58)$$

Let us introduce the mixing angle θ . In the 2×2 case the mixing matrix U can be chosen to be real. Hence we set:

$$U_{\alpha 2} = \sin \theta, \quad U_{\alpha' 2} = \cos \theta.$$

The amplitude $A_{\alpha';\alpha}$ is then given by

$$A_{\alpha';\alpha} = \sin^2 2\theta,$$

and the two-neutrino transition probability takes the standard form

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \Delta m^2 \frac{L}{2E} \right) \quad (\alpha' \neq \alpha). \quad (59)$$

From Eqs. (57) and (58) it is obvious that in the two-neutrino case the following relations are valid:

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\nu_{\alpha'} \rightarrow \nu_\alpha) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}). \quad (60)$$

Thus, the CP violation in the lepton sector cannot be revealed in the case of transitions between two types of neutrinos.

The survival probability $P(\nu_\alpha \rightarrow \nu_\alpha)$ is determined by the condition of conservation of probability:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - P(\nu_\alpha \rightarrow \nu_{\alpha'}) = 1 - \frac{1}{2} \sin^2 2\theta \left(1 - \cos \Delta m^2 \frac{L}{2E} \right). \quad (61)$$

From Eqs. (60) and (61) it follows that the two-neutrino survival probabilities satisfy the following relation:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\nu_{\alpha'} \rightarrow \nu_{\alpha'}). \quad (62)$$

Thus, only two oscillation parameters, $\sin^2 2\theta$ and Δm^2 , characterize all transition probabilities in the case of a transition between two types of neutrinos.

The expressions (59) and (61) describe periodic transitions between two types of neutrinos (neutrino oscillations). They are widely used in the analysis of experimental data*.

Let us notice that the expression (59) for the two-neutrino transition probability can be recast in the form

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_0} \right), \quad (63)$$

where

$$L_0 = 4\pi \frac{E}{\Delta m^2} \quad (64)$$

is the oscillation length.

Finally, the two-neutrino transition probability and the oscillation length can be rewritten as

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2.53 \Delta m^2 \frac{L}{E} \right) \quad (65)$$

and

$$L_0 \simeq 2.48 \frac{E}{\Delta m^2}, \quad (66)$$

where E is the neutrino energy in MeV (GeV); L is the distance in m (km), and Δm^2 is the neutrino mass-squared difference in eV^2 .

6. NEUTRINO OSCILLATIONS IN MATTER

In the previous sections we have considered neutrino oscillations in the vacuum. However, if neutrinos pass through the Sun, the Earth, supernova, etc., matter can significantly alter the neutrino mixing and the probabilities of the transitions between different types of neutrinos. Here we will discuss the effects of matter on the neutrino transition probabilities (see [45, 52–55]).

The refraction index of a particle with momentum p is given by the following classical expression

$$n = 1 + \frac{2\pi}{p^2} f(0)\rho, \quad (67)$$

*As we shall see later, in the case of three-neutrino mixing the neutrino oscillations in different ranges of Δm^2 are described, in leading approximation, by the two-neutrino formulas.

where ρ is the number density of matter and $f(0)$ is the amplitude for elastic scattering in the forward direction. The second term in Eq. (65) is due to the coherent scattering of the particle in matter.

In the case of neutrinos the amplitude of the process $\nu_e e \rightarrow \nu_e e$ is different from the amplitude of the processes $\nu_{\mu,\tau} e \rightarrow \nu_{\mu,\tau} e$. This is related to the fact that to the matrix element of the process $\nu_e e \rightarrow \nu_e e$, the diagrams with the exchange of the W and Z bosons give contribution, while to the matrix element of the processes $\nu_{\mu,\tau} e \rightarrow \nu_{\mu,\tau} e$ only the diagram with the exchange of the Z boson contributes. Thus, the refraction indexes of ν_e and $\nu_{\mu,\tau}$ are different. If neutrino mixing occurs, this difference between the refraction indexes leads to important effects for the neutrino transitions in matter.

In the flavour representation the evolution equation of a neutrino with momentum \mathbf{p} in matter has the following general form:

$$i \frac{\partial a_\alpha(t)}{\partial t} = \sum_{\alpha'} (H_0 + H_I)_{\alpha\alpha'} a_{\alpha'}(t). \quad (68)$$

Here H_0 is the free Hamiltonian; H_I is the effective Hamiltonian of the interaction of neutrino with matter and $a_\alpha(t)$ is the amplitude of the probability to find ν_α at a time t . We have

$$(H_0)_{\alpha\alpha'} = \langle \alpha | H_0 | \alpha' \rangle = \sum_i U_{\alpha i} E_i U_{\alpha' i}^*, \quad (69)$$

where U is the neutrino mixing matrix in vacuum and $E_i \simeq p + m_i^2/(2p)$.

Let us consider the effective Hamiltonian of the interaction of neutrinos with matter. We will limit ourselves to the case of the flavour neutrinos.

Due to the $\nu_e - \nu_\mu - \nu_\tau$ universality of the NC, the NC part of the effective Hamiltonian is proportional to the unit matrix. This part of the Hamiltonian can be excluded from the evolution equation through a phase transformation of the function $a(t)$. Thus, we need to take into account only the CC part of the Hamiltonian of $\nu_e - e$ interaction. We have:

$$(H_I)_{ee} = \langle \nu_e \text{ mat} | \frac{G_F}{\sqrt{2}} 2\bar{\nu}_{eL} \gamma^\alpha \nu_{eL} \bar{e} \gamma_\alpha (1 - \gamma_5) e | \nu_e \text{ mat} \rangle \simeq \sqrt{2} G_F \rho_e, \quad (70)$$

where $|\nu_e \text{ mat}\rangle$ is the state vector of a neutrino with momentum \mathbf{p} in matter and ρ_e is the electron number density*.

We will consider now the simplest case of two flavour neutrinos. The mixing matrix has the form

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (71)$$

*The same expression for the effective Hamiltonian of neutrinos in matter can be obtained from Eq. (67) (see [52]).

where θ is the *vacuum* mixing angle. From Eqs. (69)–(71) the total effective Hamiltonian of neutrinos with momentum \mathbf{p} in matter can then be written as*:

$$H = \frac{1}{4p} \begin{pmatrix} -\Delta m^2 \cos 2\theta + A & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & \Delta m^2 \cos 2\theta - A \end{pmatrix}, \quad (72)$$

where

$$\Delta m^2 = m_2^2 - m_1^2, \quad A = 2\sqrt{2}G_F\rho_e p.$$

The total Hamiltonian (72) can be easily diagonalized. We have

$$H = U^m E^m U^{m+}, \quad (73)$$

where E_i^m ($i = 1, 2$) are the eigenvalues of the matrix H and

$$U^m = \begin{pmatrix} \cos \theta^m & \sin \theta^m \\ -\sin \theta^m & \cos \theta^m \end{pmatrix} \quad (74)$$

is the orthogonal mixing matrix in matter.

For the eigenvalues $E_{1,2}^m$ we find the following expressions:

$$E_{1,2}^m = \mp \frac{1}{4p} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}, \quad (75)$$

while the mixing angle θ^m is given by the relations

$$\begin{aligned} \tan 2\theta^m &= \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}, \\ \cos 2\theta^m &= \frac{\Delta m^2 \cos 2\theta - A}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}. \end{aligned} \quad (76)$$

Let us consider now the simplest case of constant electron density. The probability of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ ($\alpha \neq \alpha'$) in matter is given in this case by the expression

$$P^m(\nu_\alpha \rightarrow \nu_{\alpha'}) = P^m(\nu_{\alpha'} \rightarrow \nu_\alpha) = \frac{1}{2} \sin^2 2\theta^m (1 - \cos \Delta E^m L), \quad (77)$$

where L is the distance which the neutrino travels in matter, and

$$\Delta E^m = E_2^m - E_1^m = \frac{1}{2p} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}. \quad (78)$$

*We omit the irrelevant unit matrix.

From Eqs. (77) and (78) it follows that the oscillation length in matter is given by

$$L_0^m = \frac{4\pi p}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}. \quad (79)$$

By putting $\rho_e = 0$, we can easily see that the expressions (77) and (79) reduce to the corresponding vacuum expressions (63) and (64). Again the survival probability of ν_α ($\nu_{\alpha'}$) is given by the condition of conservation of the total probability:

$$P^m(\nu_\alpha \rightarrow \nu_\alpha) = P^m(\nu_{\alpha'} \rightarrow \nu_{\alpha'}) = 1 - P^m(\nu_\alpha \rightarrow \nu_{\alpha'}). \quad (80)$$

The neutrino mixing angle and oscillation length in matter can be significantly different from the vacuum values. Let us assume that the condition

$$\Delta m^2 \cos 2\theta = A = 2\sqrt{2}G_F \rho_e p \quad (81)$$

is satisfied. Then from (76) it follows that the mixing in matter is maximal ($\theta^m = \pi/4$), independently of the value of the vacuum mixing angle θ . In this case the oscillation length in matter turns out to be:

$$L_0^m = \frac{L_0}{\sin 2\theta},$$

where L_0 is the vacuum oscillation length.

In general the electron density is not a constant. For example, in the Sun the density is maximal in the centre of the Sun and it decreases practically exponentially to its periphery.

Hence in the general case the evolution equation of neutrinos in matter has the form

$$i \frac{\partial a(t)}{\partial t} = H^m(t) a(t), \quad (82)$$

where $H^m(t) = H_0 + H_I(t)$ is the total effective Hamiltonian in the flavour representation.

This equation can be easily solved in the case of slowly changing density of electrons ρ_e . In fact, the hermitian Hamiltonian $H^m(t)$ can be diagonalized by the unitary transformation

$$H^m(t) = U^m(t) E^m(t) U^{m+}(t), \quad (83)$$

where $U^m(t) U^{m+}(t) = 1$ and $E_i^m(t)$ is the eigenvalue of the Hamiltonian. Let us now introduce the function

$$a'(t) = U^{m+}(t) a(t); \quad (84)$$

from Eqs. (82)–(84) we have

$$i\frac{\partial a'(t)}{\partial t} = \left(E^m(t) - iU^{m+}(t)\frac{\partial U^m(t)}{\partial t} \right) a'(t). \quad (85)$$

If we assume that the function $\rho_e(t)$ depends so weakly on t that we can neglect the second term in Eq. (85), then in this approximation (the so-called adiabatic approximation) the solution of the evolution equation is obviously given by

$$a'(t) = e^{-i\int_{t_0}^t E^m(t)dt} a'(t_0), \quad (86)$$

where t_0 is the initial time.

From Eqs. (84) and (86) for the solution of the evolution equation in the flavour representation we have

$$a(t) = U^m(t) e^{-i\int_{t_0}^t E^m(t)dt} U^{m+}(t_0) a(t_0). \quad (87)$$

Hence, in the adiabatic approximation, the amplitude of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ during a time interval $(t - t_0)$ is given by the expression

$$A(\nu_\alpha \rightarrow \nu_{\alpha'}) = \sum_i U_{\alpha'i}^m(t) e^{-i\int_{t_0}^t E_i^m(t)dt} U_{\alpha i}^{m*}(t_0). \quad (88)$$

The latter is similar in form to the expression (39) for the amplitude of transitions in vacuum: this is connected with the fact that in the adiabatic approximation neutrino remains on the same energy level. The expression (88) has a simple meaning: $U_{\alpha i}^{m*}(t_0)$ is the amplitude of the transition from the state of the initial ν_α to the state with energy $E_i(t_0)$; the factor $e^{-i\int_{t_0}^t E_i^m(t)dt}$ describes the propagation in the state with definite energy; $U_{\alpha'i}^m(t)$ is the amplitude of the transition from the state with energy $E_i^m(t)$ to the state of the final $\nu_{\alpha'}$. The coherent sum over all i must be performed.

The transition probability must be averaged over the region where neutrinos are produced, over the energy resolution, etc. Oscillatory terms in the transition probability usually disappear after the averaging. In this case from Eq. (88) we obtain ($x \simeq t$):

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \sum_i |U_{\alpha'i}^m(x)|^2 |U_{\alpha i}^m(x_0)|^2. \quad (89)$$

Hence, in the adiabatic approximation the averaged transition probability is determined by the elements of the mixing matrix in matter at the initial and final points.

In the case of two neutrino flavours, from Eqs. (74) and (89) we obtain the following expression for the ν_e survival probability:

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} [1 + \cos 2\theta^m(x) \cos 2\theta^m(x_0)]. \quad (90)$$

Let us now consider solar neutrinos in the two-neutrino case. If at some point $x = x_R$ the MSW [52,53] resonance condition

$$\Delta m^2 \cos 2\theta = A(x_R) \quad (91)$$

is satisfied, neutrino mixing at this point is maximal ($\theta(x_R) = \pi/4$).

If the resonance condition is fulfilled, $\cos 2\theta > 0$ (neutrino masses are labeled in such a way that $\Delta m^2 > 0$). Let us assume that the resonance point is in the region between the Sun core, where neutrinos are produced, and the surface of the Sun: then at the initial point x_0 the electron density will be larger than at the resonance point x_R and we have $A(x_0) > \Delta m^2 \cos 2\theta$. In this case from Eq. (76) it follows that $\cos 2\theta^m(x_0) < 0$ and Eq. (90) predicts:

$$P(\nu_e \rightarrow \nu_e) < \frac{1}{2}.$$

If the condition

$$A(x_0) \gg \Delta m^2 \quad (92)$$

is satisfied, then from Eq. (76) we have

$$\cos 2\theta^m(x_0) \simeq -1. \quad (93)$$

Taking into account that $\theta^m(x) = \theta$ on the surface of the Sun, for the ν_e survival probability we obtain the following relation

$$P(\nu_e \rightarrow \nu_e) \simeq \frac{1}{2}(1 - \cos 2\theta). \quad (94)$$

This relation implies that if the vacuum angle θ is small, the ν_e survival probability is close to zero and practically all ν_e 's are transferred into other neutrino states.

The MSW resonance condition (91) was written in units $\hbar = c = 1$. It can be rewritten as follows

$$\Delta m^2 \cos 2\theta \simeq 0.7 \cdot 10^{-7} E \rho \text{ eV}^2, \quad (95)$$

where ρ is the density of matter in $\text{g} \cdot \text{cm}^{-3}$ and E is neutrino energy in MeV. In the central region of the Sun $\rho \simeq 10^2 \text{ g} \cdot \text{cm}^{-3}$; the energy of solar neutrinos

is $E \simeq 1$ MeV. Thus, the resonance condition is satisfied for the solar neutrinos if $\Delta m^2 \simeq 10^{-5} \text{ eV}^2$.

In the general case of nonadiabatic transitions, for the averaged transition probability we have

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \sum_{i,k} |U_{\alpha'k}^m(x)|^2 P_{ki} |U_{\alpha i}^m(x_0)|^2, \quad (96)$$

where P_{ki} is the probability of transition from the energy level i to the energy level k .

If we limit ourselves to the two-neutrino case, from the conservation of the total probability we have

$$P_{11} = 1 - P_{21}, \quad P_{22} = 1 - P_{12}, \quad (97)$$

taking into account that $P_{21} = P_{12}$, from Eqs. (74), (96), and (97) we obtain the following general expression for the ν_e survival probability [56]

$$P(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_{12} \right) \cos 2\theta^m(x) \cos 2\theta^m(x_0). \quad (98)$$

Different approximate expressions for the transition probability P_{12} exist in the literature. In the Landau-Zenner approximation, which is based on the assumption that transitions occur mainly in the resonance region, we have

$$P_{12} = \exp \left(-\frac{\pi}{2} \gamma_R F \right), \quad (99)$$

where

$$\gamma_R = \frac{\Delta m^2 \sin^2 2\theta}{2p \cos 2\theta \left| \frac{d}{dx} \ln \rho_e(x_R) \right|}. \quad (100)$$

For an exponential density $F = 1 - \tan^2 \theta$; for a linear density $F = 1$. Let us notice that the adiabatic approximation is valid if $\gamma_R \gg 1$. In this case $P_{12} \simeq 0^*$.

7. NEUTRINO OSCILLATION DATA

7.1. Evidence in Favour of Oscillations of Atmospheric Neutrinos. Atmospheric neutrinos mainly originate from the decays of charged pions and consequent decays of muons:

$$\pi \rightarrow \mu + \nu_\mu, \quad \mu \rightarrow e + \nu_\mu + \nu_e. \quad (101)$$

*The exact solution for an exponentially varying matter density can be found in Ref. 57; see also Ref. 45 for a more detailed discussion on neutrino oscillations and transitions in matter.

Pions are produced in the processes of interaction of the cosmic rays in the atmosphere. In the Super-Kamiokande (S-K) experiment [29] electron and muon were detected via the observation of the Cherenkov light in the large water-Cherenkov detector (50 kt of H₂O).

At relatively small energies ($\lesssim 1$ GeV) practically all muons decay in the atmosphere and from (101) it follows that the ratio of the numbers of muon and electron events $R_{\mu/e}$ must be equal to 2 (if there are no neutrino oscillations). At higher energies, not all muons have enough time to decay in atmosphere and the expected ratio $R_{\mu/e}$ is larger than two. It can be predicted, however, with an accuracy better than 5%.

The ratio $R_{\mu/e}$, measured in the S-K [29] and SOUDAN2 [30] atmospheric neutrino experiments is significantly smaller than the ratio $(R_{\mu/e})_{MC}$, predicted under the assumption of no neutrino oscillations. In the S-K experiment for the ratio of ratios in the sub-GeV region ($E_{vis} \leq 1.33$ GeV) and multi-GeV region ($E_{vis} > 1.33$ GeV) the values

$$\frac{(R_{\mu/e})_{meas}}{(R_{\mu/e})_{MC}} = 0.638 \pm 0.016 \pm 0.050;$$

$$\frac{(R_{\mu/e})_{meas}}{(R_{\mu/e})_{MC}} = 0.658 \pm 0.030 \pm 0.078$$

were obtained, respectively.

The fact that the ratio $(R_{\mu/e})_{meas}$ is significantly smaller than the predicted ratio $(R_{\mu/e})_{MC}$ was known from the results of the previous atmospheric neutrino experiments, Kamiokande [58] and IMB [59]. For many years this «atmospheric neutrino anomaly» was considered as an indication in favour of the disappearance of muon neutrinos due to neutrino oscillations.

The compelling evidence in favour of neutrino oscillations has been obtained recently by the S-K collaboration [29], from the observation of the large up-down asymmetry of the atmospheric high-energy muon events. In the S-K experiment [29] the zenith angle dependence of the numbers of electron and muon events was measured. In the absence of neutrino oscillations, the number of the electron (muon) events in the multi-GeV region must obey the following relation*:

$$N_l(\cos \theta_z) = N_l(-\cos \theta_z) \quad (l = e, \mu), \quad (102)$$

where θ_z is the zenith angle.

The results of the S-K experiment on the measurement of the zenith-angle distribution of the atmospheric neutrino events is shown in Fig. 1. As is seen from

*For the multi-GeV events the effect of the magnetic field of the Earth can be neglected.

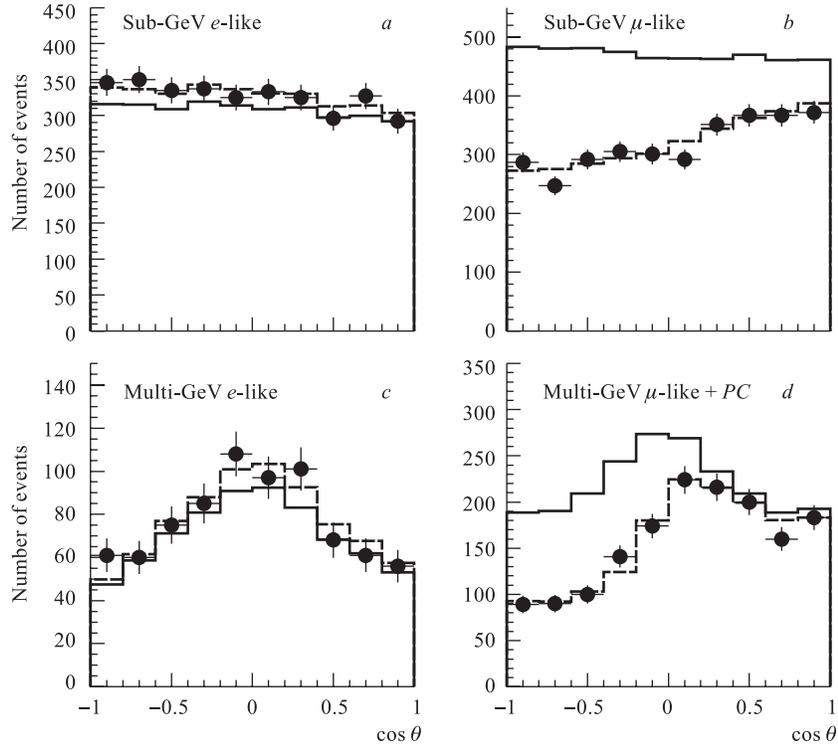


Fig. 1. Zenith angle distribution of Super-Kamiokande sub-GeV (*a*, *b*) and multi-GeV (*c*, *d*) electron and muon events. The black histogram shows the Monte Carlo prediction (under the assumption of no neutrino oscillations). The grey histogram was obtained for $\nu_\mu \rightarrow \nu_\tau$ oscillations with the best-fit values $\Delta m^2 = 2.5 \cdot 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta = 1.0$. The figure is taken from Ref. 60

this figure, for the electron events there is a good agreement between the data and relation (102). Instead, for the multi-GeV muon events a significant violation of relation (102) was observed. The ratio of the number of up-going muons ($-1 \leq \cos \theta_z \leq -0.2$) to the number of down-going muons ($0.2 \leq \cos \theta_z \leq 1$) was found to be

$$\left(\frac{U}{D}\right)_\mu = 0.54 \pm 0.04 \pm 0.01.$$

At high energies the direction of leptons practically coincides with the direction of neutrinos. The up-going muons are produced by neutrinos which travel distances

from $\simeq 500$ to $\simeq 13000$ km, and the down-going muons are produced by neutrinos which travel distances from $\simeq 20$ to $\simeq 500$ km. Hence the observation of the up-down asymmetry clearly demonstrates the dependence of the number of muon neutrinos on the distance which they travel from the production point in the atmosphere to the detector.

The S-K data [29] and the data of the other atmospheric neutrino experiments, SOUDAN 2 [30] and MACRO [31], are perfectly described if we assume that the two-neutrino $\nu_\mu \rightarrow \nu_\tau$ oscillations take place. From the analysis of the S-K data it was found [29] that at 90% CL the neutrino oscillation parameters Δm_{atm}^2 and $\sin^2 2\theta_{\text{atm}}$ are in the range

$$1.6 \cdot 10^{-3} \leq \Delta m_{\text{atm}}^2 \leq 3.9 \cdot 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} > 0.92,$$

the best-fit values of the parameters being equal to

$$\Delta m_{\text{atm}}^2 = 2.5 \cdot 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{\text{atm}} = 1.0 \quad (\chi_{\text{min}}^2 = 163.2/170 \text{ d. o. f.}). \quad (103)$$

7.2. Indications in Favour of Neutrino Oscillations Obtained in the K2K Experiment. Neutrino oscillations in the atmospheric range of Δm^2 are investigated in the first long baseline accelerator experiment K2K [61]. In this experiment neutrinos, originated mainly from the decay of pions, produced at the 12 GeV KEK accelerator, are recorded by the S-K detector at a distance of about 250 km from the accelerator. The average neutrino energy is $\simeq 1.3$ GeV.

Two near detectors at a distance of about 300 m from the beam-dump target are used in the K2K experiment: a 1 kt water-Cherenkov detector and a fine-grained detector. The total number and spectrum of muon neutrinos, observed in the S-K detector, are compared to the total number and spectrum calculated from the results of the near detectors under the assumption of the absence of neutrino oscillations. For the measurement of the energy of neutrinos in the S-K detector, quasi-elastic one-ring events $\nu_\mu + n \rightarrow \mu^- + p$ are selected.

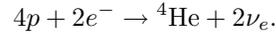
The first results of the K2K experiment were recently published [61]. The total number of muon events observed in the S-K detector is equal to 56: it must be compared with an expected number of events equal to $80.1_{-5.4}^{+6.2}$. The observed number of the one-ring muon events which was used for the calculation of neutrino spectrum is equal to 29, while the expected number of the one-ring events is equal to 44.

Thus, in the long baseline accelerator K2K experiment, indications in favour of the disappearance of the accelerator ν_μ were obtained. From the maximum likelihood two-neutrino analysis of the data the following best-fit values of the oscillation parameters were found

$$\sin^2 2\theta_{\text{K2K}} = 1, \quad \Delta m_{\text{K2K}}^2 = 2.8 \cdot 10^{-3} \text{ eV}^2. \quad (104)$$

These values are in agreement with the values of the oscillation parameters found from the analysis of the S-K atmospheric neutrino data (see (103)). The first K2K results were obtained with $4.8 \cdot 10^{19}$ protons on target (POT). It is expected that 10^{20} POT will be utilized in the experiment.

7.3. Evidence in Favour of the Transitions of Solar ν_e into ν_μ and ν_τ . The energy of the Sun is produced in the reactions of the thermonuclear pp and CNO cycles, in which protons and electrons are converted into Helium and electron neutrinos:



The reactions, which are most important for the solar neutrino experiments, are listed in the Table. From this Table one can see that the largest part of the solar neutrino flux draws up low energy pp neutrinos. According to the SSM BP00 [62], the medium energy monoenergetic ${}^7\text{Be}$ neutrinos make up about 10 % of the total flux, while the high energy ${}^8\text{B}$ neutrinos constitute only about 10^{-2} % of the total flux. However, the ${}^8\text{B}$ decay is a very important source of the solar neutrinos: in the S-K [35] and SNO [36–38] experiments, due to the high energy thresholds, practically only neutrinos from ${}^8\text{B}$ decay can be detected*. The ${}^8\text{B}$ neutrinos also give the dominant contribution to the event rate measured in the Homestake experiment [32].

The main sources of the solar neutrinos. The maximum neutrino energies and SSM BP00 [62] fluxes are also given

Reaction	Neutrino energy, MeV	SSM BP00 flux, $\text{cm}^{-2} \cdot \text{s}^{-1}$
$pp \rightarrow de^+\nu_e$	≤ 0.42	$5.95 \cdot 10^{10}$
$e^- + {}^7\text{Be} \rightarrow \nu_e {}^7\text{Li}$	0.86	$4.77 \cdot 10^9$
${}^8\text{B} \rightarrow {}^8\text{Be}^* e^+\nu_e$	≤ 15	$5.05 \cdot 10^6$

The event rates measured in all solar neutrino experiments are significantly smaller than the ones predicted by the Standard Solar models.

In the Homestake experiment [32] solar neutrinos are detected through the observation of the Pontecorvo–Davis reaction $\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$; in the GALLEX-GNO [33] and SAGE [34] experiments, through the reaction $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$; and in the S-K [35] experiment solar neutrinos are detected via the observation of the process $\nu + e \rightarrow \nu + e$.

*According to the SSM BP00 the flux of the high energy hep neutrinos, produced in the reaction ${}^3\text{He} + p \rightarrow {}^4\text{He} + e^+ + \nu_e$, is about three orders of magnitude smaller than the flux of the ${}^8\text{B}$ neutrinos.

For the ratio R of the observed and the predicted by SSM BP00 [62] rates the following values were obtained:

$$R = 0.34 \pm 0.03 \text{ (Homestake),}$$

$$R = 0.58 \pm 0.05 \text{ (GALLEX-GNO),}$$

$$R = 0.60 \pm 0.05 \text{ (SAGE),}$$

$$R = 0.465 \pm 0.018 \text{ (S-K).}$$

If there is neutrino mixing, the original solar ν_e 's, due to neutrino oscillations or matter MSW transitions, are transferred into another type of neutrinos, which cannot be detected in the radiochemical Homestake, GALLEX-GNO and SAGE experiments. In the S-K experiment mainly ν_e are detected: the sensitivity of the experiment to ν_μ and ν_τ is about six times smaller than the sensitivity to ν_e (see below). Thus, neutrino oscillations or MSW transition in matter provide a natural explanation for the observed depletion of the fluxes of solar ν_e .

Recently, a strong model independent evidence in favour of the transition of the solar ν_e into ν_μ and ν_τ was obtained in the SNO experiment [36–38]. The detector in the SNO experiment is a heavy water Cherenkov detector (1 kt of D_2O). Neutrinos from the Sun are detected via the observation of the following three reactions:

$$1. \text{ CC reaction } \quad \nu_e + d \rightarrow e^- + p + p, \quad (105)$$

$$2. \text{ NC reaction } \quad \nu + d \rightarrow \nu + n + p, \quad (106)$$

$$3. \text{ ES process } \quad \nu + e \rightarrow \nu + e. \quad (107)$$

During 306.4 days of running $1967_{-60.9}^{+61.9}$ CC events, $576.5_{-48.9}^{+49.5}$ NC events, and $263.6_{-25.6}^{+26.4}$ ES events were recorded in the SNO experiment. The kinetic energy threshold for the detection of electrons was equal to 5 MeV, the NC threshold to 2.2 MeV. Thus, practically only neutrinos from the decay ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$ are detected in the SNO experiment. The important point is that the initial spectrum of ${}^8\text{B}$ neutrinos is known [63].

The total CC-event rate can be presented in the form

$$R_{\nu_e}^{\text{CC}} = \langle \sigma_{\nu_e d}^{\text{CC}} \rangle \Phi_{\nu_e}^{\text{CC}}, \quad (108)$$

where $\langle \sigma_{\nu_e d}^{\text{CC}} \rangle$ is the cross section of the CC process (105), averaged over the initial spectrum of ${}^8\text{B}$ neutrinos, and $\Phi_{\nu_e}^{\text{CC}}$ is the flux of ν_e on the Earth, which is given by the relation

$$\Phi_{\nu_e}^{\text{CC}} = \langle P(\nu_e \rightarrow \nu_e) \rangle_{\text{CC}} \Phi_{\nu_e}^0, \quad (109)$$

where $\Phi_{\nu_e}^0$ is the total initial flux of ν_e and $\langle P(\nu_e \rightarrow \nu_e) \rangle_{\text{CC}}$ is the averaged ν_e survival probability.

All flavour neutrinos ν_e , ν_μ , and ν_τ are recorded via the detection of the NC process (106). Taking into account the $\nu_e - \nu_\mu - \nu_\tau$ universality of the neutral current interaction, for the total NC event rate we have

$$R_\nu^{\text{NC}} = \langle \sigma_{\nu d}^{\text{NC}} \rangle \Phi_\nu^{\text{NC}}, \quad (110)$$

where $\langle \sigma_{\nu d}^{\text{NC}} \rangle$ is the cross section of the NC process (106), averaged over the initial spectrum of the ^8B neutrinos, and Φ_ν^{NC} is the total flux of all flavour neutrinos on the Earth. We have

$$\Phi_\nu^{\text{NC}} = \sum_{l=e,\mu,\tau} \Phi_{\nu_l}^{\text{NC}}, \quad (111)$$

where

$$\Phi_{\nu_l}^{\text{NC}} = \langle P(\nu_e \rightarrow \nu_l) \rangle_{\text{NC}} \Phi_{\nu_e}^0, \quad (112)$$

$\langle P(\nu_e \rightarrow \nu_l) \rangle_{\text{NC}}$ being the averaged probability of the transition $\nu_e \rightarrow \nu_l$ ($l = e, \mu, \tau$).

All flavour neutrinos are detected also via the observation of the ES process (107). However, the cross section of the NC process $\nu_{\mu,\tau} + e \rightarrow \nu_{\mu,\tau} + e$ is about six times smaller than the cross section of the CC and NC process $\nu_e + e \rightarrow \nu_e + e$.

The total ES event rate can be presented in the form

$$R_\nu^{\text{ES}} = \langle \sigma_{\nu ee} \rangle \Phi_\nu^{\text{ES}}. \quad (113)$$

Here $\langle \sigma_{\nu ee} \rangle$ is the cross section of the process $\nu_e e \rightarrow \nu_e e$, averaged over initial spectrum of the ^8B neutrinos,

$$\Phi_\nu^{\text{ES}} = \Phi_{\nu_e}^{\text{ES}} + \frac{\langle \sigma_{\nu_\mu e} \rangle}{\langle \sigma_{\nu_e e} \rangle} \Phi_{\nu_{\mu,\tau}}^{\text{ES}}, \quad (114)$$

where $\Phi_{\nu_e}^{\text{ES}}$ ($\Phi_{\nu_{\mu,\tau}}^{\text{ES}}$) is the flux of ν_e (ν_μ and ν_τ) and

$$\frac{\langle \sigma_{\nu_\mu e} \rangle}{\langle \sigma_{\nu_e e} \rangle} \simeq 0.154. \quad (115)$$

We have

$$\Phi_{\nu_l}^{\text{ES}} = \langle P(\nu_e \rightarrow \nu_l) \rangle_{\text{ES}} \Phi_{\nu_e}^0, \quad (116)$$

where $\langle P(\nu_e \rightarrow \nu_l) \rangle_{\text{ES}}$ is the averaged probability of the transition $\nu_e \rightarrow \nu_l$.

In the SNO experiment it was found [38]

$$(\Phi_\nu^{\text{ES}})_{\text{SNO}} = (2.39_{-0.23}^{+0.24} \text{ (stat.)} \pm 0.12 \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}. \quad (117)$$

This value is in good agreement with the value of the ES flux, obtained in the Super-Kamiokande experiment.

In the S-K solar neutrino experiment [35] neutrinos are detected via the observation of the ES process. During 1496 days of running a large number 22400 ± 800 of solar neutrino events with recoil total energy threshold of 5 MeV were recorded. From the data of the S-K experiment the value

$$(\Phi_{\nu}^{\text{ES}})_{\text{S-K}} = (2.35 \pm 0.02 \text{ (stat.)} \pm 0.08 \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \text{ s}^{-1} \quad (118)$$

was obtained.

In the S-K experiment the spectrum of the recoil electrons was measured: no sizable distortion was observed with respect to the spectrum, expected under the assumption of no neutrino oscillations.

The spectrum of electrons, produced in the CC process (105), was measured in the SNO experiment [38]. Also in this experiment no distortion of the electron spectrum was observed.

Thus, the data of the S-K and SNO experiments are compatible with the assumption that in the high-energy ${}^8\text{B}$ region the probability of solar neutrinos to survive is practically a constant:

$$P(\nu_e \rightarrow \nu_e) \simeq \text{const.} \quad (119)$$

The constant ν_e survival probability, Eq. (119), implies

$$\langle P(\nu_e \rightarrow \nu_e) \rangle_{\text{CC}} \simeq \langle P(\nu_e \rightarrow \nu_e) \rangle_{\text{NC}} \simeq \langle P(\nu_e \rightarrow \nu_e) \rangle_{\text{ES}}.$$

Taking into account these relations, from (109), (112) and, (116) in the high energy ${}^8\text{B}$ region we obtain

$$\Phi_{\nu_e}^{\text{CC}} \simeq \Phi_{\nu_e}^{\text{NC}} \simeq \Phi_{\nu_e}^{\text{ES}}. \quad (120)$$

In the SNO experiment for the flux of ν_e on the Earth it was found

$$(\Phi_{\nu_e}^{\text{CC}})_{\text{SNO}} = (1.76_{-0.05}^{+0.06} \text{ (stat.)}_{-0.09}^{+0.09} \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}. \quad (121)$$

For the flux of all flavour neutrinos Φ_{ν}^{NC} from the NC measurement, there was obtained the value

$$(\Phi_{\nu}^{\text{NC}})_{\text{SNO}} = (5.09_{-0.43}^{+0.44} \text{ (stat.)}_{-0.43}^{+0.46} \text{ (syst.)}) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}, \quad (122)$$

which is about three times larger than the flux of electron neutrinos on the Earth.

Obviously, the NC flux is given by

$$\Phi_{\nu}^{\text{NC}} = \Phi_{\nu_e}^{\text{NC}} + \Phi_{\nu_{\mu,\tau}}^{\text{NC}}, \quad (123)$$

where $\Phi_{\nu_e}^{\text{NC}}$ is the flux of ν_e , and $\Phi_{\nu_{\mu,\tau}}^{\text{NC}}$ is the flux of ν_{μ} and ν_{τ} .

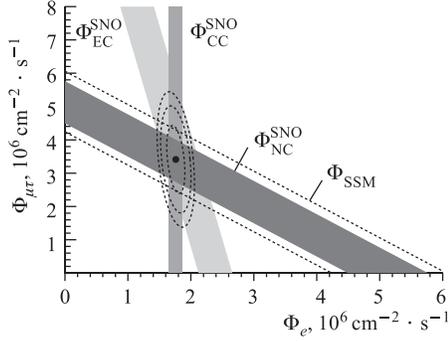


Fig. 2. Flux of ν_μ and ν_τ vs flux of ν_e in the ${}^8\text{B}$ -energy region deduced from the three neutrino reactions in SNO. The diagonal bands show the total ${}^8\text{B}$ flux as predicted by the BP00 SSM [62] (dashed lines) and that measured in SNO experiment via the observation of NC (solid band). The interception of these bands with the axes represent the $\pm 1\sigma$ errors. The bands intersect at the fit values for Φ_{ν_e} and $\Phi_{\nu_{\mu,\tau}}$. The figure is taken from Ref. 37

By combining now CC and NC fluxes and using the relation (120), we can determine the flux $\Phi_{\nu_{\mu,\tau}}^{\text{NC}}$. In Refs. 37, 38 the ES flux (117) was also taken into account (see Fig. 2). For the flux of ν_μ and ν_τ on the Earth the following value

$$(\Phi_{\nu_{\mu,\tau}})_{\text{SNO}} = (3.41_{-0.45}^{+0.45}(\text{stat.})_{-0.45}^{+0.48}(\text{syst.})) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1} \quad (124)$$

was then obtained. Thus, the detection of solar neutrinos via the simultaneous observation of CC, NC, and ES processes allowed the SNO collaboration to obtain a *direct model-independent 5.3 σ evidence of the presence of ν_μ and ν_τ in the flux of the solar neutrinos on the Earth.*

The total flux of the ${}^8\text{B}$ neutrinos, predicted by SSM BP00 [62], is equal to

$$(\Phi_{\nu_e}^0)_{\text{SSM}} = (5.05_{-0.81}^{+1.01}) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1}. \quad (125)$$

This value is compatible with the value of total flux of all flavour neutrinos (122), determined from the data of the SNO experiment.

The flux of ν_μ and ν_τ on the Earth can be also obtained from the SNO CC data and the S-K ES data. In the first SNO publication [36] the value

$$(\Phi_{\nu_{\mu,\tau}})_{\text{S-K,SNO}} = (3.69 \pm 1.13) \cdot 10^6 \text{ cm}^{-2} \cdot \text{s}^{-1} \quad (126)$$

was found, which is in good agreement with the value (124).

The data of all solar neutrino experiments can be described if one assumes that there are transitions of solar ν_e into ν_μ and ν_τ , and that the ν_e survival probability is given by the two-neutrino expression, which is characterized by two oscillation parameters, Δm_{sol}^2 and $\tan^2 \theta_{\text{sol}}$. From the global χ^2 fit of the total event rates, measured in all solar neutrino experiments, several allowed regions (solutions) in the plane of the oscillation parameters were obtained (see, for example, Ref. 64): LMA, LOW, SMA, VO and other regions. The situation changed after the recoil electron spectrum was measured in the S-K experiment [35] and

the SNO results [36–38] were obtained. Analyses of all solar neutrino data showed that the large mixing angle LMA MSW region is the most plausible one (see [65] and references therein). In Ref. 38 the following best-fit LMA values of the solar neutrino oscillation parameters were found:

$$\Delta m_{\text{sol}}^2 = 5 \cdot 10^{-5} \text{ eV}^2, \quad \tan^2 \theta_{\text{sol}} = 0.34, \quad \chi_{\text{min}}^2 = 57/72 \text{ d.o.f.} \quad (127)$$

Recent data of the KamLAND experiment [39] (see below) allow one to exclude all solutions of the solar neutrino problem, except the LMA one.

7.4. Reactor Experiments CHOOZ and Palo Verde. In the long baseline reactor experiments CHOOZ [41] and Palo Verde [42] the disappearance of the reactor $\bar{\nu}_e$'s in the atmospheric range of Δm^2 was searched for. In spite of the fact that in these experiments no indication in favour of neutrino oscillations were found, their results are very important for the neutrino mixing.

In the CHOOZ experiment $\bar{\nu}_e$'s from two reactors at a distance of about 1 km from the detector were recorded via the observation of the process

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The value of the ratio R of the total number of the detected $\bar{\nu}_e$ events to the expected number was found to be

$$R = 1.01 \pm 2.8\% \text{ (stat.)} \pm 2.7\% \text{ (syst.) (CHOOZ)}.$$

In the similar Palo Verde experiment it was obtained:

$$R = 1.01 \pm 2.4\% \text{ (stat.)} \pm 5.3\% \text{ (syst.) (Palo Verde)}.$$

The data of the experiments were analyzed in [41,42] in the framework of two-neutrino oscillations, and exclusion plots in the plane of the oscillation parameters Δm^2 and $\sin^2 2\theta$ were obtained. From the CHOOZ exclusion plot at $\Delta m^2 = 2.5 \cdot 10^{-3} \text{ eV}^2$ (the S-K best-fit value) one gets

$$(\sin^2 2\theta)_{\text{CHOOZ}} \lesssim 1.5 \cdot 10^{-1}.$$

7.5. The KamLAND Evidence in Favour of the Disappearance of the Reactor $\bar{\nu}_e$. The first results of the KamLAND experiment [39] were published recently. In this experiment $\bar{\nu}_e$'s from many reactors in Japan and Korea are detected via the observation of the classical process

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The threshold of this process is $\simeq 1.8 \text{ MeV}$. About 80 % of the total number of the events is due to $\bar{\nu}_e$ from 26 reactors within the distances of 138–214 km.

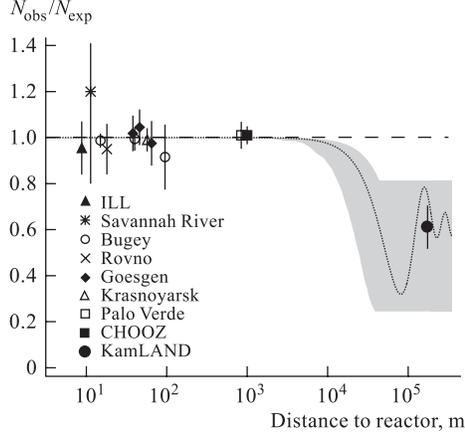


Fig. 3. The ratio of measured-to-expected numbers of $\bar{\nu}_e$ events for different reactor neutrino experiments. The shaded region indicates the range of the predictions corresponding to the 95 % CL. LMA region found from the global analysis of the solar neutrino data [66]. The dotted curve corresponds to the best-fit values $\Delta m_{\text{sol}}^2 = 5.5 \cdot 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{\text{sol}} = 0.83$ found in Ref. 66. The Figure is taken from Ref. 39

The 1 kt liquid scintillator detector of the KamLAND experiment is located in the Kamioka mine at a depth of about 1 km. Both prompt photons from the annihilation of e^+ in the scintillator and 2.2 MeV delayed photons from the neutron capture $n + p \rightarrow d + \gamma$ are detected. The mean neutron capture time is $188 \pm 23 \mu\text{s}$. In order to avoid background, mainly from decay of ^{238}U and ^{232}Th in the Earth, the cut $E_{\text{prompt}} > 2.6 \text{ MeV}$ was applied.

During 145.1 days of running there were observed 54 $\bar{\nu}_e$ events. The number of the events expected in the absence of neutrino oscillations is equal to 86.8 ± 5.6 . For the ratio of observed and expected $\bar{\nu}_e$ events the following value

$$\frac{N_{\text{obs}} - N_{\text{BG}}}{N_{\text{exp}}} = 0.611 \pm 0.085 \pm 0.041 \quad (128)$$

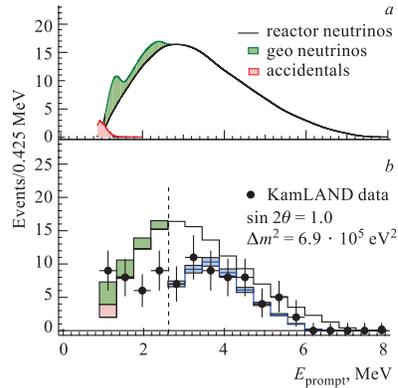
was obtained.

In Fig. 3, for all reactor neutrino experiments, the dependence of the ratio of the observed and expected $\bar{\nu}_e$ events on the average distance between reactors and detectors is plotted. The dotted curve was calculated with the best-fit solar neutrino LMA values of the oscillation parameters $\Delta m^2 = 5.5 \cdot 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta = 0.83$, obtained in Ref. 66.

In the KamLAND experiment the prompt energy spectrum was also measured (see Fig. 4). The prompt energy is connected with the energy of $\bar{\nu}_e$ by the relation $E_{\text{prompt}} = E_{\bar{\nu}_e} - 0.8 \text{ MeV} - \bar{E}_n$ (\bar{E}_n being the average energy of the neutron). From the two-neutrino analysis of the KamLAND data the following best-fit values of the oscillation parameters were obtained

$$(\Delta m^2)_{\text{KamLAND}} = 6.9 \cdot 10^{-5} \text{ eV}^2, \quad (\sin^2 2\theta)_{\text{KamLAND}} = 1.$$

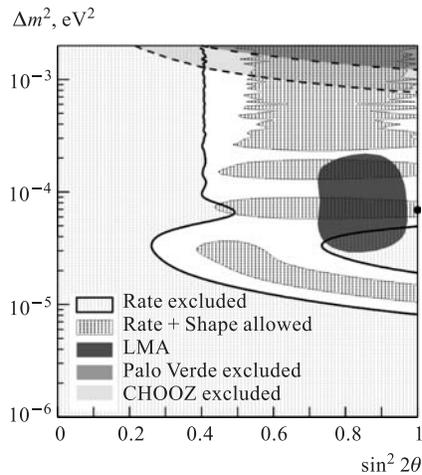
Fig. 4. *a*) Expected reactor $\bar{\nu}_e$ energy spectrum with contributions of $\bar{\nu}_{\text{geo}}$ (antineutrinos produced in ^{238}U and ^{232}Th decays in the Earth) and accidental background. *b*) Energy spectrum of the observed prompt events (solid circles with error bars), along with the expected no oscillation spectrum (upper histogram, with $\bar{\nu}_{\text{geo}}$ and accidentals shown) and best fit (lower histogram) in the case of neutrino oscillations. The shaded band indicates the systematic error in the best-fit spectrum. The vertical dashed line corresponds to the threshold of 2.6 MeV. The figure is taken from Ref. 39



These values are compatible with the values of the oscillation parameters in the solar neutrino LMA MSW region. The allowed region in the plane of the oscillation parameters, obtained from the analysis of the measured rate and measured spectrum is shown in Fig. 5 (95 % CL). The region outside the solid line is excluded from the rate analysis. The dark region is the solar neutrino LMA allowed region, obtained in Ref. 66.

The KamLAND results provide a strong evidence for neutrino masses and oscillations, obtained for the first time in an experiment with terrestrial antineutrinos with the expected flux well under control. It allows us to exclude the SMA, LOW, and VAC regions of neutrino oscillation parameters. The only viable solution of the solar neutrino problem appears to be the LMA MSW solution.

Fig. 5. KamLAND excluded regions of neutrino oscillation parameters Δm^2 and $\sin^2 2\theta$ for the rate analysis and allowed regions for the combined rate and energy spectrum analysis at 95 % CL. At the top is the 95 % CL excluded region from CHOOZ [41] and Palo Verde [42] experiments, respectively. The dark area is the 95 % CL LMA allowed region obtained in Ref. 66. The thick dot indicates the best-fit values of the oscillation parameters. The figure is taken from Ref. 39



8. NEUTRINO OSCILLATIONS IN THE FRAMEWORK OF THREE-NEUTRINO MIXING

8.1. Neutrino Oscillations in the Atmospheric Range of the Neutrino Mass-Squared Difference. We have discussed evidences in favour of neutrino oscillations, which were obtained in the solar, atmospheric and reactor KamLAND neutrino experiments. At present there exists also an indication in favour of the transition $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, that was obtained in the single accelerator LSND experiment [67]. The LSND data can be explained by neutrino oscillations. From the analysis of the data, the following ranges for the values of the oscillation parameters

$$\begin{aligned} 2 \cdot 10^{-1} &\lesssim (\Delta m^2)_{\text{LSND}} \lesssim 1 \text{ eV}^2, \\ 3 \cdot 10^{-3} &\lesssim (\sin^2 2\theta)_{\text{LSND}} \lesssim 4 \cdot 10^{-2} \end{aligned}$$

were obtained.

In order to describe the data of the solar, atmospheric, KamLAND and LSND experiments, which require three different values of neutrino mass-squared differences, it is necessary to assume that there are (at least) four massive and mixed neutrinos. This means that in addition to the three flavour neutrinos (at least) one sterile neutrino must exist (see, for example, Ref. 45). The result of the LSND experiment requires, however, confirmation. The MiniBooNE experiment at Fermilab [68], which started in 2002, is aimed at checking the LSND result.

We will consider here the *minimal* scheme of three-neutrino mixing

$$\nu_{\alpha L} = \sum_{i=1}^3 U_{\alpha i} \nu_{iL}, \quad (129)$$

where U is the unitary 3×3 PMNS mixing matrix. This scheme provides two independent Δm^2 's and allows us to describe solar, atmospheric, KamLAND and other neutrino oscillation data.

Let us start with the consideration of neutrino oscillations in the atmospheric range of Δm^2 , which can be explored in the atmospheric and long baseline accelerator and reactor neutrino experiments. In the framework of the three-neutrino mixing (129), with $m_1 < m_2 < m_3$, there are two possibilities:

I. Hierarchy of neutrino mass-squared differences

$$\Delta m_{21}^2 \simeq \Delta m_{\text{sol}}^2, \quad \Delta m_{32}^2 \simeq \Delta m_{\text{atm}}^2, \quad \Delta m_{21}^2 \ll \Delta m_{32}^2. \quad (130)$$

II. Inverted hierarchy of neutrino mass-squared differences

$$\Delta m_{32}^2 \simeq \Delta m_{\text{sol}}^2, \quad \Delta m_{21}^2 \simeq \Delta m_{\text{atm}}^2, \quad \Delta m_{32}^2 \ll \Delta m_{21}^2. \quad (131)$$

We will first assume that the neutrino mass spectrum is of the type I. The values of L/E , relevant for neutrino oscillations in the atmospheric range of neutrino mass-squared difference, satisfy the inequality:

$$\Delta m_{21}^2 \frac{L}{E} \ll 1.$$

Thus, we can neglect the contribution of Δm_{21}^2 to the transition probability, Eq. (40). In this case, for the probability of the transition $\nu_\alpha \rightarrow \nu_{\alpha'}$ we obtain the following expression

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) \simeq \left| \delta_{\alpha'\alpha} + U_{\alpha'3} U_{\alpha 3}^* \left(\exp \left(-i \Delta m_{32}^2 \frac{L}{2E} \right) - 1 \right) \right|^2. \quad (132)$$

Hence, in the leading approximation, the transition probabilities in the atmospheric range of Δm^2 are determined by the largest neutrino mass-squared difference Δm_{32}^2 and by the elements of the third column of the neutrino mixing matrix, which connect the flavour neutrino fields $\nu_{\alpha L}$ with the field of the heaviest neutrino ν_{3L} .

For the appearance probability we obtain from Eq. (132) the expression

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \frac{1}{2} A_{\alpha';\alpha} \left(1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right) \quad (\alpha \neq \alpha'), \quad (133)$$

where the oscillation amplitude is given by

$$A_{\alpha';\alpha} = 4 |U_{\alpha'3}|^2 |U_{\alpha 3}|^2. \quad (134)$$

The survival probability can be obtained from the condition of conservation of probability and from Eqs. (133) and (134) we have

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sum_{\alpha' \neq \alpha} P(\nu_\alpha \rightarrow \nu_{\alpha'}) = \\ &= 1 - \frac{1}{2} B_{\alpha;\alpha} \left(1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right). \end{aligned} \quad (135)$$

Taking into account the unitarity of the mixing matrix, the amplitude $B_{\alpha;\alpha}$ can be written as:

$$B_{\alpha;\alpha} \equiv \sum_{\alpha' \neq \alpha} A_{\alpha';\alpha} = 4 |U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2). \quad (136)$$

Obviously, in the case of inverted hierarchy of the neutrino mass-squared differences (case II above) the transition probabilities can be obtained from Eqs. (133)–(136) with the replacements $\Delta m_{32}^2 \rightarrow \Delta m_{21}^2$ and $|U_{\alpha 3}|^2 \rightarrow |U_{\alpha 1}|^2$.

We notice that the transition probability (133) depends only on $|U_{\alpha 3}|^2$ and Δm_{32}^2 . The CP phase does not enter into this equation and in the leading approximation the relation

$$P(\nu_\alpha \rightarrow \nu_{\alpha'}) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) \quad (137)$$

is automatically satisfied.

Thus, the investigation of the effects of CP violation in the lepton sector in the future long baseline neutrino oscillation experiments will be a difficult problem: possible effects are suppressed due to the smallness of the parameter $\Delta m_{12}^2/\Delta m_{32}^2$. High precision experiments on the search for effects of CP violation in the lepton sector are planned for the future neutrino facility JHF [69] and Neutrino Factories [70,71].

The transition probabilities (133) and (135) have a *two-neutrino form* in each channel. This is the obvious consequence of the fact that only the largest mass-squared difference Δm_{32}^2 contributes to the transition probabilities. The elements $|U_{\alpha 3}|^2$, which determine the oscillation amplitudes, satisfy the unitarity condition $\sum_\alpha |U_{\alpha 3}|^2 = 1$. Hence, in leading approximation, transition probabilities are characterized by three parameters. We can choose the latter to be:

$$\Delta m_{32}^2, \quad \sin^2 \theta_{23}, \quad |U_{e3}|^2.$$

Then, from Eqs. (52) and (134), for the amplitudes of $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_e$ transitions we obtain the expressions

$$A_{\tau;\mu} = (1 - |U_{e3}|^2)^2 \sin^2 2\theta_{23},$$

$$A_{e;\mu} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \sin^2 \theta_{23}.$$

8.2. Oscillations in the Solar Range of Neutrino Mass-Squared Difference.

Let us consider now, in the framework of the three-neutrino mixing, neutrino oscillations in the solar range of Δm^2 . The ν_e survival probability in vacuum can be written in the form

$$P(\nu_e \rightarrow \nu_e) = \left| \sum_{i=1,2} |U_{ei}|^2 \exp\left(-i\Delta m_{i1}^2 \frac{L}{2E}\right) + |U_{e3}|^2 \exp\left(-i\Delta m_{31}^2 \frac{L}{2E}\right) \right|^2. \quad (138)$$

We are interested in the ν_e survival probability averaged over the region where neutrinos are produced, over energy resolution, etc. Because of the hierarchy $\Delta m_{32}^2 \gg \Delta m_{21}^2$, in the expression for the averaged survival probability the

interference between the first and the second terms in Eq. (138) disappears. The averaged survival probability can then be presented in the form

$$P(\nu_e \rightarrow \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 P^{(1,2)}(\nu_e \rightarrow \nu_e). \quad (139)$$

Here $P^{(1,2)}(\nu_e \rightarrow \nu_e)$ is given by the expression

$$P^{(1,2)}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} A^{(1,2)} \left(1 - \cos \Delta m_{21}^2 \frac{L}{2E} \right), \quad (140)$$

where

$$A^{(1,2)} = 4 \frac{|U_{e1}|^2 |U_{e2}|^2}{(1 - |U_{e3}|^2)^2}. \quad (141)$$

With the help of Eqs. (43), Eq. (141) simplifies to

$$A^{(1,2)} = \sin^2 2\theta_{12}. \quad (142)$$

Thus, the probability $P^{(1,2)}(\nu_e \rightarrow \nu_e)$ is again characterized by two parameters only and has the standard two-neutrino form.

The expression (139) is also valid for oscillations in matter (see [45, 72]). In this case $P^{(1,2)}(\nu_e \rightarrow \nu_e)$ is the two-neutrino ν_e survival probability in matter, calculated under the condition that the density of electrons $\rho_e(x)$ in the effective Hamiltonian of the interaction of neutrino with matter is changed by $(1 - |U_{e3}|^2) \rho_e(x)$.

Hence, the ν_e survival probability is characterized, in the solar range of Δm^2 , by three parameters

$$\Delta m_{21}^2, \quad \tan^2 \theta_{12}, \quad |U_{e3}|^2.$$

The only common parameter for the atmospheric and solar ranges of Δm^2 is $|U_{e3}|^2$. As we will see in the next subsection, from the data of the reactor CHOOZ and Palo Verde experiments, this parameter turns out to be small.

8.3. The Upper Bound of $|U_{e3}|^2$ from the CHOOZ Data. Decoupling of Oscillations in the Solar and Atmospheric Ranges of Δm^2 . The long baseline reactor experiments CHOOZ [41] and Palo Verde [42] are sensitive to the atmospheric range of Δm^2 . No indications in favour of the disappearance of reactor $\bar{\nu}_e$ was obtained in these experiments. From the analysis of their data a stringent bound on the parameter $|U_{e3}|^2$ was obtained, as it is explained below.

In the framework of the three-neutrino mixing the probability of $\bar{\nu}_e$ to survive in the atmospheric range of Δm^2 is given by the expression

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} B_{e;\bar{e}} \left(1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right), \quad (143)$$

where

$$B_{e;\bar{e}} = 4|U_{e3}|^2 (1 - |U_{e3}|^2). \quad (144)$$

In Refs. 41, 42 exclusion plots in the plane of the parameters $\Delta m^2 \equiv \Delta m_{32}^2$ and $\sin^2 2\theta \equiv B_{e;e}$ were obtained, from which one has

$$B_{e;e} \leq B_{e;e}^0, \quad (145)$$

where the upper bound $B_{e;e}^0$ depends on Δm_{32}^2 . For the S-K allowed values of the parameter Δm_{32}^2 , from the CHOOZ exclusion plot we have

$$1 \cdot 10^{-1} \leq B_{e;e}^0 \leq 2.4 \cdot 10^{-1}. \quad (146)$$

From Eqs. (144) and (146) the following bounds on the parameter $|U_{e3}|^2$ can be easily obtained:

$$|U_{e3}|^2 \leq \frac{1}{2}(1 - \sqrt{1 - B_{e;e}^0}) \lesssim \frac{1}{4}B_{e;e}^0 \quad (147)$$

or

$$|U_{e3}|^2 \gtrsim \frac{1}{2}(1 + \sqrt{1 - B_{e;e}^0}) \geq 1 - \frac{1}{4}B_{e;e}^0. \quad (148)$$

Thus, the parameter $|U_{e3}|^2$ can be either small or large (close to one). This last possibility is excluded, however, by the solar neutrino data. In fact, if $|U_{e3}|^2$ is large, from Eq. (139) it follows that in the whole range of the solar neutrino energies the probability of ν_e to survive is close to one in obvious contradiction with the solar neutrino data. Hence, the upper bound of the parameter $|U_{e3}|^2$ is given by Eq. (147). At the S-K best-fit value $\Delta m_{32}^2 = 2.5 \cdot 10^{-3} \text{ eV}^2$ we get:

$$|U_{e3}|^2 \leq 4 \cdot 10^{-2} \text{ (95 \% CL)}. \quad (149)$$

Taking into account the accuracies of the present-day experiments, one can neglect $|U_{e3}|^2$ in the expressions for the transition probabilities. In this approximation, neutrino oscillations in the atmospheric range of Δm^2 are $\nu_\mu \rightarrow \nu_\tau$ oscillations, with $\Delta m_{32}^2 \simeq \Delta m_{\text{atm}}^2$ and $A_{\tau;\mu} \simeq \sin^2 2\theta_{23} \simeq \sin^2 2\theta_{\text{atm}}$. In the solar range of Δm^2 we have

$$P(\nu_e \rightarrow \nu_e) \simeq P^{(1,2)}(\nu_e \rightarrow \nu_e), \quad (150)$$

where the two-neutrino survival probability $P(\nu_e \rightarrow \nu_e)$ depends on the parameters $\Delta m_{21}^2 \simeq \Delta m_{\text{sol}}^2$ and $\tan^2 2\theta_{12} \simeq \tan^2 2\theta_{\text{sol}}$.

Thus, due to the smallness of the parameter $|U_{e3}|^2$ and the hierarchy of neutrino mass-squared differences $\Delta m_{12}^2 \ll \Delta m_{32}^2$, neutrino oscillations in the atmospheric and solar ranges of Δm^2 , in the leading approximation, are decoupled [45, 73] and are described by the two-neutrino formulas, which are characterized, respectively, by the oscillation parameters

$$\Delta m_{32}^2, \quad \sin^2 2\theta_{23} \quad \text{and} \quad \Delta m_{21}^2, \quad \tan^2 \theta_{12}.$$

From the CHOOZ data only the upper bound of the parameter $|U_{e3}|^2$ can be obtained. The possibilities to investigate effects of the three-neutrino mixing and in particular important effects of CP violation in the lepton sector depend on the value of this parameter. The value of the parameter $|U_{e3}|^2$ will be probed in MINOS [74] and ICARUS [75] experiments and in neutrino experiments at JHF [76] and Neutrino Factories (see [70,71]).

From the data of the experiments on the investigation of neutrino oscillations only neutrino mass-squared differences Δm_{21}^2 and Δm_{32}^2 can be determined. Neutrino masses m_2 and m_3 are given by the relations

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{32}^2 + \Delta m_{21}^2}. \quad (151)$$

In the next section we will discuss the results of experiments which allow us to obtain information about the absolute values of neutrino masses.

9. β -DECAY EXPERIMENTS ON THE MEASUREMENT OF NEUTRINO MASS

The standard method for the measurement of the absolute value of the neutrino mass is based on the detailed investigation of the high-energy part of the β spectrum of the decay of tritium



This decay is the super-allowed one. Thus, the nuclear matrix element is a constant and the electron spectrum is determined by the phase space. The decay (152) has a small energy release ($E_0 \simeq 18.6$ keV) and a convenient time of life ($T_{1/2} \simeq 12.3$ y).

The standard effective Hamiltonian of the β decay is given by

$$\mathcal{H}_I^{\text{CC}} = \frac{G_F}{\sqrt{2}} 2\bar{e}_L \gamma_\alpha \nu_{eL} j^\alpha + \text{h. c.} \quad (153)$$

Here j^α is the hadronic charged current; G_F is the Fermi constant and

$$\nu_{eL} = \sum_i U_{ei} \nu_{iL}, \quad (154)$$

where ν_i is the field of neutrino with mass m_i , and U is the unitary mixing matrix.

Neglecting the recoil of the final nucleus, from Eqs. (153) and (154) for the spectrum of electrons we obtain the following expression

$$\frac{d\Gamma}{dE} = \sum_i |U_{ei}|^2 \frac{d\Gamma_i}{dE}, \quad (155)$$

where

$$\frac{d\Gamma_i}{dE} = Cp(E + m_e)(E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2} \times F(E)\theta(E_0 - E - m_i), \quad (156)$$

E being the kinetic energy of the electron; E_0 — the energy released in the decay and m_e is the mass of the electron. The Fermi function $F(E)$ takes into account the Coulomb interaction of the final particles and the constant C is given by the expression

$$C = G_F^2 \frac{m_e^5}{2\pi^3} \cos^2 \theta_C |M|^2,$$

where θ_C is the Cabibbo angle and M — the nuclear matrix element (which is a constant).

The sensitivity to the neutrino mass of the present-day tritium experiments Troitsk [77] and Mainz [78] is 2–3 eV. In the future experiment KATRIN [79], a sensitivity of 0.25 eV is expected. Taking into account that $\sqrt{\Delta m_{\text{atm}}^2} \simeq 5 \cdot 10^{-2}$ eV is much smaller than the sensitivities of the present and future tritium experiments, from the relations (151) we can conclude that the neutrino mass can be measured in these experiments only if the neutrino mass spectrum is practically degenerate: $m_1 \simeq m_2 \simeq m_3$. In this case, from the unitarity of the mixing matrix we have:

$$\frac{d\Gamma}{dE} = Cp(E + m_e)(E_0 - E)\sqrt{(E_0 - E)^2 - m_1^2} F(E). \quad (157)$$

Let us discuss now the results of the tritium experiments. In the Mainz experiment [78] the target is molecular tritium condensed on a graphite substrate. The spectrum of the electron is measured by the integral electrostatic spectrometer, which combines high luminosity with high resolution. The resolution of the Mainz spectrometer is equal to 4.8 eV. In the analysis of the experimental data four free variable parameters are used: the normalization C , the background B , the released energy E_0 and the neutrino mass-squared m_1^2 . The analysis of 1998, 1999, and 2001 data gave the following result:

$$m_1^2 = (-1.2 \pm 2.2 \pm 2.1) \text{ eV}^2. \quad (158)$$

This value corresponds to the upper bound

$$m_1 < 2.2 \text{ eV (95\% CL)}. \quad (159)$$

The integral electrostatic spectrometer is also used in the Troitsk neutrino experiment [77]: its resolution is 3.5–4 eV. In the Troitsk experiment the source is a gaseous molecular tritium source. From the four-parameter fit of the Troitsk

data, for the parameter m_1^2 large negative values, in the range $(-10 \div -20) \text{ eV}^2$, were obtained. The investigation of the character of the measured spectrum suggests that the effect of obtaining a negative m_1^2 is due to a step function superimposed on the integral continuous spectrum: the step function in the integral spectrum corresponds to a narrow peak in the differential spectrum.

In the analysis of the data, the authors of the Troitsk experiment added to the theoretical integral spectrum a step function with two additional variable parameters (the position of the step E_{step} and its height). From the six-parameter fit of the data, the following value for the parameter m_1^2

$$m_1^2 = (-2.3 \pm 2.5 \pm 2.0) \text{ eV}^2 \quad (160)$$

was found. From (160) the upper bound on the neutrino mass m_1

$$m_1 < 2.2 \text{ eV} \quad (95 \% \text{ CL}) \quad (161)$$

can be deduced.

The analysis of the Troitsk data shows that the position of the step $E_0 - E_{\text{step}}$ is periodically changed in the interval 5–15 eV and the average value of the height of the step is about $6 \cdot 10^{-11}$. The existence of this anomaly was not confirmed by the Mainz experiment [78].

A new tritium experiment, KATRIN [79], is now under preparation. In this experiment gaseous molecular source and frozen tritium source are planned to be used. The integral electrostatic spectrometer will have two parts: the prespectrometer, which will select electrons in the last $\simeq 100$ eV of the spectrum, and the main spectrometer. The latter will have a resolution of $\simeq 1$ eV. It is expected that the KATRIN experiment will start to collect data in 2007. After three years of running the accuracy of the measurement of the neutrino mass will reach $\simeq 0.25$ eV.

In order to understand the origin of the small neutrino masses we need to know the nature of the massive neutrinos: are they Majorana or Dirac particles? As we have seen, neutrino oscillation experiments cannot answer this fundamental question. The nature of the massive neutrinos can be revealed in experiments on the search for neutrinoless double β decay of some even–even nuclei. The next section will be devoted to the discussion of such experiments.

10. NEUTRINOLESS DOUBLE β DECAY

The search for neutrinoless double β decay

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \quad (162)$$

of some even–even nuclei is the most sensitive and direct way of investigating the nature of neutrinos with definite masses. The total lepton number in the process

(162) is violated and this is allowed only if the massive neutrinos ν_i are Majorana particles.

We will assume that the Hamiltonian of the process has the standard form, Eq. (153), and that the flavour field ν_{eL} is given by

$$\nu_{eL} = \sum_i U_{ei} \nu_{iL}, \quad (163)$$

where ν_i are Majorana fields.

The neutrinoless double β decay ($(\beta\beta)_{0\nu}$ decay) is a process of second order in the Fermi constant G_F , with virtual neutrinos. For small neutrino masses the neutrino propagator is given by the expression

$$\langle 0 | T(\nu_{eL}(x_1) \nu_{eL}^T(x_2)) | 0 \rangle \simeq m_{\beta\beta} \frac{i}{(2\pi)^4} \int \frac{d^4 p}{p^2} e^{-ip(x_1-x_2)} \frac{1-\gamma_5}{2} C, \quad (164)$$

where

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i. \quad (165)$$

The total matrix element of the $(\beta\beta)_{0\nu}$ decay is a product of $m_{\beta\beta}$ and the nuclear matrix element, which does not depend on neutrino masses and mixing.

Results of many experiments on the search for $(\beta\beta)_{0\nu}$ decay are available at present (see [80, 81]). No indication in favour of $(\beta\beta)_{0\nu}$ decay was obtained so far*. The most stringent lower bounds for the time of life of $(\beta\beta)_{0\nu}$ decay were obtained in the Heidelberg–Moscow [85] and IGEX [86] ^{76}Ge experiments:

$$T_{1/2}^{0\nu} \geq 1.9 \cdot 10^{25} \text{ y (90 \% CL) Heidelberg–Moscow,}$$

$$T_{1/2}^{0\nu} \geq 1.57 \cdot 10^{25} \text{ y (90 \% CL) IGEX.}$$

Taking into account different calculations of the nuclear matrix element, from these results the following upper bounds were obtained for the effective Majorana mass:

$$|m_{\beta\beta}| \leq (0.35 - 1.24) \text{ eV.} \quad (166)$$

Many new experiments on the search for the neutrinoless double β decay are in preparation at present (see Ref. 80). In these experiments the sensitivities

$$|m_{\beta\beta}| \simeq (1 \cdot 10^{-1} - 1.5 \cdot 10^{-2}) \text{ eV}$$

are expected to be achieved.

The evidence for neutrinoless double β decay would be a proof that neutrinos with definite masses are Majorana particles. The value of the effective Majorana

*The recent claim [82] of evidence of a $(\beta\beta)_{0\nu}$ decay, obtained from the reanalysis of the data of the Heidelberg–Moscow experiment, has been strongly criticized in Refs. 83, 84.

mass $|m_{\beta\beta}|$ combined with the values of the neutrino oscillation parameters, obtained from the results of neutrino oscillation experiments, would enable us to obtain important information about the character of the neutrino mass spectrum, the minimal neutrino mass m_1 and the Majorana CP phase (see Ref. 87 and references therein).

We will consider here three typical neutrino mass spectra.

1. The hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$. In this case for the effective Majorana mass we have the following upper bound:

$$|m_{\beta\beta}| \leq \sin^2 \theta_{\text{sol}} \sqrt{\Delta m_{\text{sol}}^2} + |U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2}. \quad (167)$$

Using the best-fit values of the oscillation parameters and the CHOOZ limit on $|U_{e3}|^2$ (see Eqs. (53), (77), and (149)), we obtain from (167) the upper bound

$$|m_{\beta\beta}| \leq 3.8 \cdot 10^{-3} \text{ eV}, \quad (168)$$

which is significantly smaller than the expected sensitivities of the future $(\beta\beta)_{0\nu}$ experiments.

2. Inverted hierarchy of neutrino masses: $m_1 \ll m_2 < m_3$. The effective Majorana mass is given, in this case, by the expression

$$|m_{\beta\beta}| \simeq (1 - \sin^2 2\theta_{\text{sol}} \sin^2 \alpha)^{1/2} \sqrt{\Delta m_{\text{atm}}^2}, \quad (169)$$

where $\alpha = \alpha_3 - \alpha_2$ is the difference of the Majorana CP phases ($U_{ei} = |U_{ei}| e^{i\alpha_i}$). From this expression it follows that

$$\sqrt{\Delta m_{\text{atm}}^2} |\cos 2\theta_{\text{sol}}| \lesssim |m_{\beta\beta}| \lesssim \sqrt{\Delta m_{\text{atm}}^2}, \quad (170)$$

where the upper and lower bounds correspond to the case of CP conservation with equal and opposite CP parities of ν_3 and ν_2 , respectively.

Using the best-fit value of the parameter $\tan^2 \theta_{\text{sol}}$ (see Eq. (127)), we have

$$\frac{1}{2} \sqrt{\Delta m_{\text{atm}}^2} \lesssim |m_{\beta\beta}| \lesssim \sqrt{\Delta m_{\text{atm}}^2}, \quad (171)$$

Thus, in the case of the inverted mass hierarchy, the scale of $|m_{\beta\beta}|$ is determined by $\sqrt{\Delta m_{\text{atm}}^2}$. Should the value of $|m_{\beta\beta}|$ be in the range (171) (which can be reached in the future experiments on the search for $(\beta\beta)_{0\nu}$ decay), then we would have an argument in favour of the inverted neutrino mass hierarchy.

3. Practically degenerate neutrino-mass spectrum: $m_2 \simeq m_3 \simeq m_1 \gg \sqrt{\Delta m_{\text{atm}}^2}$. The effective Majorana mass in this case is given by the expression

$$|m_{\beta\beta}| \simeq m_1 \left| \sum_{i=1}^3 U_{ei}^2 \right|. \quad (172)$$

Neglecting the small contribution of $|U_{e3}|^2$ ($|U_{e1}|^2$ in the case of the inverted hierarchy), for $|m_{\beta\beta}|$ one obtains relations Eqs. (169)–(171) in which $\sqrt{\Delta m_{\text{atm}}^2}$ is replaced by m_1 . Thus, a signature of the degenerate neutrino-mass spectrum would be the occurrence of a value $|m_{\beta\beta}| \gg \sqrt{\Delta m_{\text{atm}}^2}$.

It is obvious that for neutrino mass m_1 we have the following bound

$$|m_{\beta\beta}| \leq m_1 \leq \frac{|m_{\beta\beta}|}{|\cos 2\theta_{\text{sol}}|} \simeq 2|m_{\beta\beta}|, \quad (173)$$

and the parameter $\sin^2 \alpha$, which characterizes the violation of the CP invariance in the lepton sector [87], is given by the relation

$$\sin^2 \alpha \simeq \left(1 - \frac{|m_{\beta\beta}|^2}{m_1^2}\right) \frac{1}{\sin^2 2\theta_{\text{sol}}}. \quad (174)$$

Thus, if the mass m_1 is measured in the future experiments and the value of the parameter $\sin^2 2\theta_{\text{sol}}$ is determined with high accuracy in the KamLAND [39], BOREXINO [89] and other neutrino experiments, from the results of the future $(\beta\beta)_{0\nu}$ experiments information on the Majorana CP phase can be inferred.

All previous conclusions are based on the assumption that the value of the effective Majorana mass $|m_{\beta\beta}|$ can be obtained from the measurement of the $(\beta\beta)_{0\nu}$ -decay lifetime. However, the determination of the parameter $|m_{\beta\beta}|$ from the experimental data requires the knowledge of the nuclear matrix elements. At present there are large uncertainties in the calculation of these quantities (see, for example, Refs. 90–92). Different calculations of the $(\beta\beta)_{0\nu}$ -decay lifetime differ by about one order of magnitude.

In Ref. 93 a method was proposed, which allows one to check the results of the calculations of the $(\beta\beta)_{0\nu}$ -decay nuclear matrix elements in a model-independent way. This method can be applied if $(\beta\beta)_{0\nu}$ decay of *different* nuclei is observed.

CONCLUSION

About forty years after the original idea of B. Pontecorvo, compelling evidence in favour of neutrino oscillations was obtained in the S-K [29], SNO [36–38], KamLAND [39] and other neutrino experiments. These findings opened a new field of research in particle physics and astrophysics: *the physics of massive and mixed neutrinos*.

From the results of the experiments it follows that neutrino masses are many orders of magnitude smaller than the masses of leptons and quarks. Tiny neutrino masses are a first signature of a new physics, beyond the Standard Model.

There remain many unsolved problems in the physics of massive and mixed neutrinos. The problem concerning $|U_{e3}|^2$ is an urgent one: the possibility of

studying the effects of the three-neutrino mixing and, in particular, the effects of CP violation in the lepton sector depends on the value of $|U_{e3}|^2$.

Another problem is the one connected with the LSND experiment [67]. If the LSND result will be confirmed by the MiniBOONE experiment [68], this will imply that the number of light neutrinos is more than three and in addition to the three flavour neutrinos sterile neutrino(s) must exist. If, on the contrary, LSND result is refuted, the minimal scheme with three massive and mixed neutrinos will be a plausible possibility.

The problem of the nature of massive neutrinos (Dirac or Majorana?) is the most fundamental one. This problem can be solved by the experiments on the search for neutrinoless double β decay. From the existing data the following bound on the effective Majorana mass was found: $|m_{\beta\beta}| \leq (0.3-1.3)$ eV. In future experiments, now in preparation, a significant improvement of the sensitivity is expected.

Due to the interference nature of the neutrino oscillation phenomenon and to the possibilities of exploring large values of L/E , neutrino oscillation experiments are sensitive to very small values of the neutrino mass-squared differences. The determination of the absolute value of the neutrino masses requires the so-called direct measurements and it is a challenging problem. From the data of the tritium experiments the upper bound $m_1 \leq 2.2$ MeV was obtained. The future experiment KATRIN [79] is expected to be sensitive to $m_1 \simeq 0.25$ MeV.

The progress in cosmological measurements, achieved in the last years, allows one to reach $\simeq 1$ eV sensitivity for the sum of the neutrino masses $\sum_i m_i$. The most stringent bound,

$$\sum_i m_i < 0.71 \text{ eV},$$

was recently obtained in Ref. 95. For a degenerate neutrino mass spectrum this bound implies

$$m_i < 0.23 \text{ eV}.$$

In Ref. 95 the 2dF Galaxy Redshift Survey [94] data, recent WMAP high precision data and other cosmological data were used. A significant improvement of this limit is expected with the future Sloan Digital Sky Survey [96], future WMAP and other data.

We have discussed here mainly the phenomenology of neutrino mixing and oscillations, and the most recent experimental data. For models of neutrino masses and mixing we refer the reader to the recent reviews [46–48].

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