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LEPTON POLARIZATION IN NEUTRINO–NUCLEON INTERACTIONS

K. S. Kuzmin[1] , *V. V. Lyubushkin*[1,2] , *V. A. Naumov*[1,3]

[1]Joint Institute for Nuclear Research, Dubna
[2]Physics Department of Irkutsk State University, Irkutsk, Russia
[3]Dipartimento di Fisica and Sezione INFN di Firenze, Sesto Fiorentino, Italy

We derive generic formulas for the polarization density matrix of leptons produced in νN and $\overline{\nu} N$ collisions and briefly consider some important particular cases. Next we employ the general formalism in order to include the final lepton mass and spin into the Rein–Sehgal model for single pion neutrino production.

Получены общие формулы для поляризационной матрицы лептонов, образованных в νN- и $\overline{\nu} N$-соударениях, и кратко рассматриваются важные частные случаи. Далее мы используем общий формализм для включения массы и спина лептона в модель Rein–Sehgal для образования одиночного пиона.

INTRODUCTION

Polarization of leptons generated in νN and $\overline{\nu} N$ collisions is important for studying neutrino oscillations and relevant phenomena in experiments with atmospheric and accelerator neutrino beams. Let us shortly touch upon a few illustrative examples.

• Contained τ-lepton events provide the primary signature for $\nu_\mu \to \nu_\tau$ oscillations. Besides they are a source of unavoidable background to the future proton decay experiments. But a low or intermediate energy τ lepton generated inside a water Cherenkov detector is unobservable in itself and may only be identified through the τ-decay secondaries whose momentum configuration is determined by the τ-lepton helicity.

• In case of $\nu_\mu - \nu_\tau$ mixing, the leptonic decays of τ's generated inside the Earth yield an extra contribution into the flux of through-going upward-going muons (TUM) and stopping muons (SM) which is absent in case of $\nu_\mu - \nu_s$ or $\nu_\mu - \nu_e$ mixing. The absolute value and energy spectrum of the «$\tau_{\mu 3}$ muons» are affected by the τ-beam polarization. The contribution is evidently small but measurable in future large-scale experiments, particularly those with magnetized tracking calorimeters (like in the experiments NuMI–MINOS and MONOLITH). Note that the energy and angular distributions of the charge ratio for the «$\tau_{\mu 3}$

muons» considerably differ from those for the «direct» TUM and SM since the longitudinal polarizations of τ^+ and τ^- have opposite signs.

• Decay of ν_μ or $\overline{\nu}_\mu$ induced muons with energy below the detection threshold may produce detectable electrons whose energy distributions are affected by the muon polarization. Such events, being classified as «e-like» (for a water detector) or «showering» (for an iron detector), mimic the ν_e or $\overline{\nu}_e$ induced events.

In this paper, we derive general formulas for the lepton polarization density matrix by applying a covariant method (Sec. 1) and briefly consider their applications to deep inelastic, quasi-elastic and resonance neutrino interactions. We explicitly demonstrate that the perpendicular and transverse polarizations depend on an intrinsically indeterminate phase and thus are unobservable in contrast with the longitudinal polarization and degree of polarization. In Sec. 2 we discuss a generalization of the Rein–Sehgal model for single pion production which takes into account the final lepton mass and spin.

1. POLARIZATION DENSITY MATRIX

The lepton polarization vector $\boldsymbol{\mathcal{P}} = (\mathcal{P}_P, \mathcal{P}_T, \mathcal{P}_L)$ is defined through the polarization density matrix $\boldsymbol{\rho} = \frac{1}{2}(1 + \boldsymbol{\sigma}\boldsymbol{\mathcal{P}})$ whose matrix elements are given by contracting the leptonic tensor $L^{\alpha\beta}_{\lambda\lambda'}$ with the spin-averaged hadronic tensor $W_{\alpha\beta}$. The leptonic tensor is given by

$$L^{\alpha\beta}_{\lambda\lambda'} = \begin{cases} j^{\alpha}_{\lambda}\left(j^{\beta}_{\lambda'}\right)^* \ \text{ with } \ j^{\alpha}_{\lambda} = \overline{u}\,(k', s)\,\gamma^\alpha\left(\dfrac{1-\gamma_5}{2}\right)u(k) \ \text{ for } \ \nu_\ell, \\ \overline{j}^{\,\alpha}_{\lambda}\left(\overline{j}^{\,\beta}_{\lambda'}\right)^* \ \text{ with } \ \overline{j}^{\,\alpha}_{\lambda} = \overline{v}(k)\gamma^\alpha\left(\dfrac{1-\gamma_5}{2}\right)v(k', s) \ \text{ for } \ \overline{\nu}_\ell, \end{cases} \tag{1}$$

where k and k' are the 4-momenta of ν_ℓ or $\overline{\nu}_\ell$ and lepton ℓ^- or ℓ^+ ($\ell = e, \mu, \tau$); λ and λ' are the lepton helicities; s is the axial 4-vector of the lepton spin.

It can be shown that the weak leptonic currents j^{α}_{λ} and $\overline{j}^{\,\alpha}_{\lambda}$ are given by

$$\begin{gathered} j^{\alpha}_{\lambda} = N_\lambda\left[mk^\alpha + k'^\alpha(ks) - s^\alpha(kk') - i\epsilon^{\alpha\beta\gamma\delta}s_\beta k_\gamma k'_\delta\right] \quad \text{and} \\ \overline{j}^{\,\alpha}_{\lambda} = -\lambda(j^{\alpha}_{-\lambda})^*. \end{gathered} \tag{2}$$

Here m is the lepton mass and the normalization constant N_λ is expressed in terms of the kinematic variables and of two intrinsically indeterminate phases φ_+ and φ_-:

$$N_\lambda = \frac{(1+\lambda)\ \mathrm{e}^{i\varphi_+} + (1-\lambda)\ \mathrm{e}^{i\varphi_-}}{2\sqrt{\upsilon_\lambda}},\ \upsilon_\lambda = (kk') \pm m\,(ks) = \frac{m^2 E_\nu\,(1 \mp \lambda\cos\theta)}{E_\ell \mp \lambda P_\ell},$$

where E_ν, E_ℓ, and P_ℓ denote the neutrino energy, lepton energy and momentum, respectively; θ is the scattering angle in lab. frame; the upper (lower) signs are for ν_ℓ ($\overline{\nu}_\ell$).

We use the generally accepted representation of the hadronic tensor (see, e.g., Ref. 1)

$$W_{\alpha\beta} = -g_{\alpha\beta} W_1 + \frac{p_\alpha p_\beta}{M^2} W_2 - \frac{i\,\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma}{2M^2} W_3 + \\ + \frac{q_\alpha q_\beta}{M^2} W_4 + \frac{p_\alpha q_\beta + q_\alpha p_\beta}{2M^2} W_5 + i\,\frac{p_\alpha q_\beta - q_\alpha p_\beta}{2M^2} W_6 \quad (3)$$

which includes 6 nucleon structure functions, W_n, whose explicit form is defined by the particular subprocess (QE, RES or DIS). Here p and M are the nucleon 4-momentum and mass, respectively; $q = k - k'$ is the W boson 4-momentum. By applying Eqs. (1), (2), and (3), we obtain

$$\rho_{\lambda\lambda'} \propto L^{\alpha\beta}_{\lambda\lambda'} W_{\alpha\beta} = E_\nu^2 m^2 N_\lambda N^*_{\lambda'} \sum_{n=1}^{6} A^n_{\lambda\lambda'} W_n,$$

$$A^1_{\lambda\lambda'} = 2\left(\eta_{\lambda\lambda'} \mp \eta_{-\lambda\lambda'}\right)\sin^2\theta,$$

$$A^2_{\lambda\lambda'} = 4\left(\eta_{\pm\lambda}\eta_{\pm\lambda'}\sin^4\frac{\theta}{2} + \eta_{\mp\lambda}\eta_{\mp\lambda'}\cos^4\frac{\theta}{2}\right) \pm \eta_{-\lambda\lambda'}\sin^2\theta,$$

$$A^3_{\lambda\lambda'} = \pm\sin^2\theta\left(\eta_{\pm\lambda}\eta_{\pm\lambda'}\frac{E_\nu - P_\ell}{M} + \eta_{\mp\lambda}\eta_{\mp\lambda'}\frac{E_\nu + P_\ell}{M} \mp \eta_{-\lambda\lambda'}\frac{E_\nu}{M}\right),$$

$$A^4_{\lambda\lambda'} = 4\left[\eta_{\pm\lambda}\eta_{\pm\lambda'}\frac{(E_\nu + P_\ell)^2}{M^2}\sin^4\frac{\theta}{2} + \eta_{\mp\lambda}\eta_{\mp\lambda'}\frac{(E_\nu - P_\ell)^2}{M^2}\cos^4\frac{\theta}{2}\right] \pm \\ \pm \eta_{-\lambda\lambda'}\frac{m^2}{M^2}\sin^2\theta,$$

$$A^5_{\lambda\lambda'} = -4\left[\eta_{\pm\lambda}\eta_{\pm\lambda'}\frac{E_\nu + P_\ell}{M}\sin^4\frac{\theta}{2} + \eta_{\mp\lambda}\eta_{\mp\lambda'}\frac{E_\nu - P_\ell}{M}\cos^4\frac{\theta}{2}\right] \mp \\ \mp \eta_{-\lambda\lambda'}\frac{E_\ell}{M}\sin^2\theta,$$

$$A^6_{\lambda\lambda'} = i\left(\frac{\lambda - \lambda'}{2}\right)\frac{P_\ell}{M}\sin^2\theta,$$

where $\eta_\lambda \equiv (1+\lambda)/2$. Taking into account that $\mathrm{Tr}\,\boldsymbol{\rho} = 1$, we can find the explicit formulas for the elements of the polarization density matrix in terms of variables E_ν, P_ℓ, and θ:

$$\rho_{++}(E_\nu, P_\ell, \theta) = \rho_{--}(E_\nu, -P_\ell, \pi - \theta) = \frac{E_\ell \mp P_\ell}{2M\mathcal{R}}\,\mathcal{Z},$$

$$\rho_{+-}(E_\nu, P_\ell, \theta) = \rho^*_{-+}(E_\nu, P_\ell, \theta) = \frac{m\sin\theta}{4M\mathcal{R}}\left(\mathcal{X} - i\mathcal{Y}\right)\mathrm{e}^{i\varphi}.$$

Here we have introduced the following notation:

$$\mathcal{X} = \mp\left(2W_1 - W_2 - \frac{m^2}{M^2}W_4 + \frac{E_\ell}{M}W_5\right) - \frac{E_\nu}{M}W_3, \qquad \mathcal{Y} = -\frac{P_\ell}{M}W_6,$$

$$\mathcal{Z} = (1 \pm \cos\theta)\left(W_1 \pm \frac{E_\nu \mp P_\ell}{2M}W_3\right) + \frac{1 \mp \cos\theta}{2}\times$$
$$\times\left[W_2 + \frac{E_\ell \pm P_\ell}{M}\left(\frac{E_\ell \pm P_\ell}{M}\,W_4 - W_5\right)\right],$$

$$\mathcal{R} = \left(\frac{E_\ell - P_\ell\cos\theta}{M}\right)\left(W_1 + \frac{m^2}{2M^2}W_4\right) + \left(\frac{E_\ell + P_\ell\cos\theta}{2M}\right)W_2\pm$$
$$\pm\left[\left(\frac{E_\nu + E_\ell}{M}\right)\left(\frac{E_\ell - P_\ell\cos\theta}{2M}\right) - \frac{m^2}{2M^2}\right]W_3 - \frac{m^2}{2M^2}W_5,$$

and $\varphi = \varphi_+ - \varphi_-$. Finally the projections of the lepton polarization vector are given by

$$\begin{pmatrix}\mathcal{P}_P\\ \mathcal{P}_T\end{pmatrix} = \frac{m\sin\theta}{2M\mathcal{R}}\begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}\mathcal{X}\\ \mathcal{Y}\end{pmatrix}, \tag{4a}$$

$$\mathcal{P}_L = \mp 1 \pm \frac{m^2}{M^2\mathcal{R}}\left\{\left[\left(\frac{2M}{E_\ell + P_\ell}\right)W_1 \pm \left(\frac{E_\nu - P_\ell}{E_\ell + P_\ell}\right)W_3\right]\cos^2\frac{\theta}{2} + \right.$$
$$\left. + \left[\left(\frac{M}{E_\ell + P_\ell}\right)W_2 + \left(\frac{E_\ell + P_\ell}{M}\right)W_4 - W_5\right]\sin^2\frac{\theta}{2}\right\}. \tag{4b}$$

By putting $\varphi = 0^*$, the formulas for $\mathcal{P}_P$ and $\mathcal{P}_L$ exactly coincide with those of Ref. 2 (obtained within a noncovariant approach under assumption $W_6 = 0$).

Several simple conclusions immediately follow from Eqs. (4). First, the perpendicular and transverse projections are unobservable quantities in contrast with the longitudinal projection of $\boldsymbol{\mathcal{P}}$ and the degree of polarization $|\boldsymbol{\mathcal{P}}|$. Second, supposing that $W_6 = 0$ (as is probably the case) one can force the polarization vector to lie in the production plane. Third, a massless lepton is fully polarized, $\boldsymbol{\mathcal{P}} = (0, 0, \mp 1)$. In particular, at the energies of our interest, electron is always fully polarized while, in general, this is not the case for muon and τ lepton.

We end this section with a short description of the structure functions relevant to the three fundamental subprocesses, DIS, QE, and RES.

1.1. Deep Inelastic Scattering (DIS). In this case the relation between the structure functions $W_n^{(\mathrm{DIS})}\left(x, Q^2\right)$ and measurable quantities $F_n\left(x, Q^2\right)$ is obtained in a straightforward manner:

$$W_1^{(\mathrm{DIS})}\left(x, Q^2\right) = F_1\left(x, Q^2\right),\ W_n^{(\mathrm{DIS})}\left(x, Q^2\right) = w^{-1}F_n\left(x, Q^2\right),\ n = 2, \ldots, 6.$$

*We adopt this convention from here on. Therefore, according to Eq. (4a), $\mathcal{P}_P \propto \mathcal{X}$ and $\mathcal{P}_T \propto \mathcal{Y}$.

Here $Q^2 = -q^2$, $x = Q^2/2(pq)$ is the Bjorken scaling variable and $w = (pq)/M^2$.

The generally accepted relations between the functions F_1, F_2, F_4, and F_5 are

$$F_1 = F_5 = \frac{F_2}{2x(1+R)}\left(1+\frac{x^2}{x'}\right), \quad F_4 = \frac{1}{2x}\left(\frac{F_2}{2x} - F_1\right),$$

where R is the ratio of longitudinal to transverse cross sections in DIS and $x' = Q^2/(4M^2)$.

1.2. Quasielastic Scattering (QE). The charged currents $\langle p, p'|\widehat{J}_\alpha^+|n, p\rangle = \overline{u}(p')\Gamma_\alpha\, u(p)$ and $\langle n, p'|\widehat{J}_\alpha^-|p, p\rangle = \overline{u}(p')\overline{\Gamma}_\alpha\, u(p)$ for the QE reactions are defined through 6 (in general complex) form factors. The vertex is

$$\Gamma_\alpha = \cos\theta_C \left[\gamma_\alpha F_V + i\sigma_{\alpha\beta}\frac{q_\beta}{2M}F_M + \frac{q_\alpha}{M}F_S + \right.$$
$$\left. + \left(\gamma_\alpha F_A + \frac{p_\alpha + p'_\alpha}{M}F_T + \frac{q_\alpha}{M}F_P\right)\gamma_5\right],$$

where θ_C is the Cabibbo mixing angle and $p' = p + q$. A standard calculation yields

$$W_n^{(\mathrm{QE})}\left(x, Q^2\right) = \cos^2\theta_C w^{-1}\omega_n\left(Q^2\right)\delta(1-x), \quad n = 1, \ldots, 6.$$

The functions ω_n are the bilinear combinations of the form factors:

$$\omega_1 = |F_A|^2 + x'\left(|F_A|^2 + |F_V + F_M|^2\right),$$
$$\omega_2 = |F_V|^2 + |F_A|^2 + x'\left(|F_M|^2 + 4\,|F_T|^2\right),$$
$$\omega_3 = -2\,\mathrm{Re}\left[F_A^*\left(F_V + F_M\right)\right],$$
$$\omega_4 = \mathrm{Re}\left[F_V^*\left(F_S - \frac{1}{2}F_M\right) - F_A^*\left(F_T + F_P\right)\right] +$$
$$+ x'\left(\frac{1}{2}|F_M - F_S|^2 + |F_T + F_P|^2\right) - \frac{1}{4}\left(1 + x'\right)|F_M|^2 + \left(1 + \frac{1}{2}x'\right)|F_S|^2,$$
$$\omega_5 = 2\,\mathrm{Re}\left[F_S^*\left(F_V - x'F_M\right) - F_T^*\left(F_A - 2x'F_P\right)\right] + \omega_2,$$
$$\omega_6 = 2\,\mathrm{Im}\left[F_S^*\left(F_V - x'F_M\right) + F_T^*\left(F_A - 2x'F_P\right)\right].$$

The only difference between this result and that from Ref. 1 is in the sign of the term $\propto F_T^*$ in ω_6*. Assuming all the form factors to be real we have $\omega_6 = 0$ and thus $\mathcal{P}_T = 0$.

*According to Ref. 1, the functions $\omega'_5 = \omega_5 - \omega_2$ and ω_6 are, respectively, the real and imaginary parts of a unique function. Our examination does not confirm this claim for the general case ($F_T \neq 0$).

1.3. Single Resonance Production (RES). Let us now consider the case of single Δ resonance neutrino production,

$$\nu_\ell + n(p) \to \ell^- + \Delta^+(\Delta^{++}), \quad \overline{\nu}_\ell + n(p) \to \ell^+ + \Delta^-(\Delta^0).$$

Assuming the isospin symmetry and applying the Wigner–Eckart theorem, the hadronic weak current matrix elements are given by [2]

$$\langle \Delta^+,\, p'|\widehat{J}_\alpha|n,\, p\rangle = \langle \Delta^0,\, p'|\widehat{J}_\alpha|p,\, p\rangle = \overline{\psi}^{\,\beta}\,(p')\,\Gamma_{\alpha\beta}\,u(p),$$
$$\langle \Delta^{++},\, p'|\widehat{J}_\alpha|p,\, p\rangle = \langle \Delta^-,\, p'|\widehat{J}_\alpha|n,\, p\rangle = \sqrt{3}\,\overline{\psi}^{\,\beta}\,(p')\,\Gamma_{\alpha\beta}\,u(p).$$

Here $\psi^\alpha\,(p')$ is the Rarita–Schwinger spin-vector for Δ resonance, $u(p)$ is the Dirac spinor for neutron or proton, and the vertex tensor $\Gamma_{\alpha\beta}$ is expressed in terms of the 8 weak transition form factors $C_n^{V,A}\left(Q^2\right)$, $n = 3, 4, 5, 6$ (assumed, for simplicity, to be real) [3]:

$$\Gamma_{\alpha\beta} = \left[C_3^V\,\frac{g_{\alpha\beta}\widehat{q} - \gamma_\alpha q_\beta}{M} + C_4^V\,\frac{g_{\alpha\beta}\,(qp') - p'_\alpha q_\beta}{M^2} + C_5^V\,\frac{g_{\alpha\beta}\,(qp) - p_\alpha q_\beta}{M^2} + \right.$$
$$\left. + C_6^V\,\frac{q_\alpha q_\beta}{M^2}\right]\gamma_5 + C_3^A\,\frac{g_{\alpha\beta}\widehat{q} - \gamma_\alpha q_\beta}{M} + C_4^A\,\frac{g_{\alpha\beta}\,(qp') - p'_\alpha q_\beta}{M^2} + C_5^A g_{\alpha\beta} + C_6^A\,\frac{q_\alpha q_\beta}{M^2}.$$

After accounting for the explicit form of a spin-3/2 projection operator (see, e.g., Ref. 3) and computing the proper convolutions, we arrive at the following expressions for $W_n^{(\text{RES})}$:

$$W_n^{(\text{RES})} = \kappa\cos^2\theta_C M M_\Delta \mathfrak{D}_\Delta \sum_{j,k=3\text{ to }6}\left(V_n^{jk} C_j^V C_k^V + A_n^{jk} C_j^A C_k^A\right),\; n{=}1, 2, 4, 5,$$
$$W_3^{(\text{RES})} = 2\kappa\cos^2\theta_C M M_\Delta \mathfrak{D}_\Delta \sum_{j,k=3\text{ to }6} K_n^{jk} C_j^V C_k^A \text{ and } W_6^{(\text{RES})} = 0.$$

Here $\kappa = 2/3$ for Δ^+ and Δ^0 production or $\kappa = 2$ for Δ^{++} and Δ^- production;

$$\mathfrak{D}_\Delta = \frac{1}{\pi}\left[\frac{W\Gamma(W)}{\left(W^2 - M_\Delta^2\right)^2 + W^2\Gamma^2\,(W)}\right] \text{ and } \Gamma\,(W) = \Gamma_\Delta\sqrt{\frac{\lambda\,(W^2, M^2, m_\pi)}{\lambda\,(M_\Delta^2, M^2, m_\pi)}} \quad (5)$$

are the Breit–Wigner factor and the running width of Δ, respectively*. The coefficients V_n^{jk}, A_n^{jk}, and K_n^{jk} are found to be cubic polynomials in invariant dimensionless variables x, w and parameter $\zeta = M/M_\Delta$. Only 70 among the total 144 coefficients are nonzero; their expressions are however rather cumbersome and we omit them from this short paper.

*The running width $\Gamma\,(W)$ is as usually estimated by the S-wave $\Delta \to N\pi$ decay; in Eq. (5), $W = |p'|$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$, M_Δ and Γ_Δ are, correspondingly, the mass and width of Δ.

2. SINGLE PION PRODUCTION IN REIN–SEHGAL MODEL

In this section, we briefly describe a generalization of the famous model [4] for the neutrino induced single pion production (RS model from here on) in order to take account for the final lepton mass and polarization. The charged hadronic current in the RS approach has been derived in terms of the FKR relativistic quark model [5] and its explicit form has been written in the resonance rest frame (RRF); below we will mark this frame with asterisk ($\star$). In RRF, the energy of the incoming neutrino, outgoing lepton, target nucleon, and the 3-momentum transfer are, respectively,

$$E_\nu^\star = \frac{1}{2W}\left(2ME_\nu - Q^2 - m^2\right) = \frac{E_\nu}{W}\left[M - (E_\ell - P_\ell\cos\theta)\right], \tag{6a}$$

$$E_\ell^\star = \frac{1}{2W}\left(2ME_\ell + Q^2 - m^2\right) = \frac{1}{W}\left[ME_\ell - m^2 + E_\nu\left(E_\ell - P_\ell\cos\theta\right)\right], \tag{6b}$$

$$E_N^\star = W - (E_\nu^\star - E_\ell^\star) = \frac{M}{W}\left(M + E_\nu - E_\ell\right) \quad \text{and} \tag{6c}$$

$$\mathcal{Q}^\star = |\mathbf{q}^\star| = \frac{M}{W}\mathcal{Q} = \frac{M}{W}\sqrt{E_\nu^2 - 2E_\nu P_\ell\cos\theta + P_\ell^2}. \tag{6d}$$

It is convenient to direct the spatial axes of RRF in such a way that $\mathbf{p}^\star = (0,0,-\mathcal{Q}^\star)$ and $k_y^\star = k_y'^\star = 0$. These conditions lead to the following system of equations:

$$\begin{gathered} k_x^\star = k_x'^\star = \sqrt{\left(E_\nu^\star\right)^2 - \left(k_z^\star\right)^2}, \quad k_z^\star - k_z'^\star = \mathcal{Q}^\star, \\ k_z^\star + k_z'^\star = \frac{1}{\mathcal{Q}^\star}\left[\left(E_\nu^\star\right)^2 - \left(E_\ell^\star\right)^2 + m^2\right]. \end{gathered} \tag{7}$$

By using Eqs. (6) and (7) we find the components of the lepton spin 4-vector:

$$s_0^\star = \frac{1}{mW}\left[MP_\ell + E_\nu\left(P_\ell - E_\ell\cos\theta\right)\right], \quad s_x^\star = \frac{E_\nu E_\ell}{m\mathcal{Q}}\sin\theta, \quad s_y^\star = 0,$$

$$s_z^\star = \frac{1}{m\mathcal{Q}W}\left[(E_\nu\cos\theta - P_\ell)(ME_\ell - m^2 + E_\nu E_\ell) - E_\nu P_\ell\left(E_\nu - P_\ell\cos\theta\right)\right].$$

Then, by applying general equation (2), the components of the leptonic current in RRF with the lepton helicity λ measured in lab. frame, are expressed as

$$j_0^\star = N_\lambda m\frac{E_\nu}{W}\left(M - E_\ell - \lambda P_\ell\right)(1 - \lambda\cos\,\theta),$$

$$j_x^\star = N_\lambda m\frac{E_\nu}{\mathcal{Q}}\left(P_\ell - \lambda E_\nu\right)\sin\,\theta,$$

$$j_y^\star = i\lambda N_\lambda m E_\nu\sin\,\theta,$$

$$j_z^\star = N_\lambda m\frac{E_\nu}{\mathcal{Q}W}\left[(E_\nu + \lambda P_\ell)(M - E_\ell) + P_\ell\left(\lambda E_\nu + 2E_\nu\cos\,\theta - P_\ell\right)\right](1 - \lambda\cos\,\theta).$$

On the other hand, in the spirit of the RS model, the leptonic current may be decomposed into three polarization 4-vectors corresponding to left-handed, right-handed and scalar polarization of the intermediate W boson:

$$j_\lambda^\alpha = \frac{1}{C}\left[c_L^\lambda e_L^\alpha + c_R^\lambda e_R^\alpha + c_S^\lambda e_{(\lambda)}^\alpha\right],$$

$$e_L^\alpha = \frac{1}{\sqrt{2}}(0,1,-i,0),\ e_R^\alpha = \frac{1}{\sqrt{2}}(0,-1,-i,0),\ e_{(\lambda)}^\alpha = \frac{1}{\sqrt{Q^2}}\left(\mathcal{Q}_{(\lambda)}^\star, 0, 0, \nu_{(\lambda)}^\star\right).$$

Here the vectors e_L^α and e_R^α are the same as in Ref. 4 while $e_{(\lambda)}^\alpha$ has been modified to include the lepton mass effect. The remaining notation is

$$c_L^\lambda = \frac{C}{\sqrt{2}}\left(j_x^\star + i j_y^\star\right), \quad c_R^\lambda = -\frac{C}{\sqrt{2}}\left(j_x^\star - i j_y^\star\right), \quad c_S^\lambda = C\sqrt{|(j_0^\star)^2 - (j_z^\star)^2|},$$

$$\mathcal{Q}_{(\lambda)}^\star = \frac{C\sqrt{Q^2}}{c_S^\lambda} j_0^\star, \quad \nu_{(\lambda)}^\star = \frac{C\sqrt{Q^2}}{c_S^\lambda} j_z^\star, \quad C = \frac{\mathcal{Q}}{E_\nu\sqrt{2Q^2}}.$$

Within the generalized RS model, the elements of the polarization density matrix may be written as the superpositions of the partial cross sections $\sigma_L^{\lambda\lambda'}$, $\sigma_R^{\lambda\lambda'}$, and $\sigma_S^{\lambda\lambda'}$*:

$$\frac{d\sigma_{\lambda\lambda'}}{dQ^2 dW} = \frac{G_F^2\cos^2\theta_C}{\pi^2}\left(\frac{WQ^2}{M\mathcal{Q}^2}\right)\sum_{i=L,R,S} c_i^\lambda c_i^{\lambda'} \sigma_i^{\lambda\lambda'}\left(Q^2, W\right),$$

$$\sigma_{L,R}^{\lambda\lambda'} = \frac{\pi W}{2M}\left(A_{\pm3}^\lambda A_{\pm3}^{\lambda'} + A_{\pm1}^\lambda A_{\pm1}^{\lambda'}\right),\ \sigma_S^{\lambda\lambda'} = \frac{\pi M\mathcal{Q}^2}{2WQ^2}\left(A_{0+}^\lambda A_{0+}^{\lambda'} + A_{0-}^\lambda A_{0-}^{\lambda'}\right),$$

$$A_\varkappa^\lambda\left(p\pi^+\right) = \sqrt{3}\sum_{(I=3/2)} a_\varkappa^\lambda\left(\mathcal{N}_3^{*+}\right),$$

$$A_\varkappa^\lambda\left(p\pi^0\right) = \sqrt{\frac{2}{3}}\sum_{(I=3/2)} a_\varkappa^\lambda\left(\mathcal{N}_3^{*+}\right) - \sqrt{\frac{1}{3}}\sum_{(I=1/2)} a_\varkappa^\lambda\left(\mathcal{N}_1^{*+}\right),$$

$$A_\varkappa^\lambda\left(n\pi^+\right) = \sqrt{\frac{1}{3}}\sum_{(I=3/2)} a_\varkappa^\lambda\left(\mathcal{N}_3^{*+}\right) + \sqrt{\frac{2}{3}}\sum_{(I=1/2)} a_\varkappa^\lambda\left(\mathcal{N}_1^{*+}\right).$$

Here $\varkappa = \pm3,\ \pm1,\ 0$; only those resonances are allowed to interfere which have the same spin and orbital angular momentum. Any amplitude $a_\varkappa^\lambda\left(\mathcal{N}_i^{*+}\right)$ referring to a single resonance consists of two factors which describe the production and

*Here and below, we use the same definitions and (almost) similar notations as in Ref. 4.

subsequent decay of the resonance $\mathcal{N}_\imath^{*+}$: $a_\varkappa^\lambda(\mathcal{N}_\imath^*) = f_\varkappa^\lambda(\nu\mathcal{N} \to \mathcal{N}_\imath^*)\,\eta(\mathcal{N}_\imath^* \to \mathcal{N}\pi) \equiv f_\varkappa^{\lambda(\imath)}\,\eta^{(\imath)}$. The resonance production amplitudes, $f_\varkappa^{\lambda(\imath)}$, are collected in Table II of Ref. 4. The corresponding decay amplitudes, $\eta^{(\imath)}$, can be split into three factors, $\eta^{(\imath)} = \text{sign}\,(\mathcal{N}_\imath^*)\sqrt{\chi_\imath}\,\eta_{BW}^{(\imath)}(W)$, irrespective of isospin, charge or helicity. Here sign $(\mathcal{N}_\imath^*)$ is the pure sign (Table III of Ref. 4), $\chi_\imath$ is the elasticity of the resonance taking care of the branching ratio into the $\mathcal{N}\pi$ final state,

$$\eta_{BW}^{(\imath)}(W) = \left[\frac{1}{2\pi N_\imath}\frac{\Gamma_\imath}{(W-M_\imath)^2+\Gamma_\imath^2/4}\right]^{1/2},$$

$$N_\imath = \frac{1}{2\pi}\int_{W_{\min}}^{\infty} dW\left[\frac{\Gamma_\imath}{(W-M_\imath)^2+\Gamma_\imath^2/4}\right] \text{ and } \Gamma_\imath = \Gamma_\imath^0\left[\frac{\lambda(W^2,M^2,m_\pi)}{\lambda(M_\imath^2,M^2,m_\pi)}\right]^{L+1/2}.$$

In the generalized RS model, the structure of the vector $e_{(\lambda)}^\alpha$ has been changed by including the lepton spin dependence. Thus we have to recalculate the inner products $J_\alpha^{V,A}e_{(\lambda)}^\alpha$, where $J_\alpha^{V,A}$ are the vector and axial hadronic currents in the FKR model. The new definitions for the structures S^V, B^A, and C^A involved into the model are the following:

$$S^V = \left(\nu_{(\lambda)}^\star\nu^\star - \mathcal{Q}_{(\lambda)}^\star\mathcal{Q}^\star\right)\left(1+\frac{Q^2}{M^2}-\frac{3W}{M}\right)\frac{G^V(Q^2)}{6\mathcal{Q}^2},$$

$$B^A = \sqrt{\frac{\Omega}{2}}\left(\mathcal{Q}_{(\lambda)}^\star + \nu_{(\lambda)}^\star\frac{\mathcal{Q}^\star}{2mg^2}\right)\frac{ZG^A(Q^2)}{3W\mathcal{Q}^\star},$$

$$C^A = \left[\left(\mathcal{Q}_{(\lambda)}^\star\mathcal{Q}^\star - \nu_{(\lambda)}^\star\nu^\star\right)\left(\frac{1}{3}+\frac{\nu^\star}{2mg^2}\right) + \right.$$
$$\left. + \nu_{(\lambda)}^\star\left(\frac{2}{3}W - \frac{Q^2}{2mg^2}+\frac{N\Omega}{6mg^2}\right)\right]\frac{ZG^A(Q^2)}{2W\mathcal{Q}^\star}.$$

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