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CAN ONE UNDERSTAND BLACK-HOLE ENTROPY
WITHOUT KNOWING MUCH
ABOUT QUANTUM GRAVITY?

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CAN ONE UNDERSTAND BLACK-HOLE ENTROPY WITHOUT KNOWING MUCH ABOUT QUANTUM GRAVITY?

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It is a common belief now that the explanation of the microscopic origin of the Bekenstein–Hawking entropy of black holes should be available in quantum gravity theory, whatever this theory will finally look like. Calculations of the entropy of certain black holes in string theory do support this point of view. In the last few years there also appeared a hope that understanding of black hole entropy may be possible even without knowing the details of quantum gravity. The thermodynamics of black holes is a low energy phenomenon, so only a few general features of the fundamental theory may be really important. The aim of this review is to describe some of the proposals in this direction and the results obtained.

Сейчас достаточно широко распространено мнение, что объяснение микроскопического происхождения энтропии черных дыр Бекенштейна–Хокинга может быть получено в теории квантовой гравитации, как бы окончательно ни выглядела эта теория. Эта точка зрения действительно получает подтверждение в вычислениях энтропии определенного типа черных дыр в теории струн. В последние несколько лет возникла также надежда, что понимание энтропии черных дыр возможно даже без знания деталей теории квантовой гравитации. Термодинамика черных дыр есть явление физики низких энергий, поэтому реальное значение имеют лишь несколько общих особенностей фундаментальной теории. Цель данного обзора состоит в том, чтобы описать некоторые предложения в этом направлении и полученные результаты.

INTRODUCTION

Black holes are specific solutions of the Einstein equations which describe regions of a space-time where the gravitational field is so strong that nothing, including light signals, can escape them. The interior of a black hole is hidden from an external observer. The boundary of the unobservable region is called the horizon.

A black hole can appear as a result of the gravitational collapse of a star. In this case it quickly reaches a stationary state characterized by a certain mass M and an angular momentum J . If the collapsing matter was not electrically neutral, a black hole has an additional parameter, an electric charge Q . These are

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the only parameters a black hole in the Einstein–Maxwell theory can have. Its metric in the most general case is the Kerr–Newmann metric. This statement is known as the «no-hair» theorem*. If Ω_H is the angular velocity of the black hole at the horizon, Φ_H is the difference of the electric potential at the horizon and at infinity, then by using purely classical equations one arrives at the following variational formula [2]:

$$\delta M = T_H \delta S^{\text{BH}} + \Omega_H \delta J + \Phi_H \delta Q, \quad (1)$$

$$S^{\text{BH}} = \frac{1}{4G} \mathcal{A}, \quad T_H = \frac{\kappa}{2\pi}. \quad (2)$$

Here \mathcal{A} is the surface area of the horizon and G is the Newton gravitational constant**. The constant κ is called the surface gravity. It characterizes the strength of the gravitational field near the horizon. Relation (1) has the form of the first law of thermodynamics where S^{BH} has the meaning of an entropy, T_H is a temperature, and M is an internal energy. The quantity S^{BH} was introduced in [4–7] and is called the Bekenstein–Hawking entropy. Strictly speaking, (1) defines the entropy and the temperature up to a multiplier. This multiplier is fixed from another considerations: T_H is defined as the temperature of the Hawking radiation from a black hole [7]***.

One can also find an analogy with other laws of thermodynamics. For instance, by considering classical processes with black holes one can conclude that the area of the horizon never decreases, the observation which is reminiscent to the second law. In quantum theory this should be true if S^{BH} is considered together with the entropy of a matter outside the horizon. Black hole must have an intrinsic entropy proportional to the horizon area. Otherwise processes like a gravitational collapse would be at odds with the second law.

Thermodynamics and statistical mechanics of black holes are one of the most interesting and rapidly developing branches of black-hole physics. In the Einstein theory S^{BH} is a pure geometrical quantity. In real thermodynamical systems the entropy is the logarithm of the number of microscopic states corresponding to a given set of macroscopic parameters. This raises a natural question: Do black holes have microscopic degrees of freedom whose number is consistent with the Bekenstein–Hawking entropy?

*References on this subject as well as an introduction in black-hole physics can be found in [1].

**Here and in what follows we use the system of units where $\hbar = c = k_B = 1$ (k_B is the Boltzmann constant), and follow the notations adopted in [3]. In particular, the Lorentzian signature is $(-, +, +, +)$.

***The mechanisms which give rise to the Hawking radiation or quantum evaporation of black hole are analyzed in [8].

The main reason why this question is fundamental is because it goes beyond the black-hole physics itself. Its answer may give important insight into the as yet mysterious nature of quantum gravity.

To see this let us start with a simple estimation and consider a static neutral supermassive black hole with mass M of the order of 10^9 solar masses. Such objects are believed to occur in the centres of certain galaxies. By taking into account that $\mathcal{A} = 16\pi G^2 M^2$, one finds from (2) that the entropy of such a black hole is of the order of 10^{95} . It is eight orders of magnitude larger than the entropy of the microwave background radiation in the visible part of the Universe! What makes matters even worse is that in the classical theory a black hole is nothing but an empty space. Thus, an explanation of the Bekenstein–Hawking entropy is one of those problems which cannot be solved in classical gravity theory.

Suppose the horizon surface is covered by cells of a Planckian size $L_{\text{Pl}} \sim \sqrt{G}$. Then, according to (2), S^{BH} is of the same order as the number of ways to distribute signs «+» and «-» over these cells. The appearance of the Planck scale in this estimate is not an accident. It indicates that a reasonable resolution of the black-hole entropy problem has to be based on quantum gravity. Moreover, reproduction of S^{BH} by the methods of statistical mechanics has to be considered as a very nontrivial test for any candidate theory.

At the present moment the most promising candidate is believed to be string (D-brane) theory. A successful statistical-mechanical derivation of S^{BH} for extremal [9–11] and near-extremal black holes [12, 13] is among the most important results in this theory during the last decade. The string computations, however, do not solve the problem of the Bekenstein–Hawking entropy completely. They are not universal and, what may be worse, they are done for models in flat spacetimes which are in some sense dual to the string theory on a given black-hole background. This kind of derivation says nothing about the real microscopic degrees of freedom responsible for S^{BH} and where they are located. A review of the string computations can be found in [14–16].

Another approach to quantum gravity, loop quantum gravity, also offers an interesting explanation of S^{BH} (see [17, 18]). Loop quantum gravity is aimed at a quantum description of the geometry. The area of a surface in this approach is treated as an operator. The degeneracy of the eigenvalues of such operators can be computed. The suggestion of loop quantum gravity is that the Bekenstein–Hawking entropy is related to the degeneracy of eigenvalues of the area operator which are comparable in magnitude to the area of the black-hole horizon. However, there is a main open issue here how general relativity, coupled to quantum matter fields, is recovered from loop quantum gravity in a suitable low-energy limit [19]. Till this question is resolved, it is not clear how to describe black holes in this approach.

Let us emphasize that the thermodynamics of black holes is determined only by the Einstein equations and classical gravitational couplings. This may indicate

that an understanding of black-hole entropy is possible without knowing the details of quantum gravity. Only a few general features of the fundamental theory may be really important. If so, the question is: What are those features?

There were two main directions along which this idea was investigated during the last few years. One of them was based on the assumption that classical symmetries on a black-hole background can control the density of states in quantum gravity and in this way enable one to derive the entropy of a black hole.

The other direction of research starts from the suggestion that the origin of the Bekenstein–Hawking entropy is related to the properties of the physical vacuum in a strong gravitational field. The Bekenstein–Hawking entropy measures the loss of information about quantum states hidden inside the horizon.

In this review, we analyze the «pluses» and «minuses» of the two approaches and show that the two ways of counting S^{BH} do not necessarily contradict each other. The review is organized as follows. In Sec. 1 we discuss two-dimensional conformal theories (CFT) in relation to the problem of black-hole entropy. We start with black holes whose thermodynamical relations can be interpreted in terms of such CFT's and use these examples to introduce some properties of the conformal theory. Special attention is paid to near-extremal black holes and black holes in anti-de Sitter (AdS) gravities. After that we discuss counting of the Bekenstein–Hawking entropy by using a near-horizon conformal symmetry.

The relation of S^{BH} to the entropy of the thermal atmosphere around a black hole and an entanglement entropy is discussed in Sec. 2. We argue that in the most consistent way available at the present moment this relation can be studied in induced gravity models. The Einstein gravity in these models is entirely induced by quantum effects and the underlying theory is free from the leading ultraviolet divergences.

A possible connection of the two approaches is discussed in Sec. 3 where we show how to construct a concrete representation of the near horizon conformal algebra in induced gravity. Our conclusions are summarized in the last section.

One of our purposes is to present the material in a form suitable for nonspecialists in this field of research, thus when possible we avoid technical details. Many interesting topics related to the black-hole entropy problem are not considered here or discussed briefly. They can be found in other review works on this subject (see, for instance, [8, 14–16, 20–22] and further references below).

1. BLACK-HOLE ENTROPY AND ASYMPTOTIC SYMMETRIES

1.1. Black Holes Which Look Two-Dimensional. Before studying the problem of black-hole entropy, one may ask a simple question: Are there some familiar physical systems in flat space-time which are thermodynamically equivalent to a given black hole? The equivalence means that the relation between

the mass, temperature and other parameters of a black hole is the same, after appropriate identifications, as a relation between the energy, temperature and other parameters of the corresponding system. The answer is positive. It turns out, however, that different black holes are equivalent to completely different systems. Moreover, the dimensionalities of the black hole and the flat space-time do not coincide in general. Some black holes may have quite complicated thermodynamical properties*, some others are very simple.

Consider a Reissner–Nordström solution which describes a charged black hole in Einstein–Maxwell theory

$$ds^2 = -Bdt^2 + \frac{dr^2}{B} + r^2d\Omega^2. \quad (3)$$

Here $d\Omega^2$ is the metric on a unit sphere and

$$B = \frac{1}{r^2}(r - r_-)(r - r_+), \quad r_{\pm} = m \pm \sqrt{m^2 - q^2}. \quad (4)$$

The parameter $q = Q\sqrt{G}$ is related to the electric charge Q of the black hole, while $m = MG$, where M is its mass**. The radius of the horizon is r_+ . The Hawking temperature (2) of this black hole is

$$T_H = \frac{1}{2\pi r_+} \sqrt{m^2 - q^2}, \quad (5)$$

and the Bekenstein–Hawking entropy is $S^{\text{BH}} = \pi r_+^2/G$.

This solution has an interesting property: the Hawking temperature vanishes in the limit when $m = q$ or $M = QM_{\text{Pl}}$, where $M_{\text{Pl}} = G^{-1/2}$ is the Planck mass. Such a limiting solution is called an extremal black hole. Strictly speaking, there are no physical processes which enable one to turn a charged black hole with $m > q$ to an extremal one***. Macroscopic extremal black holes hardly exist. These solutions, however, have a theoretical interest for reasons we discuss later.

We consider now black holes which are «almost extremal» (or near-extremal) whose mass parameter is

$$m = q + E, \quad E \ll q. \quad (6)$$

*For example, charged black holes in anti-de Sitter space-times have a phase structure similar to that of the Van der Waals–Maxwell liquid–gas systems in a space-time of one dimension lower [23].

**In four dimensions the Newton constant G (in the system of units we work in) has the dimensionality (length)².

***The reason why these black holes are different can be easily seen when going in (3) from the Lorentzian to the Euclidean signature. Then in the $r - t$ plane a nonextremal black hole in a cavity has the disk topology, while an extremal black hole looks like an infinite throat.

Thermodynamical relations for these objects are very simple. If we introduce the parameter $\lambda = (2\pi^2 q^3)^{1/2}$, then

$$T_H \simeq \frac{E^{1/2}}{\lambda}, \quad (7)$$

and deviations of the mass and the entropy of the black hole from the extremal values are

$$E = m - q = \lambda^2 T_H^2, \quad S = S^{\text{BH}} - \frac{\pi}{G} q^2 = \frac{2\lambda^2}{G} T_H. \quad (8)$$

What can one say about these relations? Consider a gas of some number of massless noninteracting scalar fields ϕ_k on an interval of length b . The equations of the fields are

$$(\partial_t^2 - \partial_x^2)\phi_k(t, x) = 0, \quad \phi_k(t, 0) = \phi_k(t, b) = 0. \quad (9)$$

Suppose that this system is in a state of thermal equilibrium at some temperature T . This is a one-dimensional analog of an ideal gas of photons in a cavity. Let us denote the number of field species by c . The free energy of this model is

$$F(T, L) = cT \sum_n \ln \left(1 - e^{-\omega_n/T} \right), \quad (10)$$

where the frequencies of single-particle excitations $\omega_n = \pi n/b$, $n = 1, 2, \dots$, are determined from (9). In the thermodynamical limit, $Tb \gg 1$, series (10) can be easily calculated

$$F(T, b) \simeq -\frac{\pi c}{6} b T^2, \quad (11)$$

thus, the energy $E(T, b)$ and the entropy $S(T, b)$ of the system are

$$E(T, b) \simeq \frac{\pi c}{6} b T^2, \quad S(T, b) \simeq \frac{\pi c}{3} b T. \quad (12)$$

For $c = 1$ formula for the energy is just an analog of the Stefan–Boltzmann law. A microcanonical ensemble is characterized by the relation

$$S = S(E, b) = 2\pi \sqrt{\frac{c b E}{6 \pi}}, \quad (13)$$

which can be obtained from (12). By comparing (12) with (8) one can conclude that thermodynamical properties of a charged black hole near the extremal limit are identical to properties of an ideal gas in a flat two-dimensional space-time. If we identify in (8) and (12) the temperatures and the entropies, $T_H = T$, $S = S(T, b)$, then $cb = 12\pi Q^3 L_{\text{Pl}}$, where Q is the electric charge of the black hole and $L_{\text{Pl}} = \sqrt{G}$ is the Planck length.

1.2. Conformal Symmetry. Models (9) have an important common feature. They possess a high level of symmetry which becomes manifest if the equations are rewritten in terms of the light-cone coordinates $u = t - x$ and $v = t + x$,

$$\partial_u \partial_v \phi_k(x) = 0. \quad (14)$$

It is easy to see that equations (14) are invariant under transformations $u' = f(u)$, $v' = g(v)$, where f and g are some smooth functions. These transformations are called conformal transformations and the massless 2D quantum field model is an example of conformal field theory (CFT)*. In the Euclidean theory, an analog of these transformations is $z = f(z')$ and $\bar{z} = \bar{f}(\bar{z}')$, where z and \bar{z} are coordinates in the complex plane. Conformal transformations preserve the angle between two vectors but rescale intervals between neighboring points.

The group of conformal transformations is an infinite group. To see this it is sufficient to analyze small transformations of coordinates $x'^\mu = x^\mu + \delta x^\mu(x)$. The vector field δx^μ in the light-cone coordinates has components $\delta x^\mu = \zeta^\mu(u) + \bar{\zeta}^\mu(v)$, where $\zeta^v(u) = \bar{\zeta}^u(v) = 0$. The commutator $[\zeta_1, \zeta_2]$ of two vector fields** is again a vector field, so one can say that these fields make some algebra with certain commutation relations. As in the case of the algebra of the rotation group the algebra of diffeomorphisms can be characterized by commutation relations in some basis. Suppose for simplicity that in the model we consider the fields live on a circle, i.e., instead of the Dirichlet condition in (9) we choose a periodic condition $\phi_k(t, 0) = \phi_k(t, b)$. Then one can use Fourier decomposition for each vector

$$\begin{aligned} \zeta^\mu(u) &= \sum_n c_n \zeta_n^\mu, & \bar{\zeta}^\mu(u) &= \sum_n d_n \bar{\zeta}_n^\mu, \\ \zeta_n^v(v) &= \frac{ib}{2\pi} e^{2\pi i n v/b}, & \bar{\zeta}_n^u(u) &= \frac{ib}{2\pi} e^{2\pi i n u/b}, \end{aligned} \quad (15)$$

where n is an integer and c_n, d_n are some constants. The algebra of these vector fields has the form

$$[\zeta_n, \zeta_m] = (n - m)\zeta_{n+m}, \quad [\bar{\zeta}_n, \bar{\zeta}_m] = (n - m)\bar{\zeta}_{n+m}, \quad [\zeta_n, \bar{\zeta}_m] = 0. \quad (16)$$

In fact, one has two commuting sets of generators, each making an infinite-dimensional algebra called the Virasoro algebra.

In CFT models, the parameter c is called the central charge. Although c is not an integer in general, a number of relations, such, for example, as (13)

*For a brief introduction in CFT models see, for example, [24].

**The commutator, or a Lie bracket, $[\zeta_1, \zeta_2]$ of two vector fields, ζ_1^μ and ζ_2^μ , is a vector field with components $\zeta_3^\mu = \zeta_1^\nu \partial_\nu \zeta_2^\mu - \zeta_2^\nu \partial_\nu \zeta_1^\mu$.

are universal and applicable for any $c > 0$. The central charge is related to an important property in 2D CFT. The conformal invariance of classical equation (14) is broken in the quantized theory. This can be seen from the transformation of the uu or vv components of the renormalized stress energy tensor $T_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle$ under changes of u and v coordinates. For instance, under an infinitesimal change $\delta u = \zeta^u(u) \equiv \varepsilon(u)$, it can be shown that

$$\begin{aligned} \delta T_{uu}(u) &= T'_{uu}(u) - T_{uu}(u) = \\ &= \varepsilon(u) \partial_u T_{uu}(u) + 2 \partial_u \varepsilon(u) T_{uu}(u) + \frac{c}{24\pi} \partial_u^3 \varepsilon(u) + O(\varepsilon^2). \end{aligned} \quad (17)$$

The term proportional to $\partial_u^3 \varepsilon(u)$ is anomalous. It appears because the renormalization procedure requires subtracting the divergent part of the stress energy tensor which is not scale invariant. This property is analogous to the chiral anomaly in quantum theory.

Another way to see the conformal anomaly is the following. In quantum theory the generators of conformal transformations are some operators acting in the corresponding Fock space [24]. These operators are expressed in terms of the components of the stress-energy tensor operator $\hat{T}_{\mu\nu}$. In this way, in quantum theory each vector ζ_n ($\bar{\zeta}_m$) corresponds to some operator \hat{L}_n ($\hat{\bar{L}}_m$) which forms the following algebra:

$$[\hat{L}_n, \hat{L}_m] = (n-m)\hat{L}_{n+m} + \frac{c}{12}(n^3-n)\delta_{n+m,0}, \quad (18)$$

$$[\hat{\bar{L}}_n, \hat{\bar{L}}_m] = (n-m)\hat{\bar{L}}_{n+m} + \frac{c}{12}(n^3-n)\delta_{n+m,0}, \quad (19)$$

$[\hat{L}_n, \hat{\bar{L}}_m] = 0$. The brackets $[,]$ now are the usual commutators. Due to the conformal anomaly, the quantum algebra (18), (19) differs from the classical one (16) by the term $\frac{c}{12}(n^3-n)\delta_{n+m,0}$ which is called a central extension.

The Hamiltonian operator \hat{H} of the system, which generates the evolution along the time coordinate t can be expressed in terms of operators \hat{L}_0 and $\hat{\bar{L}}_0$ as

$$\hat{H} = \frac{2\pi}{b}(\hat{L}_0 + \hat{\bar{L}}_0). \quad (20)$$

This equation follows from the definition of coordinates u and v , which together with (15) implies that $i\partial_t = i\partial_u + i\partial_v = \frac{2\pi}{b}(\zeta_0^\mu + \bar{\zeta}_0^\mu)\partial_\mu$.

In a free quantum field theory the Fock space is constructed by using creation and annihilation operators. In the CFT theory there is an alternative way to do this by using the group algebra (18), (19). One can do it independently for each copy. Let $|0\rangle$ be the vacuum vector, such that

$$\hat{L}_k|0\rangle = \hat{\bar{L}}_k|0\rangle = 0, \quad k = 0, 1, 2, \dots \quad (21)$$

Consider a vector of the Fock space, $|h, \bar{h}\rangle$, which is an eigenvector of operators $\hat{L}_0, \hat{\bar{L}}_0$ with eigenvalues h and \bar{h} , respectively. This vector will also be an eigenvector of the Hamiltonian H with energy* $E = 2\pi(h + \bar{h})/b$. Such a vector can be constructed by acting on the vacuum with operators \hat{L}_{-k} and $\hat{\bar{L}}_{-k}$, where $k \geq 1$,

$$|h, \bar{h}\rangle = \prod_k (\hat{L}_{-k})^{\alpha_k} \prod_p (\hat{\bar{L}}_{-p})^{\beta_p} |0\rangle, \quad (22)$$

$$\sum_k k\alpha_k = h, \quad \sum_p p\beta_p = \bar{h}. \quad (23)$$

The fact that (22) is an eigenvector of \hat{L}_0 and $\hat{\bar{L}}_0$ can be easily checked by using the Virasoro algebra (18), (19).

As can be seen from (23) the states $|h, \bar{h}\rangle$ are degenerate. Their degeneracy for large h, \bar{h} can be found exactly. The degeneracy D corresponding to an eigenvalue h is

$$\ln D \simeq 2\pi \sqrt{\frac{ch}{6}} \quad (24)$$

(analogously for the degeneracy \bar{D} corresponding to an eigenvalue \bar{h}). This equation is known as the Cardy formula. It is applicable to theories with any central charge $c > 0$.

There are different ways to derive (24) by using conformal properties. For our purposes, however, it is more instructive to see how it follows from results discussed in Subsec. 1.1. Consider a state of the scalar model with $h = \bar{h} = Eb/4\pi$. Its degeneracy is related to the entropy $S(E, b)$ of the microcanonical ensemble with the given energy E ,

$$S(E, b) = \ln D + \ln \bar{D} = 2 \ln D.$$

Then the Cardy formula is just the consequence of the statistical-mechanical relation (13).

1.3. Digression about Computations in String Theory. Let us emphasize that in the considered example there is no apparent relation between a classical black hole and a quantum model (9). Suppose, however, that there is an underlying fundamental theory of quantum gravity able to provide a statistical explanation of the Bekenstein–Hawking entropy of a near-extremal black hole. Then the microscopic degrees of freedom responsible for S^{BH} are to be described by a certain CFT.

*It should be noted that the total energy of the system is $E + E_0$, where E_0 is the energy of vacuum fluctuations. In what follows we assume that E is large as compared with E_0 , so E_0 can be neglected.

String theory provides an explicit example of how this happens in the case of extremal black holes. These black holes are special solutions of an effective supergravity theory which is a low-energy limit of string theory. Typically, solutions in this theory break the supersymmetry but extremal black holes are invariant with respect to a part of the supersymmetry transformations. They are the so-called BPS solitons and the condition of extremality $m = q$ is known as a BPS bound. This bound ensures that the energy of the soliton receives no corrections in quantum theory. The Bekenstein–Hawking entropy of an extremal black hole is

$$S^{\text{BH}} = \frac{\pi q^2}{G} = \pi Q^2 \quad (25)$$

and it does not depend on the gravitational coupling. Therefore, in gravity theories with different G 's, the black holes with equal charges Q have equal entropies.

In string theory the gravitational constant $G = g^2 l^2$. It depends on the string coupling g (the value of the dilaton) and on the inverse string tension l . The parameter l determines the typical size of a closed string*. Because S^{BH} in (25) does not depend on G one can vary the string coupling g without changing the entropy of an extremal black hole. Note, however, that the size of the black hole associated with the horizon radius r_+ depends on the gravitational constant, $r_+ = MG = Q\sqrt{G} = Qgl$.

One can consider two limits. In the limit of weak coupling g , the horizon radius can be much smaller than the string size, $r_+ \ll l$. In this limit, instead of a black hole one has a dual object, a point particle in a flat space. A black hole is formed in the limit of strong coupling when $r_+ \gg l$. However, when one increases the coupling and goes from the weak limit to the strong one, S^{BH} does not change. This means, that instead of doing calculations of the entropy on the black-hole background one can consider a dual theory in a flat space-time, which is much easier. It turns out that the dual theory is a CFT similar to what we described above and counting its states gives the correct value of S^{BH} [9].

No doubt, this result is important but it is not quite satisfactory. Since computations are done in a dual theory, the physical nature and the location of the black hole degrees of freedom remain unknown. It is also not clear whether it is possible to extend this analysis to be applicable to any black hole.

1.4. Anti-de Sitter Black Holes. Another interesting example where thermodynamical relations of black holes are equivalent to relations emerging in CFT models appears in the three-dimensional (3D) gravity theory with a negative

*In quantum theory, strings with size larger than l give an exponentially small contribution to the functional integral.

cosmological constant $\Lambda = -l^{-2}$. The theory is described by the action

$$I = \frac{1}{16\pi G_3} \int \sqrt{-g} d^3x \left(R + \frac{2}{l^2} \right), \quad (26)$$

where R is the scalar curvature and G_3 is the 3D gravitational coupling (note that G_3 has the dimension of length). The important feature of gravity in three dimensions is that if $\Lambda = 0$, the space-time geometry is always locally flat. A point mass in such a theory does not have a gravitational potential but changes global properties of the geometry around itself. Another feature of 3D gravity is the absence of gravitons.

One of the solutions of 3D gravity with nonzero cosmological constant is anti-de Sitter, AdS_3 , space*

$$ds^2 = - \left(1 + \frac{r^2}{l^2} \right) dt^2 + \left(1 + \frac{r^2}{l^2} \right)^{-1} dr^2 + r^2 d\varphi^2, \quad (27)$$

where $0 \leq \varphi \leq 2\pi$. This space has a constant negative curvature and can be defined as the surface $x^2 + y^2 - z^2 - w^2 = -l^2$ in a flat 4D space with metric $ds^2 = dx^2 + dy^2 - dz^2 - dw^2$. It is denoted as AdS_3 .

There are also black-hole solutions in this theory discovered in [25] and called BTZ black holes after the authors. The metric of a BTZ black hole is simple

$$ds^2 = - \frac{r^2 - r_+^2}{l^2} dt^2 + \left(\frac{r^2 - r_+^2}{l^2} \right)^{-1} dr^2 + r^2 d\varphi^2. \quad (28)$$

The horizon is located at $r = r_+$ and is a circle. The area of the horizon is the length $2\pi r_+$ of this circle. Space (28) has locally the same geometry as (27) but differs from it by global properties. We denote the black hole space-time by \mathcal{M}_3 to distinguish it from AdS_3 .

The mass M of the BTZ black hole is defined as $M = r_+^2 / (8l^2 G_3)$. Thus, the relation between the Bekenstein–Hawking entropy and the mass is

$$S^{\text{BH}} = \frac{2\pi r_+}{4G_3} = 2\pi \sqrt{\frac{l^2 M}{2G_3}}. \quad (29)$$

This formula has the same form as Eq. (13) for the entropy of a CFT. To make the correspondence more precise note that for a black hole in AdS gravity the curvature radius l plays the role of a «size» of the black hole**. Thus, l in (29) is

*For this reason such theories are also called AdS gravities.

**To explain this analogy we take a different example and consider the Schwarzschild black hole in the Einstein theory. It is known that this black hole is thermodynamically unstable. There are two ways to solve this problem: to place the black hole in a spherical cavity of a certain radius [26] or to introduce a negative cosmological constant.

analogous to parameter b in (13). If we identify the entropy and the mass of the black hole with the entropy and the energy of the CFT, then the central charge c of this theory has to be proportional to l/G_3 . In fact, this is a very good guess: the correct value of the central charge is

$$c = \frac{3l}{2G_3}. \quad (30)$$

A remarkable property of BTZ black holes is that the corresponding conformal group is realized not in a dual theory, as in the case of extremal black holes, but as a group of the asymptotic transformations of the physical space-time background.

As one can see, at large r the BTZ metric (28) behaves like the AdS₃ metric (27). One says that BTZ geometry is asymptotically AdS. There is a group of coordinate transformations $\delta x^\mu = \zeta^\mu(x)$ which preserves this asymptotic structure. At large r the diffeomorphism vector fields are [28]:

$$\zeta^t = l(T^+ + T^-) + \frac{l^3}{2r^2}(\partial_+^2 T^+ + \partial_-^2 T^-) + O(r^{-4}), \quad (31)$$

$$\zeta^\varphi = (T^+ - T^-) - \frac{l^2}{2r^2}(\partial_+^2 T^- - \partial_-^2 T^+) + O(r^{-4}), \quad (32)$$

$$\zeta^r = -\frac{1}{2}r(\partial_+ T^+ + \partial_- T^-) + O(r^{-1}), \quad (33)$$

where $\partial_\pm = l\partial_t \pm \partial_\varphi$, T^+ is a function of a single variable $t/l + \varphi$, while T^- is a function of $t/l - \varphi$.

It is not difficult to check that the commutator of vector fields ζ_1 and ζ_2 which have the asymptotic behavior (31)–(33) with functions T_1^\pm and T_2^\pm , respectively, is a vector field ζ_3 which has the same asymptotic behavior with $T_3^\pm = T_1^\pm \partial_\pm T_2^\pm - T_2^\pm \partial_\pm T_1^\pm$. Thus, one can say that generators of these diffeomorphisms form a closed algebra. There is a natural choice of basis in this algebra, $\zeta_n, \bar{\zeta}_n$, which is singled out by the the following restrictions on the corresponding functions:

$$T_n^+ = \frac{i}{2} e^{in(t/l+\varphi)}, \quad T_n^- = 0, \quad \bar{T}_n^+ = 0, \quad \bar{T}_n^- = \frac{i}{2} e^{in(t/l-\varphi)}, \quad (34)$$

where n is an integer. The algebra of these vector fields is given by relations (16) and therefore the group of asymptotic transformations (31)–(33) is the conformal group discussed in Subsec. 1.2.

We have pointed out that the representation of the conformal algebra in quantum theory acquires a central extension due to the conformal anomaly (see (18), (19)). It is an interesting fact that the Virasoro algebras with central extension also appear in classical theory, as happens for example, in Liouville theory [27].

In a classical theory the symmetries can be realized in a phase space. If there is a symmetry group, its algebra is represented by relations where the Poisson bracket plays the role of the commutator. In the classical gravity (26) the generators of diffeomorphisms $\delta x^\mu = \zeta^\mu(x)$ have the following structure:

$$H[\zeta] = \int_{\Sigma_t} \zeta^\mu \phi_\mu d\Sigma + J[\zeta]. \quad (35)$$

The integral goes over a space-like hyper-surface Σ_t of constant time t . Quantities ϕ_μ and $J[\zeta]$ depend on canonical coordinates, g_{ij} , and momenta, π_{ij} , which are defined from the Lagrangian by standard methods.

To avoid the divergences in the theory at large radii r one has to restrict the integration over Σ_t by some large upper bound $r \leq r_0$ (r_0 can be taken to infinity in the last stage of the computation). The last term $J[\zeta]$ in (35) is a surface term defined at r_0 . Quantities $J[\zeta]$ are introduced to ensure a canonical form for variations of $H[\zeta]$,

$$\delta H[\zeta] = \int_{\Sigma_t} (A^{ij} \delta g_{ij} + B^{ij} \delta \pi_{ij}). \quad (36)$$

The form of $J[\zeta]$ in general depends on the boundary conditions at r_0 .

The equations $\phi_\mu = 0$ are constraints analogous to the Gauss law $\nabla E - \rho = 0$ in electrodynamics. Thus, when the equations of motion are satisfied, $H[\zeta]$ reduce to pure surface terms. For this reason, in particular, the energy of the system which is associated with the generator of translations $i\partial_t = i\zeta_{(t)}^\mu \partial_\mu$ along the Killing time is nontrivial because of the presence of the surface term J . The on-shell value of $H[i\zeta_{(t)}]$ is defined so that to coincide with mass M of the BTZ black hole (Eq. (28)).

One can define generators $L_n = H[\zeta_n]$, $\bar{L}_n = H[\bar{\zeta}_n]$ corresponding to the particular set of diffeomorphism vectors having the asymptotic form (31)–(33). Their canonical commutation relations were investigated by Brown and Henneaux [29] who found that L_n, \bar{L}_n form a Virasoro algebra isomorphic to (18), (19), where the constant c is the central extension given by (30).

As follows from (31)–(34), the generator of time translations is represented as $i\partial_t = i(\partial_+ + \partial_-)/2l = l^{-1}(\zeta_0^\mu + \bar{\zeta}_0^\mu)\partial_\mu$. Therefore, one has the following relation between the energy and the Virasoro generators:

$$H[i\zeta_{(t)}] = \frac{1}{l}(H[\zeta_0] + H[\bar{\zeta}_0]) = \frac{1}{l}(L_0 + \bar{L}_0). \quad (37)$$

This equation is analogous to relation (20) discussed in Subsec. 1.1. Now, however, (37) is a classical quantity defined on a phase space. Suppose that modulo the equations of motion $L_0 = h$, $\bar{L}_0 = \bar{h}$. If the energy of the system coincides with the mass of the black hole, then, by the symmetry, $h = \bar{h} = Ml/2$.

Do these observations say something about the entropy of the BTZ black hole? It is a well-known fact that there is a correspondence between the Poisson brackets in classical mechanics and commutators of operators in quantum theory. By taking this into account one can make the following suggestions:

i) There is a quantum gravity theory on AdS such that physical states of this theory yield a representation of the Virasoro algebra related to asymptotic symmetries; classical generators L_n, \bar{L}_n correspond to operators $\hat{L}_n, \hat{\bar{L}}_n$ in the quantum gravity.

ii) The central charge c of the Virasoro algebra in quantum gravity coincides with the central charge (30) of the classical theory.

iii) A quantum state for which the operators $\hat{L}_0, \hat{\bar{L}}_0$ have eigenvalues $h = \bar{h}$ corresponds to a static BTZ black hole of mass $M = 2h/l$.

As follows from iii) the mass of a black hole cannot be arbitrary but takes some discrete values which can be derived by using the commutation relations of the Virasoro algebra. The spacing between two levels is determined by the inverse radius l^{-1} of AdS₃. If M is comparable to l , the black hole is essentially a quantum object. Note that the semiclassical regime of quantum gravity theory also requires that $\mathcal{R} \gg L_{\text{Pl}}$, where \mathcal{R} is characteristic radius of the space-time curvature, $L_{\text{Pl}} = G_3$ is the Planck length in three dimensions. The geometry of the BTZ black hole has two such radii, l and $r_+ = l\sqrt{8G_3M}$. Thus, the semiclassical limit requires that $l \gg G_3$ and $M \gg G_3/l^2$. The first condition imposes a restriction on the central charge (30), $c \gg 1$. The second condition holds if the black hole mass is larger than the Planck mass, $M \gg G_3^{-1}$. Both conditions then imply that $M \gg l^{-1}$. This means that the spectrum of a semiclassical black hole can be considered as continuous.

The classical black hole is a highly degenerate object. The degeneracy D, \bar{D} of operators $\hat{L}_n, \hat{\bar{L}}_n$, can be found by using the Cardy formula (24). The total degeneracy is

$$\ln D + \ln \bar{D} = 2 \ln D \simeq 4\pi \sqrt{\frac{ch}{6}} = 2\pi \sqrt{\frac{l^2 M}{2G_3}}, \quad (38)$$

which is exactly the Bekenstein–Hawking entropy (29) of the BTZ black hole with mass M . The above derivation of (38) was first given by Strominger [28]. The result can be generalized to the case of a rotating black hole whose state has additional number, a spin. One can also investigate along these lines black holes in a 2D AdS gravity [30,31].

There are a number of technical questions in these derivations which can be addressed [32] but can hardly be resolved without more detailed information about quantum gravity on AdS₃. These questions are related to assumptions ii) and iii) which may not hold because quantum effects change classical quantities

and these changes are not always small. However, if the assumptions i)–iii) are adopted, one gets a definite answer to the question formulated in the title of this review. Note that those few properties of the quantum gravity theory relevant for the entropy counting are determined only by the low-energy constants, l and G_3 . These constants define the energy spectrum completely, all one needs to know! Therefore, the BTZ black hole certainly sets an example where one can understand black-hole entropy without knowing much about quantum gravity.

Let us emphasize that the above discussion concerns black holes in AdS_3 gravity without matter. Introduction of matter fields makes such a derivation of the entropy impossible in general. Discussion of 3D black holes with matter fields and further references can be found in [33].

1.5. AdS/CFT Correspondence. Relation (29) can be used to find the Hawking temperature of the BTZ black hole $T_H = (dS^{\text{BH}}/dM)^{-1} = (2G_3M/\pi^2l^2)^{1/2}$. If the black hole is considered as a canonical ensemble, one can introduce its free energy, $F_{\text{bh}}(T, b)$, via the standard thermodynamical relation

$$F_{\text{bh}}(T, b) = M - TS^{\text{BH}} = -M = -\frac{\pi c}{6}bT^2, \quad b = 2\pi l, \quad T = T_H, \quad (39)$$

where c is given by (30). It is instructive to compare this result with the free energy (11) of the model discussed in Subsec.1.1 and see that $F_{\text{bh}}(T, b)$ is equivalent to the free energy of c quantum fields living on a circle of the length $b = 2\pi l$ so that one can write*

$$F_{\text{bh}}(T, b) = F_{\text{CFT}}(T, b). \quad (40)$$

This result could be expected from the previous discussion. Equation (40) relates classical and quantum quantities. The conformal theory lives on a flat space-time $\tilde{\mathcal{M}}_2$ which is one dimension lower than \mathcal{M}_3 . The metric of $\tilde{\mathcal{M}}_2$ is $d\tilde{l}^2 = -dt^2 + l^2d\varphi^2$. On the other hand, the metrics of constant-radius hypersurfaces of \mathcal{M}_3 at large r have the form $dl^2 \sim (r/l)^2d\tilde{l}^2$. Thus, up to a scale, factor $\tilde{\mathcal{M}}_2$ has the same geometry as asymptotically distant sections $r = \text{const}$ of \mathcal{M}_3 . In this sense $\tilde{\mathcal{M}}_2$ can be called the asymptotic infinity of \mathcal{M}_3 or an asymptotic boundary**.

It can be shown [25] that $F_{\text{bh}}(T, b)$ in (40) can be obtained from classical gravitational action (26) on the black-hole background \mathcal{M}_3 . If one had a quantum gravity on AdS_3 , the semiclassical limit of this theory in the black-hole sector

*Let us recall that (11) is applicable in the thermodynamical limit $TL \gg 1$ which requires that the black hole is classical, $M \gg G_3^{-1}$.

**There is a conformal transformation of the AdS_3 and BTZ metrics which maps these spaces to spatially compact space-times such that in the transformed metrics the surface $r = \infty$ is located at a finite distance and defines a boundary, see [34] for details.

would be given by $F_{\text{bh}}(T, b)$. Therefore, a semiclassical limit of quantum gravity theory on AdS_3 is determined by a conformal field theory defined at the asymptotic infinity of the bulk space-time. This property is known as the AdS/CFT correspondence.

There are arguments [34–36] based on string theory that the AdS/CFT correspondence also holds for higher-dimensional AdS gravities. For a D -dimensional AdS background \mathcal{M}_D , the asymptotic boundary is a $(D - 1)$ -dimensional space-time $\tilde{\mathcal{M}}_{D-1}$. The boundary theory living on $\tilde{\mathcal{M}}_{D-1}$ is a quantum conformal theory CFT_{D-1} . It should be emphasized that if $D > 3$, the properties of the boundary theory cannot be inferred from the asymptotic symmetries of the background space-time. The asymptotic symmetry in this case is just the anti-de Sitter group which is finite-dimensional and does not admit nontrivial central extensions in general [29]. To get the energy spectrum of the CFT more data about the gauge group of the theory, its coupling constants and others are required. String theory provides an example how these data can be related to the properties of the fundamental gravity theory*.

What is important, however, is that in these examples the characteristics of the boundary CFT are expressed in terms of the low-energy parameters. For example, for five-dimensional AdS gravity the effective number of degrees of freedom of the corresponding CFT (an analog of the central charge (30)) is proportional to l^3/G_5 where l is the AdS radius and G_5 is the Newton constant. In this regard, the higher-dimensional case is similar to the BTZ black hole. It supports the idea that by using the low-energy parameters the entropy of higher-dimensional AdS black holes can be reproduced by the methods of statistical-mechanics without knowing the details of quantum gravity theory.

A final remark is in order. If the AdS/CFT correspondence holds, the information about bulk degrees of freedom in AdS is encoded into a dual boundary theory. This is an example of how a «holographic principle» first formulated by 't Hooft [37] (see also [38] and the review [39]) is realized. This property does not explain what are the bulk degrees of freedom and where are they located but it may help to resolve other problems. For instance, since the boundary theory is unitary so should be the process of black hole evaporation.

1.6. Near-Horizon Conformal Symmetry. The arguments based on the AdS/CFT correspondence are not universal because they are restricted to gravity theories with a negative cosmological constant. They are not applicable to the most interesting case of asymptotically flat black hole space-times.

*The AdS/CFT correspondence in string theory is formulated as follows [35]: IIB-type string theory on $\text{AdS}_5 \times S^5$ is dual to $\mathcal{N} = 4$, $D = 3 + 1$ super-Yang–Mills theory with $SU(N)$ group. Coupling constant, g_{YM} , in this theory is related to string coupling constant g_{st} ($g_{\text{YM}}^2 \sim g_{\text{st}}$), N equals five-form flux on S^5 (N is supposed to be large). Radius of curvature of the background is proportional to $(g_{\text{YM}}^2 N)^{1/4}$.

It is easy to understand where the difficulty comes from. The problem of the Bekenstein–Hawking entropy S^{BH} is related to the physics near the black hole horizon. The value of the entropy, the temperature of the Hawking radiation and properties of the spectrum of the radiation (the so-called gray-body factors) are determined by the space-time geometry near the horizon. These facts strongly suggest that a universal approach to S^{BH} should be related to the near horizon region* rather than to spatial infinity.

It is natural to ask whether one can derive the Bekenstein–Hawking entropy by applying the so far successful arguments based on a symmetry group to the region near the horizon. The first attempts in this direction were made by Carlip [41] and Solodukhin [42] and then continued in a large number of publications by other authors [43–56]. We will not attempt to describe these works here in full detail. This would require us to go into many technical questions which are not completely resolved**. Also there is no unique point of view as to how this approach should be realized. We focus on some general features related to the formulation of this problem.

In the region near the horizon the black hole metric takes a simple form

$$ds^2 = -\kappa^2 \rho^2 dt^2 + d\rho^2 + d\sigma^2. \quad (41)$$

The horizon is located at $\rho = 0$. The coordinate ρ is the proper distance from a point to the horizon and $d\sigma^2$ is the metric on the horizon surface. Asymptotically (41) is valid for nonextremal black holes which have a nonvanishing surface gravity constant κ (and, hence, a nonzero Hawking temperature T_H , see (2)). Formula (41) is called the Rindler approximation. If $d\sigma^2 = dx^2 + dy^2$ is a flat metric, Eq. (41) is the metric in Minkowski space written in Rindler coordinates. An observer moving along the trajectory $\rho = \text{const}$ has acceleration $1/\rho$.

By using the BTZ black hole as an example one has to look for a relevant group of coordinate transformations which preserves this form of the metric and is isomorphic to the conformal group. This can be done in many ways, but a universal approach should be applicable to black holes in different gravity theories. In particular, it must work in two-dimensional gravities where the black hole horizon is a point*** and $d\sigma^2 = 0$ in (41). Thus, it is natural to identify the conformal group with coordinate transformations in the $t - \rho$ plane, as was first proposed in [42]. In arbitrary dimensions this is a two-dimensional plane \mathcal{G} orthogonal to the horizon surface. Let us denote its metric as $d\gamma^2$. In the

*This may not be necessarily true because the black-hole entropy is a global quantity [40].

**The latest account of these results and references can be found in [56].

***More precisely, the cross section of the black-hole horizon and a constant time hypersurface in two-dimensional black holes is a point.

light-cone coordinates

$$d\gamma^2 = -\kappa^2 \rho^2 dt^2 + d\rho^2 = -\kappa^2 \rho^2 du dv, \quad (42)$$

$$u = t - x, \quad v = t + x, \quad x = \frac{1}{\kappa} \ln \rho. \quad (43)$$

The coordinate transformations which lead to the conformal group are those discussed in Subsec. 1.2, i.e., $u' = f(u)$, $v' = g(v)$. Suppose this choice of transformations is correct. Can it be used to reproduce the Bekenstein–Hawking entropy? To answer this question note that there are several key distinctions between the near-horizon approach and the approach used in the case of the BTZ black hole.

i) The thermodynamical relations for a black hole in the near-horizon region do not look like relations of a 2D CFT. An observer at rest with respect to the black-hole horizon measures a temperature of the Hawking radiation T which differs from the Hawking temperature T_H by a blue-shift factor, $T = T_H/\sqrt{B}$ where B is related to the time-component of the metric (it is the modulus of norm of the Killing vector ∂_t). Near the horizon $B \simeq \kappa^2 \rho^2$. Thus, according to (2) the local temperature is $T \simeq 1/(2\pi\rho)$. It is determined only by the acceleration of the observer and does not depend on black-hole parameters. According to York [26], if the black hole is placed in a cavity, its temperature is defined as a local-temperature T on the boundary of the cavity. The black hole is characterized by an energy E which should be consistent with the first law of thermodynamics. For instance, if the radius of the cavity is fixed, $dE = TdS^{\text{BH}}$. However, when the boundary of the cavity is moved close to the horizon, T becomes a free parameter which means that $E = TS^{\text{BH}}$ up to an additive constant. Therefore, $S^{\text{BH}} \sim E$ and this relation differs from (13)*.

ii) Approximation (42) leaves only two parameters: the surface area of the horizon \mathcal{A} and the Planck length L_{Pl} (defined by the Newton coupling constant in the given theory). For a BTZ black hole there is an extra parameter, the AdS radius l , which determines the spacing between the energy levels.

iii) The boundary conformal theory in the BTZ case is given on a circle of length $2\pi l$. Contrary to this in the near-horizon approach the light-cone coordinates are not compact. Therefore, to have a discrete basis of generators of the conformal algebra L_n, \bar{L}_n one has to impose some boundary conditions on diffeomorphisms in the $t - \rho$ plane and introduce an extra parameter b , the size of the space where the CFT theory is defined. This scale should determine the spacing in the energy spectrum.

*It is interesting to note that such a relation between the energy and the entropy is typical for string theory where the degeneracy of a level with the energy E is proportional to e^E [57].

iv) Suppose that in the gravity theory the algebra of generators $H[\zeta_n]$ of the diffeomorphisms in the $t - \rho$ plane is a Virasoro algebra (18) with a central charge c . What is the value of c in this theory? The black hole is identified with a certain quantum state with the energy E . To relate the Bekenstein–Hawking entropy to the degeneracy of the energy level E one has to use (13) and condition that $E \sim S^{\text{BH}}$. This requires that $c \sim S^{\text{BH}}$. The precise value of c is fixed when b is fixed.

The derivation of the Bekenstein–Hawking entropy along these lines was given in [41, 42] and in subsequent publications. It should be noted that these derivations used a single copy of the Virasoro algebra and the conformal transformations were not necessarily related to transformations of coordinates u and v . However, all these works despite technical differences had the basic features described above.

An attractive feature of the near-horizon approach is its universality and a certain hope to explain the black hole entropy without relying on the details of quantum gravity theory. But is this hope justified?

One of the problems is that the central charge c in the boundary CFT is proportional to the area of the black-hole horizon. This means that c depends on the background, a property which does not look natural. Let us recall that the central charge in AdS gravities is a combination of the fundamental constants, see (30). For this reason, the AdS/CFT correspondence enables one to consider different backgrounds (for example, a black hole and a pure anti-de Sitter space) as different quantum states of the same boundary CFT. As a result, black hole evaporation is equivalent to a time evolution in some CFT. There should be no loss of information in this process. In contrast to this in the near-horizon approach black holes with different masses correspond to states in *different* CFT's. The evaporation of a black hole is an evolution in a space of theories and it is not restricted by requirements of unitarity.

The other problem is the choice of the boundary conditions at the horizon and fixing the central charge. It is clear that the Rindler approximation is not enough for this purpose. Perhaps, other characteristics of the gravitational field in the vicinity of the horizon may help to define the CFT completely. Some work in this direction can be found in [58]. On the other hand, going beyond the Rindler approximation certainly puts at risk the universality of the method.

The problem may be even more serious: to fix the boundary CFT one needs to know those details of the quantum gravity theory which are not available at low energies. The approach based on the near-horizon symmetry gives at best a statistical *representation* of S^{BH} . It implies the existence of the corresponding microstates but does not prove it. Note that the AdS/CFT correspondence is supported by computations in string theory [34–36] while approach [41, 42] does not have such support so far.

Are there any examples of a microscopic realization of the near-horizon symmetry and what can one learn from them? This will be discussed in the second part of the paper.

2. BLACK-HOLE ENTROPY AS A PROPERTY OF THE PHYSICAL VACUUM

2.1. Thermal Atmosphere and Entanglement Entropy. We now turn to another approach where the origin of the black-hole entropy is related to the properties of the physical vacuum in strong gravitational fields. There are always zero-point fluctuations of physical fields in a vacuum state. An observer, who is at rest with respect to the horizon sees these vacuum excitations as a thermal atmosphere around a black hole [59–64]. The first attempts to relate the Bekenstein–Hawking entropy to the thermal atmosphere were made by Thorne and Zurek [59] and by 't Hooft [60].

Let us calculate, as an example, the entropy S of a quantum scalar field around a static black hole. First note that near the horizon the local temperature is $T = 1/(2\pi\rho)$ and it grows indefinitely when the horizon is approached (ρ goes to zero). Thus, one can use the high-temperature asymptotic form of the free energy in a gravitational field. This asymptotic form is well known. In four-dimensional static space-time the free-energy is*

$$F(\beta) \simeq -\frac{\pi^2}{90} \int \sqrt{-g} T^4 d^3x. \quad (44)$$

Here g is the determinant of the metric; $T = \beta^{-1}/\sqrt{|g_{00}|}$ is the local temperature; g_{00} is the time-component of the metric. In asymptotically flat space-times, like a Schwarzschild black hole, β^{-1} is the temperature measured by an observer at infinity. For our purposes evaluation of (44) can be done by using the Rindler approximation (41). By taking into account that $g = \kappa\rho$, $d^3x = d\rho d^2\sigma$ one can see that the integral in (44) diverges. Let us introduce a cutoff at some small distance ϵ near the horizon. The leading contribution to entropy can be found by using the standard statistical-mechanical definition

$$S = \beta^2 \frac{\partial F(\beta)}{\partial \beta} \simeq \frac{1}{360\pi\epsilon^2} \mathcal{A}. \quad (45)$$

The quantum field is supposed to be in thermal equilibrium with the black hole. This is possible when the temperature coincides with the temperature of the

*Finite-temperature quantum field theory in gravitational backgrounds including the case of black-hole backgrounds is discussed in [65–67].

Hawking radiation. Thus, when the derivative is taken, one puts $\beta = \kappa/(2\pi)$ and gets the right-hand side of (45). The quantity \mathcal{A} is the integral $\int d^2\sigma$ which is the surface area of the horizon.

It is natural to assume [60] that the cutoff parameter is comparable to the Planck length, $\epsilon \simeq \sqrt{G}$. Then S in (45) has the same order of magnitude as the Bekenstein–Hawking entropy S^{BH} of a black hole.

One may wonder how the entropy can be related to properties of the vacuum. The explanation is that static observers near a black-hole horizon perceive the vacuum as a mixed state. This happens because they cannot do measurements inside the horizon. There is a nontrivial density matrix $\hat{\rho}$ which appears because in a local quantum field theory «observable» and «nonobservable» vacuum fluctuations are correlated or entangled at the horizon. There is an information loss which can be quantified by some *entanglement* entropy $S_{\text{ent}} = -\text{Tr} \hat{\rho} \ln \hat{\rho}$. A remarkable property of black holes is that the entanglement entropy coincides with the entropy of the thermal atmosphere* because $\hat{\rho}$ is a thermal density matrix [61–63].

Can S (or S_{ent}) be the source of the Bekenstein–Hawking entropy? To answer this question one has to resolve the following problems:

i) S depends on the cutoff ϵ . Therefore, there must be some natural explanation why the cutoff is adjusted so that $S = S^{\text{BH}}$.

ii) In the general case, S receives contributions from all fields present in the Nature. It depends on the total number of fields and their spins. However, S^{BH} does not have such dependence.

Before we consider these problems one more property of the thermal entropy has to be discussed. Introduction of the cutoff ϵ means that a quantum field cannot propagate on the entire black-hole background. It cannot leak inside the horizon because of some artificial («brick wall») boundary conditions. It should be emphasized that the horizon is not a boundary and there can be no conditions in this region but regularity.

There are other regularizations of integral (44) consistent with this property. For instance, one way to get rid of the divergences would be to use dimensional regularization. In a D -dimensional space, integral (44) depends on T^D and converges if D is extrapolated to the region $D < 2$.

One can also use, as was suggested in [75], a Pauli–Villars (PV) regularization. In this method for each physical field, one introduces 5 additional auxiliary fields: two fields with masses M_k which have the same statistics as the original field and three fields with masses M'_r which have the wrong statistics. The masses can be chosen as follows: $M_{1,2} = \sqrt{3\mu^2 + m^2}$, $M'_{1,2} = \sqrt{\mu^2 + m^2}$,

*The fact that S_{ent} is proportional to the horizon area and can be related to the Bekenstein–Hawking entropy was first pointed out in [68–70]. This entropy was then studied in [71–74].

$M'_3 = \sqrt{4\mu^2 + m^2}$, where μ^2 plays the role of a regularization parameter. The leading part of the entropy of each PV field is given by formula (45) with the only difference that fields with the wrong statistics give negative contributions to the total entropy. For this reason the leading divergence is canceled. To find the entropy one has to use next-to-leading terms in the high-temperature asymptotic expressions (see [65] for details). The final result in the limit of large μ is

$$S = S(\mu) \simeq \frac{\lambda}{48\pi} \mu^2 \mathcal{A}, \quad (46)$$

where $\lambda = \ln \frac{729}{256}$. The divergence in S in the PV method appears in the limit of infinitely heavy PV fields.

Both dimensional and PV regularizations are used in quantum field theory to regularize ultraviolet divergences in Feynman diagrams. The fact they can be used for the entropy indicates that the divergences near the horizon may be related to the ultraviolet divergences. This in fact is true and, as was first suggested by Susskind and Uglum [76] and Callan and Wilczek [72], these divergences can be removed by the standard renormalization of the Newton constant.

2.2. Entanglement Entropy and Renormalization of Gravitational Couplings. Let us discuss the renormalization in more detail. Vacuum polarization in an external gravitational field $g_{\mu\nu}$ results in the appearance of a nontrivial right-hand side in the Einstein equations, the average value of the stress energy tensor of a quantum field, $\langle \hat{T}_{\mu\nu} \rangle$. Such equations can be obtained as an extremum of an effective action $\Gamma[g]$ under variation of the background metric $g_{\mu\nu}$. The effective action has the following form:

$$\Gamma[g] = I[g] + W[g], \quad (47)$$

where $I[g]$ is the classical Einstein action or its modifications and $W[g]$ is a functional related to the contribution of quantum fields. For instance, for the so-called (free) nonminimally coupled scalar field $W = \frac{1}{2} \ln \det (-\nabla^2 + \xi R + m^2)$, where ξ is the constant of the coupling with the scalar curvature R .

Computations show that $W[g]$ has ultraviolet divergences which can be absorbed by a renormalization of the couplings in the classical action $I[g]$. To this end the latter is chosen in the form

$$I(G_B, \Lambda_B, c_B^i) = \int d^4x \sqrt{g} \times \left[-\frac{\Lambda_B}{8\pi G_B} - \frac{R}{16\pi G_B} + c_B^1 R^2 + c_B^2 R_{\mu\nu} R^{\mu\nu} + c_B^3 R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \right]. \quad (48)$$

Denote by W_{div} the UV-divergent part of the quantum action W . Then the renormalized quantities are defined as

$$I_{\text{ren}} \equiv I(G_{\text{ren}}, \Lambda_{\text{ren}}, c_{\text{ren}}^i) = I(G_B, \Lambda_B, c_B^i) + W_{\text{div}}, \quad W_{\text{ren}} = W - W_{\text{div}}. \quad (49)$$

The key observation is that W_{div} has the same structure as (48) and hence W_{div} can be absorbed by simple redefinition of the coupling constants in $I(G_B, \Lambda_B, c_B^i)$. In other words, I_{ren} is identical to the initial classical action I with the only change that the bare coefficients Λ_B , G_B , and c_B^i are replaced by their renormalized versions Λ_{ren} , G_{ren} , and c_{ren}^i . The relation between bare and renormalized couplings depends on the regularization. For instance, in PV regularization the renormalization of the Newton constant for the nonminimally coupled scalar field is

$$\frac{1}{G_{\text{ren}}} = \frac{1}{G_B} + \frac{\lambda}{2\pi} \left(\frac{1}{6} - \xi \right) \mu^2, \quad (50)$$

where $\lambda = \ln \frac{729}{256}$ and μ is the PV cutoff. According to the general prescription, the observable constants are identified with the renormalized ones. Thus, the Bekenstein–Hawking entropy is $S^{\text{BH}} = S^{\text{BH}}(G_{\text{ren}}) = \mathcal{A}/(4G_{\text{ren}})$. As follows from (46) and (50) it can be written in the following form:

$$S^{\text{BH}}(G_{\text{ren}}) = S^{\text{BH}}(G_B) + S(\mu) - Q_{\text{div}}, \quad (51)$$

where $Q_{\text{div}} = \xi \lambda \mu^2 \mathcal{A}/(2\pi)$.

Equation (51) explicitly demonstrates that the «observable» Bekenstein–Hawking entropy contains the statistical-mechanical entropy $S(\mu)$ of the black hole's quantum excitations as an essential part. It can be shown [75, 77–81] that this result does not depend on the regularization procedure, or on the black-hole background and holds for the entropies of different fields. In general, equation (51) is extended to include corrections to the Bekenstein–Hawking entropy due to terms depending on curvatures [77–79].

Relation (51) removes the two problems formulated in the end of Subsec. 2.1. Indeed, if one has a gravity theory where computations are based on the renormalization procedure, the leading part of the entanglement entropy of all quantum excitations is just a part of the observable Bekenstein–Hawking entropy, no matter what kind of regularization is used and how many field species exist in the Nature.

Although this fact indicates a strong connection between the entanglement entropy and S^{BH} , it does not solve the problem of the Bekenstein–Hawking entropy. Indeed, one part of the observable entropy S^{BH} is the «bare entropy» $S^{\text{BH}}(G_B)$ in (51) which has no statistical-mechanical meaning. Another question concerns statistical meaning of quantity Q_{div} which appears due to nonminimal couplings.

2.3. Induced Gravity. As was pointed out in [82–84], the problem of the bare entropy in (51) can be resolved if Einstein gravity is entirely induced by quantum effects. The idea of induced gravity was formulated by Sakharov [85, 86] and then developed in different works, see, e.g., [87] and the review papers [88, 89]. Sakharov’s idea is very simple and physical. Its main assumption is that the dynamical equations for the gravitational field $g_{\mu\nu}$ are determined by properties of the physical vacuum which, like an ordinary medium, has a microscopic structure. The relevant example is a crystal lattice. The metric $g_{\mu\nu}$ plays the same role as the strength tensor $\sigma_{ij} = \frac{1}{2}(\xi_{i,j} + \xi_{j,i})$ which describes macroscopic deformations of a crystal (here $\xi_i = \xi_i(\mathbf{x})$ is a vector of the displacement of the lattice site at a point with the coordinates \mathbf{x}). Gravitons in this picture are analogous to phonons and are collective excitations of the microscopic degrees of freedom of the vacuum. We call these degrees of freedom *constituents*. The constituents are virtual particles of all possible fields present in Nature. The energy stored in the deformation of the crystal has the form

$$\mathcal{E}[\sigma] = \sum_{\mathbf{x}} (A\sigma_{ii}^2(\mathbf{x}) + B\sigma^{ij}(\mathbf{x})\sigma_{ij}(\mathbf{x})), \quad (52)$$

where the coefficients A and B are determined by the microscopic structure of the lattice. They are known as Young and Poisson constants. The physical vacuum responds to variations of the metric $g_{\mu\nu}$ in a similar way. Such quantum effects can be described with the help of the effective gravitational action $\Gamma[g]$,

$$e^{i\Gamma[g]} = \int [D\Phi] \exp(iI[\Phi, g]), \quad (53)$$

where the integration runs over all constituent fields (denoted by Φ). $I[\Phi, g]$ is the classical action of Φ on a classical background with the metric $g_{\mu\nu}$. Sakharov’s idea is based on the observation that the leading contribution to $\Gamma[g]$ is determined by the divergent part and has the form of the classical Einstein action

$$\Gamma[g] \simeq \frac{1}{16\pi G_{\text{ind}}} \int \sqrt{g} d^4x (R(g) - 2\Lambda_{\text{ind}}). \quad (54)$$

Here and in what follows we consider four-dimensional gravity. $G_{\text{ind}} = (\gamma/l^2)^{-1}$ is an *induced* gravitational coupling, γ is a numerical coefficient which depends on the specific set of constituents and l is a cutoff parameter in the region of high energies. Λ_{ind} is an induced cosmological constant. It follows from (52) and (54) that $\Gamma[g]$ is similar to the energy $\mathcal{E}[\sigma]$, while G_{ind} and Λ_{ind} appear in the same way as Young and Poisson constants.

There are very interesting parallels between induced gravity and condensed matter systems, such as superfluid ^3He and ^4He (see [90]).

As was pointed out by Weinberg [91], an analog of induced gravity can be also found in particle physics. It is a theory of soft pions which can be used in the limit when the masses of the u and d quarks are neglected. In this limit there is a global chiral $SU(2) \times SU(2)$ symmetry. Gravitational action (54) has the same meaning as the Lagrangian of the chiral model while the constituents are analogous to the quarks that the pions are made of.

Let us note that in the interval of low energies, as in the case of pions, one can develop a quantum theory of gravitons by using (54). This theory can be used, for instance, to calculate graviton scattering amplitudes, see [92] for discussion of this topic.

The problem of the statistical interpretation of the Bekenstein–Hawking entropy in induced gravity is resolved in the following way. The microscopic degrees of freedom responsible for S^{BH} are the constituents which live in the gravitational field of a black hole. These virtual particles have a nontrivial quantum stress-energy tensor $\langle \hat{T}_{\mu\nu} \rangle$ which can be obtained by variation of the induced effective action (53). The background metric is a solution of the equation

$$\langle \hat{T}_{\mu\nu} \rangle = 0. \quad (55)$$

In the limit when the gravitational radius is much larger than the Planck length $L_{\text{Pl}} = G_{\text{ind}}^{1/2}$ the effective action reduces to (54) and Eq. (55) reduces to the Einstein vacuum equations. Because the black hole is a solution of these equations its entropy is $S^{\text{BH}} = \mathcal{A}/4G_{\text{ind}} \sim \mathcal{A}/l^2$ and it has the same order of magnitude as the entropy of the constituents near the horizon computed with the use of the cutoff l .

2.4. Induced Gravity Models. The above explanation of the Bekenstein–Hawking entropy is rather schematic because it implies the existence of a cutoff mechanism in the region of high energies which we do not know.

To verify whether induced gravity can really explain the black-hole entropy, one needs additional assumptions and concrete models. This step was carried out in [83, 84] where the additional condition was that the theory of constituents was free from leading ultraviolet divergences. This requirement enables one to construct models where G_{ind} is a computable quantity.

Induced gravity models having this property may possess different types of constituent fields. We consider the simplest possibility. The model consists of N_s scalar constituents ϕ_s with masses m_s , some of the constituents being nonminimally coupled to the background curvature with corresponding couplings ξ_s , and N_d Dirac fields ψ_d with masses m_d . The corresponding actions in (53) are

$$I[\phi_s, g] = -\frac{1}{2} \int d^4x \sqrt{-g} [(\nabla\phi_s)^2 + \xi_s R\phi_s^2 + m_s^2\phi_s^2], \quad (56)$$

$$I[\psi_d, g] = \int d^4x \sqrt{-g} \bar{\psi}_d (i\gamma^\mu \nabla_\mu + m_d) \psi_d. \quad (57)$$

Let us impose the following constraints on parameters of the constituents:

$$p(0) = p(1) = p(2) = p'(2) = 0, \quad (58)$$

$$q(0) = q(1) = 0, \quad (59)$$

where

$$p(z) = \sum_s m_s^{2z} - 4 \sum_d m_d^{2z}, \quad q(z) = \sum_k c_k m_k^{2z}, \quad (60)$$

$k = s, d$, and $c_d = 2$, $c_s = 1 - 6\xi_s$ for spinor and scalar constituents, respectively. Constraints (58) serve to eliminate the induced cosmological constant, while (59) enable one to get rid of the ultraviolet divergences in the induced Newton constant G_{ind} . It is the second set of conditions that will be important for our analysis of black-hole entropy. Given (59), G_{ind} is defined by the formula

$$\frac{1}{G_{\text{ind}}} = \frac{1}{12\pi} q'(1) = \frac{1}{12\pi} \sum_k c_k m_k^2 \ln m_k^2. \quad (61)$$

Because G_{ind} is explicitly known one can prove that

$$S^{\text{BH}} = \frac{\mathcal{A}}{4G_{\text{ind}}} = S - Q. \quad (62)$$

Here S is a statistical-mechanical entropy of the constituents thermally distributed at the Hawking temperature in the vicinity of the horizon. The quantity Q is a quantum average of the operator

$$\hat{Q} = 2\pi \sum_s \xi_s \int_{\Sigma} d^2\sigma \hat{\phi}_s^2 \quad (63)$$

where the integration goes over the horizon surface Σ .

The reason why a quantity like Q appears in the entropy formula is the following. Constraints (59) cannot be satisfied without introduction of nonminimal couplings $\xi_s R \phi_s^2$ in the scalar sector, see (56). G_{ind} and S^{BH} depend on the nonminimal coupling constants ξ_s while the thermal entropy S does not. This disagreement in the behavior of the two entropies appeared already in the renormalization Eq. (51). What happens in (62) is that the divergence in S is compensated by the divergence in Q .

Formula (63) is rather universal: it is valid for different models including those with vector constituents [94] as well as for different kinds of black holes, rotating [95] and charged [96], in different space-time dimensions.

What can one learn from these results?

i) The induced gravity models give a physical picture of the microscopic degrees of freedom of a black hole responsible for its entropy. These degrees of

freedom are the constituents propagating near the black hole horizon. The source of the Bekenstein–Hawking entropy is the entanglement or thermal entropy of the constituents in the given black-hole background.

ii) Induced gravity is not a fundamental theory but has the key properties which an ultimate theory of quantum gravity must possess. These properties are: the generation of the equations of the gravitational field by quantum effects and the absence of the leading ultraviolet divergences. As was pointed out in [93], from the point of view of the open string theory black-hole entropy can be considered as a loop effect, in full analogy with its origin in induced gravity.

To summarize, induced gravity is an example where one can study the mechanism of generation of the Bekenstein–Hawking entropy by using very general properties of a hypothetical fundamental theory.

The question which is not completely resolved in induced gravity models is the statistical meaning of quantity Q in (62). Since this term is present it is not quite clear how to represent S^{BH} in the form $-\text{Tr } \hat{\rho} \ln \hat{\rho}$ (see, however, [97]).

The physical reason of subtracting Q in (62), as was explained in [84], is related to two inequivalent definitions of the energy in the black-hole exterior. One definition, H , is the canonical energy or the Hamiltonian. The other definition, E , is the energy expressed in terms of the stress-energy tensor $T_{\mu\nu}$ which is obtained by variation of the action over the metric tensor. The two energies correspond to different properties of a black hole. H corresponds to evolution of the system along the Killing time and for this reason the operator H in quantum theory is used for constructing the density matrix which yields the entropy S in (62). On the other hand, E is related to thermodynamical properties of a black hole. If the black hole mass measured at infinity is fixed, the change of the entropy S^{BH} caused by the change of the energy E of fields in the black-hole exterior is

$$\delta S^{\text{BH}} = -T_H \delta E, \quad (64)$$

where T_H is the Hawking temperature of the black hole. The reason why E and H are not equivalent is in the existence of the horizon. The two quantities being integrals of metrical and canonical stress tensors differ by a total derivative. This difference results in a surface term (a Noether charge) on the bifurcation surface of the horizon. This surface term does not vanish because the horizon is not a real boundary and the only requirement for fields in this region is regularity. One can show [98] that the boundary term is the Q appearing in (62). More precisely,

$$E = H - T_H Q. \quad (65)$$

According to (64) the black-hole entropy is related to the distribution over the energies E of the induced gravity constituents. Hence, the subtraction of Q in (62) accounts for the difference between E and H in (65).

It should be noted, however, that an explicit calculation of the black-hole degeneracy for a given mass M which is connected with the distribution of the constituent field states over the energies E is a problem. Two suggestions how it can be done are discussed in [84] and [97]. The difficulty is that in quantum theory a nonzero value of Q in (62) is ensured by modes which, from the point of view of a static observer, have vanishing frequencies, the so-called soft modes.

3. CFT AND INDUCED GRAVITY

3.1. Dimensional Reduction. We pointed out in Subsec.1.6 that so far there are no examples showing that the near-horizon conformal symmetry can be realized in quantum gravity theory. Before such examples are known one can investigate this question in some simple models. This is another case where induced gravity can be quite helpful in developing ones intuition. In this section we follow the work [99]. Let us note that in the considered models the induced gravity constituents are massive fields whose masses have to be comparable to the Planck mass to ensure that the induced Newton constant (61) has the correct value. The contribution to G_{ind} from the fields observable at low-energies (fields of the Standard Model) can be neglected. How can the presence of massive constituents be reconciled with conformal symmetry? The idea is simple: since the local temperature of quanta near the horizon is large, the fields living within certain distance to the horizon are effectively massless and scale invariant. The role of the masses is to introduce a scale (a correlation length) into the CFT theory.

The curvature effects near the horizon are not important and one can use approximation (41) where the metric on the horizon itself is replaced by the flat metric

$$d\sigma^2 = dy_1^2 + dy_2^2. \quad (66)$$

The conformal transformations change only the metric $d\gamma^2$ in two-dimensional plane \mathcal{G} orthogonal to the horizon surface. We will write this metric as

$$d\gamma^2 = \gamma_{\alpha\beta} dx^\alpha dx^\beta, \quad (67)$$

where $\alpha, \beta = 0, 1$ and for a moment let $\gamma_{\alpha\beta}$ be arbitrary.

In this setting the dynamics of the constituents is essentially two-dimensional. This can be easily seen if we use the Fourier decomposition in y -plane and define

$$\Phi_{\mathbf{p}}(x) = \frac{1}{2\pi a} \int d^2 y e^{-i\mathbf{p}\mathbf{y}} \Phi(x, y), \quad (68)$$

where \mathbf{p} is a momentum along the horizon, $\mathbf{p}\mathbf{y} = p_i y^i$. To avoid volume divergences related to the infinite size of the horizon we assume that the range of

coordinates y^i is restricted, $-a/2 \leq y^i \leq a/2$. This means that the horizon area \mathcal{A} is finite and equal to a^2 .

Thus, each 4D field $\Phi(x, y)$ corresponds to a tower of 2D fields $\Phi_{\mathbf{p}}(x)$ which live on \mathcal{G} . If $\Phi(x, y)$ has the mass m then the mass of $\Phi_{\mathbf{p}}(x)$ depends on the transverse momentum \mathbf{p} ,

$$m(\mathbf{p}) = \sqrt{m^2 + \mathbf{p}^2}. \quad (69)$$

It should be noted that if the induced gravity constraints (58), (59) are satisfied for the set of masses m_s, m_d , they are satisfied for the masses $m_s(\mathbf{p}), m_d(\mathbf{p})$ as well. This means that a 2D gravity theory induced in each 2D sector at a given transverse momentum \mathbf{p} is free from UV divergences. The effective action $\Gamma[g]$ of the 4D induced gravity is the sum of the actions $\Gamma_2[\gamma, p]$ of 2D gravities

$$\Gamma[g] = \sum_{\mathbf{p}} \Gamma_2[\gamma, p] \simeq \frac{a^2}{4\pi} \int_{\sigma}^{\infty} \Gamma_2[\gamma, p] dp^2. \quad (70)$$

Here $p = |\mathbf{p}|$. It is assumed in (70) that the parameter a is large, so the sum over \mathbf{p} is replaced by the integral over p . The coefficient $a^2/(4\pi)$ is related to the number of modes with the momentum square p^2 .

The two-dimensional action can be easily calculated,

$$\Gamma_2[\gamma, p] \simeq \frac{1}{4G_2(p)} \int \sqrt{-\gamma} d^2x (\mathcal{R} + 2\lambda_2(p)). \quad (71)$$

Here \mathcal{R} is the curvature of \mathcal{G} , and

$$\frac{1}{G_2(p)} = -\frac{1}{12\pi} \sum_k c_k \ln m_k^2(\mathbf{p}), \quad (72)$$

$$\frac{\lambda_2(p)}{G_2(p)} = \frac{1}{4\pi} \left[\sum_s m_s^2(\mathbf{p}) \ln m_s^2(\mathbf{p}) - 4 \sum_d m_d^2(\mathbf{p}) \ln m_d^2(\mathbf{p}) \right]. \quad (73)$$

The four-dimensional Newton constant G_{ind} can be found by summation over momenta in (70) if one takes into account that $a^2 = \int dy_1 dy_2$. It gives $\Gamma[g]$ in the form (54), where $R[g] = \mathcal{R}[\gamma]$ and

$$\frac{1}{G_{\text{ind}}} = \lim_{p \rightarrow 0} \frac{1}{G(p)}, \quad \frac{1}{G(p)} = \int_{p^2}^{\infty} \frac{d\tilde{p}^2}{G_2(\tilde{p})}, \quad (74)$$

which coincides with (61).

Let us make an additional assumption: we treat the two-dimensional field models at any momentum p not just as Fourier components but as *physical theories* in a sense that each of these theories yields a 2D induced gravity with strictly positive gravitational couplings $G_2(p)$. In this case the Bekenstein–Hawking entropy of a black hole in such a 2D gravity is positive. Examples of induced gravity models with this property are presented in [99].

3.2. Representation of the Near-Horizon CFT. The 2D constituent fields create a thermal atmosphere around a 2D black hole, see the discussion in Subsec.2.1. This entropy can be easily computed if we neglect for a moment the masses of the fields and the nonminimal couplings. Near the horizon the 2D metric $d\tilde{\gamma}^2$ is the 2D Rindler metric (42). To avoid divergences near the horizon we introduce a cutoff ϵ by imposing a restriction in (42) $\epsilon \leq \rho \leq R$. The upper cutoff R is needed to eliminate an infrared divergence at spatial infinity. Since the theory is scale invariant one can rescale the metric (42) to the form

$$d\tilde{\gamma}^2 = -dt^2 + dx^2, \quad x = \frac{1}{\kappa} \ln \rho. \quad (75)$$

Therefore, the theory we are dealing with is a massless 2D field on an interval of the length $b = \kappa^{-1} \ln(R/\epsilon)$. To find its entropy one can use the result (12)

$$S = S(T_H, b) \simeq \frac{\pi}{3} b T_H = \frac{1}{6} \ln \frac{R}{\epsilon}, \quad (76)$$

where we took into account that the temperature has to be identified with the temperature of the Hawking radiation.

How do masses and nonminimal couplings change this result? As was shown in [99], one can formulate the following rules:

i) Near the horizon each induced gravity constituent with the momentum p and mass m_k corresponds to a 2D conformal theory on an interval $b_k = \kappa^{-1} \ln(R_k(p)/\epsilon)$ where the external radius is determined as $R_k(p) = m_k(\mathbf{p})^{-1}$, $p = |\mathbf{p}|$;

ii) Each of these CFT's is characterized by a central charge c_k ; charges of spinor constituents are $c_d = 2$, while charges of scalar fields are $c_s = 1 - 6\xi_s$ and depend on the nonminimal couplings.

The first rule follows from the fact that the two-point correlator of field operators is exponentially small when fields are separated by a distance larger than their correlation length $m_k(\mathbf{p})^{-1}$. The second rule can be inferred from the transformation properties of the components of the renormalized stress-energy tensor of 2D fields. For example, for a scalar field with the nonminimal coupling ξ the uu component of the stress-energy tensor

$$T_{uu} = \langle -(\partial_u \hat{\phi})^2 + 2\xi((\partial_u \hat{\phi})^2 + \hat{\phi} \partial_u^2 \hat{\phi}) \rangle \quad (77)$$

transforms as

$$\delta T_{uu} = \varepsilon \partial_u T_{uu} + 2 \partial_u \varepsilon T_{uu} + \frac{1 - 6\xi}{24\pi} \partial_u^3 \varepsilon + O(\varepsilon^2) \quad (78)$$

under an infinitesimal change $\delta u = \varepsilon(u)$ of the light-cone coordinate u (the light-cone coordinates are introduced in (42)). Eq. (78) has the same form as transformation (17) in a CFT theory with the central charge $c = 1 - 6\xi$.

The induced gravity constraints (59) which eliminate the divergences in the induced Newton constant G_{ind} can be represented in the form

$$\sum_k c_k = 0, \quad \sum_k c_k m_k^2 = 0. \quad (79)$$

The sum $C = \sum_k c_k$ can be interpreted as a total central charge of the constituents. This charge is zero because at each momentum p the 2D theory is free from ultraviolet divergences.

Note that (79) requires that some central charges c_s are negative. Typically CFT's with negative central charges correspond to ghosts. The ghosts appear in gauge theories when the Hilbert space is enlarged during quantization. Ghosts give negative contributions to the entropy to compensate for the contribution of the extra degrees of freedom in the enlarged Hilbert space. However, if the system is unitary its total entropy is always positive. As for ghost fields, the entropy associated with the 2D constituents with negative c_k is negative, and as in gauge theories the total entropy in each 2D induced gravity sector is positive because of the requirement $G_2(p) > 0$.

Now one can construct a concrete representation of the algebra of conformal transformations in the $\rho - t$ plane in terms of the operators acting in a Fock space of the CFT's. This can be used to count the degeneracy of states corresponding to certain energy levels as is done in the near-horizon approach discussed in Subsec. 1.6. Instead of doing this we give a simpler derivation based on equation (76). According to the formulated rules, each 2D constituent gives the following contribution

$$s(c_k, b_k(p)) = \frac{c_k}{6} \ln \frac{R_k(p)}{\epsilon} = -\frac{c_k}{6} \ln \epsilon m_k(p) \quad (80)$$

to the total entropy. The entropy of all constituents in 2D induced gravity at some momentum p is

$$s(p) = \sum_k s(c_k, b_k(p)) = \frac{1}{6} \sum_k c_k \ln R_k(p) = \frac{\pi}{G_2(p)}, \quad (81)$$

where $G_2(p)$ is the 2D induced Newton constant defined in (72). The dependence on cutoff ϵ disappears because of (79). As was pointed out above, the partial entropy $s(p) > 0$ because $G_2(p) > 0$; $s(p)$ is just the entropy of a black hole in the corresponding 2D induced gravity theory. The entropy in the 4D theory is

$$S_{\text{tot}} = \frac{a^2}{4\pi} \int_0^\infty s(p) dp^2 = \frac{\mathcal{A}}{4G_{\text{ind}}}. \quad (82)$$

It coincides with the Bekenstein–Hawking entropy (61) of a four-dimensional black hole with the horizon area $\mathcal{A} = a^2$. The last equality in (81) follows from relation (74) between 4D and 2D couplings.

Several remarks about this result are in order.

i) The given analysis shows that the method based on the near-horizon CFT does reproduce the Bekenstein–Hawking entropy in the induced gravity theory and it has there a concrete realization.

ii) It shows that the dimensional parameter which defines the «size» b of the near-horizon CFT may have a dynamical origin and is related to physics at Planckian scales.

iii) The near-horizon CFT's are effective theories because they are obtained as a result of dimensional reduction. The definition of 2D fields depends on the horizon radius (see (68)). Thus, black holes with different horizon areas are described by different CFT's. This property is similar to that one has in the approach [41, 42]. The question of whether black hole evaporation may result in the information loss should be addressed in the original theory of 4D constituents.

Apart from these similarities the near-horizon CFT in induced gravity has several features which do not appear in the approach discussed in Subsec. 1.6.

i) The total central charge in this theory vanishes (see (79)). This property is related to cancellation of the leading ultraviolet divergences.

ii) Because the masses of constituents are different, there is a set of correlation lengths $m_k^{-1}(\mathbf{p})$. Thus, such a theory may possess several different scales.

iii) What is important for understanding of the black-hole entropy is not only the conformal symmetry itself but also the way it is broken.

iv) Interpretation of induced gravity in terms of a near-horizon CFT requires further restrictions on the parameters of constituents to ensure positivity of 2D gravitational coupling $G_2(p)$ at each transverse momentum.

v) Each 2D induced gravity sector contains negative central charges. As for ghost fields, the entropy associated with the corresponding degrees of freedom has to be subtracted from the total entropy. This property requires further analysis.

Finally, it should be noted that the computations of S^{BH} we discussed in this section can be done not only in four-dimensional space-times (see [99]). It would be very interesting to investigate other possibilities of realizing the near horizon CFT in induced gravity.

SUMMARY

We discussed several examples that strongly support the idea that a microscopic origin of the Bekenstein–Hawking entropy of black holes can be understood by using a few general properties of a fundamental quantum gravity theory. These properties may be gleaned entirely from the low-energy physics.

One of the possibilities is that finding a proper place for a group of 2D conformal symmetries will make it possible to control completely the density of states in quantum gravity. The other possibility is that the entropy of a black hole

can be considered as a measure of the information loss inside the horizon provided that the gravity is entirely induced by quantum effects and the underlying theory is ultraviolet finite. These two points of view may complement each other.

It is fair to say that although these possibilities are very promising, both approaches have unresolved problems. It is a matter for future research to see whether the discussed problems are technical or whether they are more fundamental and, hence, require something which we cannot know about quantum gravity at low energies.

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