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ON NEUTRINO OSCILLATIONS AND TIME-ENERGY UNCERTAINTY RELATION

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We consider neutrino oscillations as nonstationary phenomenon based on the Schrödinger evolution equation and mixed neutrino states with definite flavor. It is demonstrated that for such states invariance under translations in time does not take place. It is shown that time-energy uncertainty relation plays a crucial role in neutrino oscillations. Neutrino oscillations are compared with $K^0 \leftrightarrows \bar{K}^0$, $B_d^0 \leftrightarrows \bar{B}_d^0$ and other oscillations.

Нейтринные осцилляции рассмотрены как нестационарное явление, основанное на уравнении Шредингера и смешанных состояниях нейтрино с определенным ароматом. Мы обращаем внимание на то, что для таких состояний нарушается трансляционная инвариантность во времени. Показано, что соотношение неопределенности энергия–время играет ключевую роль в возникновении нейтринных осцилляций. Проанализирована связь нейтринных осцилляций с осцилляциями $K^0 \leftrightarrows \bar{K}^0$ и $B_d^0 \leftrightarrows \bar{B}_d^0$.

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INTRODUCTION

Evidence for neutrino oscillations obtained in the atmospheric Super-Kamiokande [1], solar SNO [2], reactor KamLAND [3] and other neutrino experiments [4–8] is an important signature of a new beyond the Standard Model Physics.

In spite of the fact that the existence of neutrino oscillations is established, the basics of this new phenomenon are still a subject of different opinions and active discussions (see [9] and references therein). We will consider here neutrino oscillations as nonstationary phenomenon based on the Schrödinger evolution equation and notion of mixed states for flavor neutrinos ν_e, ν_μ, and ν_τ, which are produced in CC weak processes together with, correspondingly, e, μ, and τ. We will discuss flavor neutrino states in some detail. We will show that for usual neutrino beams with neutrino energies many orders of magnitude larger than neutrino masses, flavor lepton numbers L_e, L_μ, and L_τ are effectively conserved in the neutrino-production and neutrino-detection SM weak processes, and states of produced (and detected) flavor neutrinos are mixed states.

The basic evolution equation of the quantum field theory is the Schrödinger equation. According to this equation if at $t = 0$ flavor neutrino (antineutrino) is produced, at the time t the neutrino (antineutrino) state is *nonstationary one*. The time-energy uncertainty relation is a characteristic feature of such states (see, for example, [10–13]). We will show that this relation plays a crucial role in neutrino oscillations (see [14]). Neutrino oscillations, which are characterized by finite time during which the state of the system is significantly changed, in accordance with time-energy uncertainty relation *require uncertainty in energy*. In fact, we will demonstrate that for flavor neutrino states invariance under translations in time does not hold.

Neutrino oscillations have the same quantum-mechanical origin as $B_d \leftrightarrows \bar{B}_d$, $K^0 \leftrightarrows \bar{K}^0$, etc., oscillations. We will compare here neutrino oscillations with $B_d \leftrightarrows \bar{B}_d$ oscillations which were studied recently in detail at asymmetric B factories.

1. ON THE STATUS OF NEUTRINO OSCILLATIONS

The probabilities of the transitions $\nu_\alpha \to \nu_{\alpha'}$ and $\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}$ in vacuum in the general case of n neutrinos with definite masses are given by the following expressions (see [15–18]):

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \left| \sum_{i=1}^{n} U_{\alpha' i} \exp\left(-i\,\Delta m^2_{1i} \frac{L}{2E}\right) U^*_{\alpha i} \right|^2 \tag{1}$$

and

$$\mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_{\alpha'}) = \left| \sum_{i=1}^{n} U^*_{\alpha' i} \exp\left(-i\,\Delta m^2_{1i} \frac{L}{2E}\right) U_{\alpha i} \right|^2. \tag{2}$$

Here L is the distance between production and detection points; E is the neutrino energy, $\Delta m^2_{ik} = m^2_k - m^2_i$. Indices α, α' take the values $e, \mu, \tau, s_1, s_2, \ldots$, indices s_i label sterile neutrinos.

All existing neutrino oscillation data (with the exception of the LSND data [19])* are in good agreement with the assumption that the number of neutrinos with definite masses is equal to the number of the flavor neutrinos (three), determined from the measurement of the width of the decay of Z^0 boson into neutrino–antineutrino pairs at the LEP experiments. In the following we will consider the three-neutrino mixing.

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*Indication in favor of $\bar{\nu}_\mu \leftrightarrows \bar{\nu}_e$ oscillations obtained several years ago in the accelerator short-baseline LSND experiment are going to be checked by the running at the Fermilab MiniBooNE experiment [20].

The three-neutrino probabilities of the transitions $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$ in vacuum $(l, l' = e, \mu, \tau)$ can be presented in the following form (see, for example, [18]):

$$P(\nu_l \to \nu_{l'}) = \left| \delta_{l'l} + U_{l'2} \left(\exp\left(-i\,\Delta m_{12}^2 \frac{L}{2E}\right) - 1 \right) U_{l2}^* + \right. \\ \left. + U_{l'3} \left(\exp\left(-i\,\Delta m_{13}^2 \frac{L}{2E}\right) - 1 \right) U_{l3}^* \right|^2 \quad (3)$$

and

$$P(\bar{\nu}_l \to \bar{\nu}_{l'}) = \left| \delta_{l'l} + U_{l'2}^* \left(\exp\left(-i\,\Delta m_{12}^2 \frac{L}{2E}\right) - 1 \right) U_{l2} + \right. \\ \left. + U_{l'3}^* \left(\exp\left(-i\,\Delta m_{13}^2 \frac{L}{2E}\right) - 1 \right) U_{l3} \right|^2. \quad (4)$$

Here U is 3×3 PMNS [21, 22] mixing matrix.

In the case of the Dirac neutrinos ν_i, the matrix U is characterized by three mixing angles and one CP phase and in the standard parameterization has the form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e\,\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (5)$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$.

In the case of Majorana neutrinos ν_i, the mixing matrix is given by

$$U^M = U\,S(\alpha), \quad (6)$$

where $S_{ik}(\alpha) = \mathrm{e}^{i\alpha_i}\delta_{ik}; \alpha_3 = 0$. Majorana phases $\alpha_{2,3}$ do not enter into expressions (3) and (4) for neutrino and antineutrino transition probabilities. Thus, investigation of neutrino oscillations cannot make it possible to reveal the nature of neutrinos with definite masses (Majorana or Dirac?) [23] (see recent discussion in [24]).

The probabilities (3) and (4) depend on six parameters (two neutrino mass-squared differences Δm_{12}^2 and Δm_{23}^2, three mixing angles θ_{12}, θ_{23}, and θ_{13} and CP phase δ) and have rather complicated form. Taking into account the accuracy of the present-day neutrino oscillation experiments, we can consider, however, neutrino oscillations in the *leading approximation* (see, review [18]).

This approximation is based on the smallness of two parameters

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq 3.3 \cdot 10^{-2}; \quad \sin^2\theta_{13} \leqslant 5 \cdot 10^{-2}. \tag{7}$$

The value of the parameter $\frac{\Delta m_{12}^2}{\Delta m_{23}^2}$ can be inferred from the analysis of solar, atmospheric and KamLAND neutrino oscillation data. The upper bound of the parameter $\sin^2\theta_{13}$ can be obtained from the data of the reactor CHOOZ experiment [25].

Let us consider first the atmospheric-accelerator long baseline region of $\frac{L}{E}$ in which

$$\Delta m_{23}^2 \, \frac{L}{E} \gtrsim 1. \tag{8}$$

In this region in the transition probabilities (3) and (4) we can neglect the contribution of the Δm_{12}^2 term. If we neglect also the term proportional to $\sin^2\theta_{13}$, we come to the conclusion that the only possible transitions in this region are $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$. For the probabilities of ν_μ ($\bar{\nu}_\mu$) to survive from (3) and (4) we find the standard two-neutrino expression

$$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu) = 1 - \frac{1}{2}\,\sin^2 2\theta_{23}\left(1 - \cos\Delta m_{23}^2\,\frac{L}{2E}\right). \tag{9}$$

Existing atmospheric and K2K data are perfectly described by (9). From the analysis of the data of the atmospheric Super-Kamiokande experiment the following 90% CL ranges of the oscillation parameters were obtained [1]:

$$1.5 \cdot 10^{-3} \leqslant \Delta m_{23}^2 \leqslant 3.4 \cdot 10^{-3}\ \mathrm{eV}^2; \quad \sin^2 2\theta_{23} > 0.92. \tag{10}$$

For solar and reactor KamLAND experiments small Δm_{12}^2 is relevant. In the corresponding transition probabilities contributions of the «large» Δm_{23}^2 are averaged. For ν_e ($\bar{\nu}_e$) survival probabilities in vacuum (or in matter) the following general expression can be obtained [26]:

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^4\theta_{13} + (1 - \sin^2\theta_{13})^2\, P^{(12)}(\nu_e \to \nu_e), \tag{11}$$

where $P^{(12)}(\nu_e \to \nu_e)$ is the two-neutrino ν_e ($\bar{\nu}_e$) survival probability in vacuum (or in matter).

If we neglect the contribution of $\sin^2\theta_{13}$, for the probability of reactor $\bar{\nu}_e$ to survive in vacuum we find the following expression:

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\,\sin^2 2\,\theta_{12}\left(1 - \cos\Delta m_{12}^2\,\frac{L}{2E}\right). \tag{12}$$

The probability of solar ν_e to survive in matter in the approximation $\sin^2\theta_{13} \to 0$ is given by the standard two-neutrino expression which depends on Δm_{12}^2, $\tan^2\theta_{12}$ and electron number density $\rho_e(x)$ (see, for example, [27]).

From global analysis of solar and KamLAND data it was found [2]

$$\Delta m_{12}^2 = 8.0_{-0.4}^{+0.6} \cdot 10^{-5}\ \mathrm{eV}^2; \quad \tan^2\theta_{12} = 0.45_{-0.07}^{+0.09}. \tag{13}$$

The existing neutrino oscillation data are compatible with two different types of neutrino-mass spectra:

1. normal spectrum $m_1 < m_2 < m_3$; $\Delta m_{12}^2 \ll \Delta m_{23}^2$;
2. inverted spectrum* $m_3 < m_1 < m_2$; $\Delta m_{12}^2 \ll |\Delta m_{13}^2|$.

In the case of the normal spectrum neutrino masses are given by

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}; \quad m_3 = \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}. \tag{14}$$

For the inverted spectrum we have

$$m_1 = \sqrt{m_3^2 + |\Delta m_{13}^2|}; \quad m_2 = \sqrt{m_3^2 + |\Delta m_{13}^2| + \Delta m_{12}^2}. \tag{15}$$

Neutrino mass-squared differences are known from neutrino oscillation data. Only upper bound of the lightest neutrino mass is known at present. From the data of the Mainz [28] and Troitsk [29] tritium β-decay experiments it was found

$$m_{1(3)} \leqslant 2.3\ \mathrm{eV}. \tag{16}$$

Future KATRIN [30] tritium experiment will be sensitive to

$$m_{1(3)} \simeq 0.2\ \mathrm{eV}. \tag{17}$$

From cosmological data for the sum of neutrino masses upper bounds in the range

$$\sum_i m_i \leqslant 0.4\text{–}1.7\ \mathrm{eV} \tag{18}$$

can be inferred (see [31]). The precision of the cosmological measurements will significantly increase in future. It is expected that the future sensitivity to the sum of neutrino masses will reach $\sum_i m_i \simeq 0.05$ eV [32].

*In order to keep for the solar-KamLAND neutrino mass-squared difference notation $\Delta m_{12}^2 > 0$, neutrino masses are usually labeled differently in the cases of normal and inverted neutrino spectra. In the case of the normal spectrum $\Delta m_{23}^2 > 0$ and in the case of the inverted spectrum $\Delta m_{13}^2 < 0$. Thus, with such notations for the neutrino masses the character of the neutrino-mass spectrum is determined by the sign of atmospheric neutrino mass-squared difference.

The accuracies of future neutrino oscillation experiments are planned to be much higher than today. In the experiments of the next generation one of the major efforts will be dedicated to the measurement of the important parameter $\sin^2\theta_{13}$. In the accelerator T2K experiment, $\nu_\mu \to \nu_e$ oscillations in the atmospheric range of the neutrino mass-squared difference will be searched for. The sensitivity $\sin^2\theta_{13} \simeq 1.5 \cdot 10^{-3}$ will be reached in this experiment [33]. In the reactor DOUBLE CHOOZ experiment the sensitivity $\sin^2\theta_{13} \simeq 1 \cdot 10^{-2}$ is planned to be achieved [34]. If it will occur that the parameter $\sin^2\theta_{13}$ is not too small, the character of the neutrino-mass spectrum and CP violation in the lepton sector can be probed at the Super Beam [35] , β-beam [36] and Neutrino Factory [37] facilities.

2. FLAVOR NEUTRINO STATES

From the point of view of the field theory, neutrino oscillations are based on the mixing relation for the fields

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x) \quad (l = e, \mu, \tau), \tag{19}$$

here $\nu_i(x)$ is (Majorana or Dirac) field of neutrino with mass m_i, U is the unitary PMNS mixing matrix, and $\nu_{lL}(x)$ is the so-called flavor field. The flavor fields $\nu_{lL}(x)$ enter into the standard CC and NC Lagrangians

$$\mathcal{L}_I^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}\, W^\alpha + \text{h.c.}; \quad j_\alpha^{\mathrm{CC}} = 2\sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_\alpha l_L \tag{20}$$

and

$$\mathcal{L}_I^{\mathrm{NC}} = -\frac{g}{2\cos\theta_W}\, j_\alpha^{\mathrm{NC}}\, Z^\alpha; \quad j_\alpha^{\mathrm{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}\gamma_\alpha \nu_{lL}, \tag{21}$$

where g is the $SU(2)$ gauge constant and θ_W is the weak angle.

The relation (19) is the result of the diagonalization of a neutrino mass term of the total Lagrangian. There are two possible types of the neutrino mass terms: Majorana and Dirac (see [15–18]). In the case of the Majorana mass term, $\nu_i(x)$ is the field of truly neutral Majorana particles which satisfies the condition

$$\nu_i(x) = \nu_i^c(x) = C\ \bar{\nu}_i^T(x), \tag{22}$$

where C is the matrix of the charge conjugation.

In the case of the Dirac mass term, $\nu_i(x)$ is the field of particles ν_i and antiparticles $\bar{\nu}_i$ which differ by the conserved total lepton number $L = L_e + L_\mu + L_\tau$ $(L(\nu_i) = -L(\bar{\nu}_i) = 1)$.

As we have mentioned before, investigation of neutrino oscillations does not allow one to establish the nature of ν_i. In order to reveal the nature of the massive neutrinos it is necessary to study precesses in which the total lepton number L is violated. The most sensitive to the nature of neutrino process is neutrinoless double β decay of nuclei (see [38])

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{23}$$

The nature of neutrinos with definite masses will be not important for our discussion. In this section we will consider the production of neutrinos (and antineutrinos) in the case of the neutrino mixing (see [39]). Neutrinos (and antineutrinos) are produced in CC decays and reactions. Let us consider the production of neutrinos in a CC decay

$$a \to b + l^+ + \nu_i, \tag{24}$$

where a and b are some hadrons.

Neutrino mass-squared differences (10) and (13) are so small that due to Heisenberg uncertainty relation it is impossible to distinguish momenta of produced neutrinos with different masses. For the state of the final neutrinos we have

$$|\nu_f\rangle = \sum_i |\nu_i\rangle\ \langle \nu_i,\, l^+\, b\,|S|\, a\rangle, \tag{25}$$

where $\langle \nu_i\, l^+\, b\,|S|\, a\rangle$ is the matrix element of the process (24), and $|\nu_i\rangle$ is the state of left-handed neutrino with mass m_i*, momentum $\mathbf{p}$ and energy $E_i = \sqrt{\mathbf{p}^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$. We have

$$H_0\,|\nu_i\rangle = E_i\ |\nu_i\rangle, \tag{26}$$

where H_0 is the free Hamiltonian.

In neutrino experiments energies of neutrinos E are much larger than neutrino masses: in solar and reactor experiments $E \gtrsim 1$ MeV, in atmospheric and accelerator long-baseline experiments $E \gtrsim 1$ GeV, etc. Taking into account that $m_i \lesssim 1$ eV, we have $\frac{m_i^2}{E^2} \leqslant 10^{-12}$. Thus, neutrino masses can be safely neglected in matrix elements of neutrino production processes. From (19) and (20) we find

$$\langle \nu_i\, l^+\, b\,|S|\, a\rangle \simeq U_{li}^*\ \langle \nu_l\, l^+\, b\,|S|\, a\rangle_{\rm SM}, \tag{27}$$

*Contributions of the states with positive helicity are proportional to $\frac{m_i}{E}$ and are negligibly small.

where $\langle \nu_l \, l^+ \, b \, |S| \, a \rangle_{\rm SM}$ is the Standard Model matrix element of the process of the emission of massless flavor neutrino ν_l in the decay

$$a \to b + l^+ + \nu_l. \tag{28}$$

We have

$$\langle \nu_l \, l^+ \, b \, |S| \, a \rangle_{\rm SM} = \\ = -i \, \frac{G_F}{\sqrt{2}} \, N \, 2 \, \bar{u}_L(p) \, \gamma_\alpha \, v_L(p') \, \langle b | \, J^\alpha(0) \, |a\rangle \, (2\pi)^4 \, \delta(P' - P). \tag{29}$$

Here N is the product of the standard normalization factors; p is neutrino momentum; p' is the momentum of l^+; P and P' are total initial and final momenta, and J^α is hadronic charged current*.

From (25) and (27) for the final neutrino state we find the expression

$$|\nu_f\rangle = |\nu_l\rangle \langle \nu_l \, l^+ \, b \, |S| \, a \rangle_{\rm SM}, \tag{30}$$

where

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^* \; |\nu_i\rangle \tag{31}$$

is normalized left-handed neutrino state. It follows from (30) that the probability of the decay (28) is given by the Standard Model (assuming that ν_l is massless).

Neutrino which is produced in a CC weak decay together with l^+ is called flavor neutrino ν_l. We have shown that the state of flavor neutrino is given by *coherent superposition* of states of neutrinos with definite masses.

Analogously, in CC processes together with lepton l^- right-handed flavor antineutrino $\bar{\nu}_l$ is produced. The state of $\bar{\nu}_l$ is given by the expression

$$|\bar{\nu}_l\rangle = \sum_{i=1}^{3} U_{li} |\nu_i\rangle, \tag{32}$$

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*The arguments presented above are not applicable to high-energy part of β spectrum of the decay $^3{\rm H} \to {}^3{\rm He} + e^- + \bar{\nu}_e$ which corresponds to the emission of neutrino with *energy comparable with neutrino mass*. For the spectrum we have

$$\frac{d\,\Gamma}{d\,E} = C \, p \, (E + m_e) \, (E_0 - E) \sum_i |U_{ei}|^2 \, \sqrt{(E_0 - E)^2 - m_i^2} \, F(E) \, \theta(E_0 - E - m_i),$$

where E_0 is the energy, released in the decay; m_e is the mass of the electron; $F(E)$ is the Fermi function which takes into account the Coulomb interaction of the final particles, and C is a constant. From the measurement of the electron spectrum in the high-energy region the bound (16) was obtained in the Troitsk and Mainz experiments.

where $|\nu_i\rangle$ is the state of the right-handed neutrino (or right-handed antineutrino in the Dirac case) with mass m_i, momentum $\mathbf{p}$, and energy $E_i \simeq p + \frac{m_i^2}{2p}$.

We will consider now detection of neutrinos with energies much larger than neutrino masses. Neutrinos are detected via the observation of CC and NC weak processes. Let us consider, for example, inclusive process

$$\nu_{l'} + N \to l^- + X. \quad (33)$$

Neglecting neutrino masses, for the matrix element of the process we have

$$\langle lX|S|\nu_{l'}N\rangle = \sum_i \langle lX|S|\nu_i N\rangle U_{l'i}^* = \\ = \langle lX|S|\nu_l\ N\rangle_{\rm SM} \sum_i U_{l'i}^* U_{li} = \langle lX|S|\nu_l N\rangle_{\rm SM}\delta_{l'l}. \quad (34)$$

Here

$$\langle lX|S|\nu_l N\rangle_{\rm SM} = \\ = -i\frac{G_F}{\sqrt{2}} N 2\bar{u}_L(p')\gamma_\alpha u_L(p)\langle X|J^\alpha(0)|N\rangle(2\pi)^4\delta(P'-P), \quad (35)$$

where p' is the momentum of final lepton, and p is the neutrino momentum.

It follows from (34) that due to unitarity of the neutrino mixing matrix the matrix element $\langle lX|S|\nu_{l'}N\rangle$ is different from zero only if $l' = l$. Thus, the lepton l^- is produced in CC process (33) by the left-handed flavor neutrino ν_l. Analogously, the lepton l^+ can be produced in inclusive CC process

$$\bar{\nu}_l + N \to l^+ + X \quad (36)$$

by the right-handed flavor antineutrino $\bar{\nu}_l$. We come to the conclusion that in CC processes flavor lepton numbers are effectively conserved.

Let us summarize previous discussion. For neutrinos with energies many orders of magnitude larger than neutrino masses in matrix elements of neutrino-production and neutrino-detection processes neutrino masses can be neglected. As a result of that

• Flavor lepton numbers L_e, L_μ, and L_τ are conserved in such processes: together with l^- right-handed flavor antineutrino $\bar{\nu}_l$ is produced, left-handed flavor neutrinos ν_l in the processes of interaction with nucleon produce l^-, etc.

Nonconservation of the flavor lepton numbers can be revealed only in such processes in which effects of neutrino masses are relevant. Such processes are neutrino oscillations in vacuum and neutrino transitions in matter.

• Matrix elements of neutrino-production and neutrino-detection processes are given by the Standard Model expressions (in which neutrino masses can be neglected).

• States of flavor neutrino ν_l and antineutrino $\bar{\nu}_l$ are given by coherent superpositions (31) and (32).

Let us stress that states of flavor neutrino ν_l and flavor antineutrino $\bar{\nu}_l$ are the superpositions of the states of neutrinos with definite masses ν_i with coefficients U^*_{li} and U_{li}, respectively. Because of this difference in the case of the CP violation

$$P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'}); \quad l' \neq l.$$

The mixed flavor neutrinos and antineutrinos states (31) and (32) are different from usual states of particles in the Quantum Field Theory. We will show now that for such state invariance under translation in time is not valid.

Let us consider translations in space and time (see, for example, [40])

$$x'_\alpha = x_\alpha + a_\alpha, \tag{37}$$

where a is a constant vector. In the case of the invariance under translation we have

$$|\Psi\rangle' = \mathrm{e}^{iPa}|\Psi\rangle \tag{38}$$

and

$$O(x+a) = \mathrm{e}^{iPa}O(x)\mathrm{e}^{-iPa}, \tag{39}$$

where P_α is the operator of the total momentum; $O(x)$ is an operator, and vectors $|\Psi\rangle$ and $|\Psi\rangle'$ describe *the same* physical state.

If $|\Psi\rangle$ is a state with total momentum p, vectors $|\Psi\rangle$ and $|\Psi\rangle'$ differ by the phase factor

$$|\Psi\rangle' = \mathrm{e}^{ipa}|\Psi\rangle. \tag{40}$$

Let us apply now the operator of the translations e^{iPa} to the mixed flavor neutrino state $|\nu_l\rangle$ given by Eq. (31). We have

$$|\nu_l\rangle' = \mathrm{e}^{iPa}|\nu_l\rangle = \mathrm{e}^{-i\mathbf{pa}} \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i}\mathrm{e}^{iE_i a^0} U^*_{li}. \tag{41}$$

The vector $|\nu_l\rangle'$ describes *superposition of different flavor states*. Thus, initial and transformed vectors describe *different states*. We come to the conclusion that in the case of the states which describe mixed flavor neutrinos with definite momentum $\mathbf{p}$ there is no invariance under translation in time. This means that in transitions between different flavor neutrinos (and antineutrinos) energy is not conserved.

3. NEUTRINO OSCILLATIONS IN VACUUM AND TIME-ENERGY UNCERTAINTY RELATION

The basic evolution equation of the quantum field theory is the Schrödinger equation

$$i\,\frac{\partial|\Psi(t)\rangle}{\partial t} = H\,|\Psi(t)\rangle, \tag{42}$$

where H is the total Hamiltonian. The general solution of equation (42) has the form

$$|\Psi(t)\rangle = \mathrm{e}^{-iHt}\,|\Psi(0)\rangle, \tag{43}$$

where $|\Psi(0)\rangle$ is an initial state.

Let us consider the evolution in vacuum of the states of flavor neutrinos ν_l and flavor antineutrinos $\bar{\nu}_l$ produced in weak processes. We have in this case

$$|\Psi(0)\rangle = |\nu_l\rangle; \quad \text{or} \quad |\Psi(0)\rangle = |\bar{\nu}_l\rangle; \quad H = H_0, \tag{44}$$

where H_0 is the free Hamiltonian and the states $|\nu_l\rangle$ and $|\bar{\nu}_l\rangle$ are given by Eq. (31) and Eq. (32). Taking into account (43) for neutrino and antineutrino states at the time $t \geqslant 0$ we have

$$|\nu_l\rangle_t = \sum_{i=1}^{3} |\nu_i\rangle\,\mathrm{e}^{-iE_i t} U_{li}^* \tag{45}$$

and

$$|\bar{\nu}_l\rangle_t = \sum_{i=1}^{3} |\nu_i\rangle\,\mathrm{e}^{-iE_i t} U_{li}. \tag{46}$$

Thus, flavor neutrinos ν_l and antineutrinos $\bar{\nu}_l$, produced in weak processes at $t = 0$, at $t > 0$ are described by *nonstationary states*.

It is a general property of quantum theory that for nonstationary states the time-energy uncertainty relation

$$\Delta E \Delta t \geqslant 1 \tag{47}$$

takes place (see, for example, [10–13, 41]). In this relation ΔE is uncertainty in energy and Δt is time interval during which significant changes in the system happen.

In the neutrino case

$$(\Delta E)_{ik} = E_k - E_i \simeq \frac{\Delta m_{ik}^2}{2E}, \tag{48}$$

and time-energy uncertainty relation takes the form

$$\frac{\Delta m_{ik}^2}{2E}\; t \geqslant 1, \tag{49}$$

where t is the time interval during which flavor content of neutrino state is changed.

Neutrinos are detected through the observation of weak processes. Let us develop the state $|\nu_l\rangle_t$ over flavor states $|\nu_{l'}\rangle$. We have

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \mathbf{A}(\nu_l \to \nu_{l'}; t), \tag{50}$$

where

$$\mathbf{A}(\nu_l \to \nu_{l'}; t) = \sum_{i=1}^{3} U_{l'i}\, \mathrm{e}^{-iE_i t} U_{li}^{*} \tag{51}$$

is the amplitude of the transition $\nu_l \to \nu_{l'}$ during the time t*.

Analogously in the case of the antineutrino we have

$$|\bar{\nu}_l\rangle_t = \sum_{l'} |\bar{\nu}_{l'}\rangle \mathbf{A}(\bar{\nu}_l \to \bar{\nu}_{l'}; t), \tag{52}$$

where the amplitude of the transition $\bar{\nu}_l \to \bar{\nu}_{l'}$ is given by

$$\mathbf{A}(\bar{\nu}_l \to \bar{\nu}_{l'}; t) = \sum_{i=1}^{3} U_{l'i}^{*}\, \mathrm{e}^{-iE_i t} U_{li}. \tag{53}$$

Expressions (51) and (53) have a simple meaning: $U_{li}^{*}(U_{li})$ is the amplitude of the transition from initial flavor neutrino (antineutrino) state $|\nu_l\rangle(|\bar{\nu}_l\rangle)$ to the state $|\nu_i\rangle$; the factor $\mathrm{e}^{-iE_i t}$ describes propagation in the state with definite energy E_i, and $U_{l'i}(U_{l'i}^{*})$ is the amplitude of the transition from the state $|\nu_i\rangle$ to the final flavor state $|\nu_{l'}\rangle(|\bar{\nu}_{l'}\rangle)$. Because neutrino masses cannot be resolved in the production and detection processes, in the amplitudes (51) and (53) sum over all i is performed.

It is instructive to derive expressions (51) and (54) starting from the flavor representation. We have

$$|\Psi(t)\rangle = \sum_{l=e,\mu,\tau} |\nu_l\rangle a_l(t), \tag{54}$$

*We can obtain the same result in another way. Let us consider the process $\nu_i + N \to l' + X$. For neutrinos with energies many orders of magnitude larger than neutrino masses we have

$$\langle lX|S|\nu_i N\rangle \simeq \langle l'X|S|\nu_{l'} N\rangle_{\mathrm{SM}} U_{l'i}.$$

From (45) and this relation we will find the expression (51) for $\nu_l \to \nu_{l'}$ transition amplitude.

where $a_l(t) = \langle \nu_l | \Psi(t) \rangle$ is the wave function of neutrino in the flavor representation. From (42) for the equation of motion in vacuum we find

$$i \frac{\partial a(t)}{\partial t} = H_0 a(t), \tag{55}$$

where

$$(H_0)_{l'l} = \langle \nu_{l'} | H_0 | \nu_l \rangle = \sum_i U_{l'i} E_i U_{li}^* = (U E U^\dagger)_{l'l} \tag{56}$$

is the free Hamiltonian in the flavor representation. In order to obtain the solution of Eq. (55) let us introduce the function

$$a'(t) = U^\dagger a(t). \tag{57}$$

From (55) and (57) we have

$$i \frac{\partial a'(t)}{\partial t} = E a'(t). \tag{58}$$

The solution of this equation is obvious:

$$a'(t) = \mathrm{e}^{-iEt} a'(0). \tag{59}$$

From (57) and (59) we find that solution of Eq. (55) is given by

$$a(t) = U \,\mathrm{e}^{-iEt} U^\dagger a(0). \tag{60}$$

Assuming that $a_{l''}(0) = \delta_{l''l}$ for the amplitude of the transition $\nu_l \to \nu_{l'}$ we find the expression

$$a_{l'}(t) = (U \,\mathrm{e}^{-iEt} U^\dagger)_{l'l}, \tag{61}$$

which coincides with (51).

Analogously, in the case of antineutrino we have

$$|\Psi(t)\rangle = \sum_{l=e,\mu,\tau} |\bar{\nu}_l\rangle b_l(t), \tag{62}$$

where $b_l(t) = \langle \bar{\nu}_l | \Psi(t) \rangle$ is the wave function of antineutrino in the flavor representation. The function $b(t)$ satisfies the following evolution equation:

$$i \frac{\partial b(t)}{\partial t} = \bar{H}_0 b(t), \tag{63}$$

where

$$(\bar{H}_0)_{l'l} = \langle \bar{\nu}_{l'} | H_0 | \bar{\nu}_l \rangle = (U^* E U^T)_{l'l} \tag{64}$$

is the free Hamiltonian in the flavor representation. The solution of this equation is given by

$$b(t) = U^* \, \mathrm{e}^{-iEt} U^T b(0). \tag{65}$$

If we assume that $b_{l''}(0) = \delta_{l''l}$ for the amplitude of the transition $\bar{\nu}_l \to \bar{\nu}_{l'}$ we find the expression

$$b_{l'}(t) = (U^* \, \mathrm{e}^{-iEt} U^T)_{l'l}, \tag{66}$$

which coincides with (53).

From (51) and (53) for the probabilities of the transitions $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$ in vacuum we obtain the following standard expressions:

$$P(\nu_l \to \nu_{l'}) = |\mathbf{A}(\nu_l \to \nu_{l'}; t)|^2 = \\ = \left| \delta_{l'l} + \sum_{i=2,3} U_{l'i} \left(\exp\left(-i\Delta m^2_{1i} \frac{L}{2E} \right) - 1 \right) U^*_{li} \right|^2 \tag{67}$$

and

$$P(\bar{\nu}_l \to \bar{\nu}_{l'}) = |\mathbf{A}(\bar{\nu}_l \to \bar{\nu}_{l'}; t)|^2 = \\ = \left| \delta_{l'l} + \sum_{i=2,3} U^*_{l'i} \left(\exp\left(-i\Delta m^2_{1i} \frac{L}{2E} \right) - 1 \right) U_{li} \right|^2 . \tag{68}$$

Taking into account the unitarity of the neutrino mixing matrix it is easy to check that $P(\nu_l \to \nu_{l'})$ and $P(\nu_l \to \nu_{l'})$ are normalized probabilities*:

$$\sum_{l'=e,\mu,\tau} P(\nu_l \to \nu_{l'}) = 1; \quad \sum_{l'=e,\mu,\tau} P(\bar{\nu}_l \to \bar{\nu}_{l'}) = 1. \tag{69}$$

In (67) and (68) we have used the relation

$$t \simeq L, \tag{70}$$

*We assumed that states $|\nu_i\rangle$ are the states with the same momentum $\mathbf{p}$. Correspondingly, mixed states $|\nu_l\rangle$ and $|\bar{\nu}_l\rangle$ are characterized by momentum $\mathbf{p}$. Notice that this is the standard procedure, which corresponds to conditions of experiments with beams of particles. If we assume, however, that $|\nu_i\rangle$ are states with different momenta $\mathbf{p}_i$, for oscillation phases in (67) and (68) we will have $(E_i - E_1)t \simeq \left[(p_i - p_1)L + \Delta m^2_{1i} \frac{L}{2E} \right]$, where the first term $p_i - p_i$ is proportional to Δm^2_{1i} with some unknown coefficient which could vary from one experiment to another. There is no such term in the oscillation phases: data of different neutrino oscillation experiments are compatible if the standard expressions (67) and (68) for transition probabilities are used. In the framework of the considered formalism with mixed flavor neutrino states and Schrödinger evolution equation, «the equal momentum assumption» is the only possibility to obtain oscillation phases which do not depend on experimental conditions.

where L is the distance between neutrino-production and neutrino-detection points. There were many discussions in literature connected with the relation (70) (see [9, 42]). After K2K experiment [4] there is no reasons for such discussions. In this experiment this relation was confirmed and used to select neutrino events.

In the K2K experiment, neutrinos were produced by protons from the KEK accelerator in 1.1 μs spills. Protons were extracted from the accelerator every 2.2 s. The difference of the time of the detection of neutrinos in the Super-Kamiokande detector ($t_{\rm SK}$) and the time of the production of neutrinos at KEK ($t_{\rm SK}$) $t = t_{\rm SK} - t_{\rm KEK}$ was measured in the K2K experiment. Let us determine

$$\Delta t \simeq t - t_{\rm TOF}, \tag{71}$$

where $t_{\rm TOF} = L/c$. In the K2K experiment, muon neutrino events which satisfy the criteria

$$-0.2 \leqslant \Delta t \leqslant 1.3\,\mu\text{s}$$

were selected. Notice that in the K2K experiment $L \simeq 250$ km and $L/c \simeq 0.83 \cdot 10^3\,\mu$s.

It follows from (67) and (68) that neutrino oscillations can be observed if at least for one value of i the following inequality is satisfied (see [15, 16])*:

$$\frac{\Delta m^2_{1i}}{2E}\, L \geqslant 1. \tag{72}$$

Comparing (49) and (72) we conclude that in the case of neutrino oscillations time-energy uncertainty relation coincides with the condition of the observation of neutrino oscillations [14].

Let us stress that finite time during which a significant change of the flavor content of neutrino state happens (oscillation time) in accordance with time-energy uncertainty relation requires uncertainty in energy. This corresponds to the violation of the invariance under translation in time in the case of the mixed neutrino states, which we discussed in the previous section.

We will make the following remark. It was stated in some papers (see [42–44]) that neutrino oscillations can take place only if energies of different neutrinos ν_i are equal. This statement is based on the assumption that in experiments on the study of neutrino oscillations only distance L is relevant. The time is not measured and transition probability must be averaged over time.

We do not see any reasons for such assumption. Of course, the time of the traveling of neutrinos from production to detection points is not measured in the solar, atmospheric, and reactor experiments. However, from the K2K experiment,

*It is obvious that inequality (72) is only necessary condition of the observation of neutrino oscillations. For the transition $\nu_l \to \nu_{l'}$ to be observed, corresponding elements of the neutrino mixing matrix have to be not small.

in which this time is measured, we know that traveling time and distance between production and detection points are equal.

We will present another argument against «equal energy» assumption. Let us consider the propagation of neutrino in matter. The evolution equation in the flavor representation has the form

$$i\,\frac{\partial a(t)}{\partial t} = Ha(t). \tag{73}$$

Here

$$H = H_0 + H_I, \tag{74}$$

where H_0 is the free Hamiltonian; and H_I, the effective Hamiltonian of interaction of neutrino with matter.

The free Hamiltonian in the flavor representation is given by

$$(H_0)_{l'l} = \langle \nu_{l'} | H_0 | \nu_l \rangle = \sum_i U_{l'i} E_i U_{li}^*. \tag{75}$$

The refraction indices of flavor neutrinos in matter are determined by amplitudes of elastic neutrino scattering in forward direction and target particles densities. Taking into account $\nu_e - \nu_\mu - \nu_\tau$ universality of NC for the effective Hamiltonian of interaction of neutrino with matter we have [45]

$$(H_I)_{l'l} = \sqrt{2}\,G_F \rho_e \eta_{l'l}, \tag{76}$$

where ρ_e is the electron number density, $\eta_{ee} = 1$, other elements of $\eta_{l'l}$ are equal to zero. Let us stress that H_I is determined by the CC part of the Standard Model amplitude of the $\nu_e e \to \nu_e e$ forward scattering. Neutrino masses in the interaction Hamiltonian do not enter.

If energies of neutrinos with definite masses are equal ($E_i = E$), in this case the free Hamiltonian is unit matrix

$$(H_0)_{l'l} = E\delta_{l'l}. \tag{77}$$

With such free Hamiltonian it will be no matter effect [46] which was observed in solar neutrino experiments [47]. Thus, assumption of «equal energies» is not compatible with data of solar neutrino experiments.

4. COMPARISON OF NEUTRINO OSCILLATIONS WITH $B_d^0 \leftrightarrows \bar{B}_d^0$ OSCILLATIONS

Neutrino oscillations and flavor oscillations of neutral mesons ($K^0 \leftrightarrows \bar{K}^0$, $B_d^0 \leftrightarrows \bar{B}_d^0$, etc.) have the same quantum-mechanical origin*.

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*In fact, the existence of $K^0 \leftrightarrows \bar{K}^0$ oscillations was a major argument for B. Pontecorvo [21] to propose neutrino oscillations in 1957.

We will compare here neutrino oscillations with $B_d \leftrightarrows \bar{B}_d$ oscillations which were studied recently in detail at asymmetric B factories [48].

The states of B_d and $\bar{B}_d$ mesons are eigenstates of the Hamiltonian H_0 which is the sum of the free Hamiltonian and Hamiltonians of the strong and electromagnetic interactions. Assuming CPT invariance of the strong interaction in the rest frame of $B_d^0(\bar{B}_d^0)$ we have

$$H_0|B_d^0\rangle = m_B|B_d^0\rangle; \quad H_0|\bar{B}_d^0\rangle = m_B|\bar{B}_d^0\rangle, \tag{78}$$

where m_B is the mass of $B_d(\bar{B}_d)$ meson.

B_d^0 and $\bar{B}_d^0$ mesons are produced in the decays of $\Upsilon(4S)$ and other strong processes in which quark flavor is conserved. Effects of weak interaction can be neglected in production processes. After B_d^0 and $\bar{B}_d^0$ mesons are produced, weak interaction plays the major role: due to weak interaction particles decay, eigenstates of the total effective Hamiltonian have different masses, etc.

Let us assume that at $t = 0$ $B_d^0(\bar{B}_d^0)$ was produced. At $t > 0$ for the vector of the state we have

$$|\Psi(t)\rangle = \sum_{\alpha=B_d,\bar{B}_d} a_\alpha(t)|\alpha\rangle + \sum_i b_i(t)|i\rangle, \tag{79}$$

where $a_\alpha(t)$ is the wave function of $B_d^0 - \bar{B}_d^0$ system in the flavor representation, and the states $|i\rangle$ describe products of the decay of B_d^0 and $\bar{B}_d^0$. The vector $|\Psi(t)\rangle$ satisfies the Schrödinger equation (42). From (79) and (42) it can be shown (see [49]) that in the Waiskopf–Wigner approximation the function $a(t)$ satisfies the evolution equation

$$i\,\frac{\partial a(t)}{\partial t} = Ha(t). \tag{80}$$

Here H is the total effective non-Hermitian Hamiltonian of $B_d^0 - \bar{B}_d^0$ system in the flavor representation. The Hamiltonian H has the form

$$H = M - \frac{i}{2}\,\Gamma, \tag{81}$$

where $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$ are 2×2 matrices of mass and width.

For the eigenstates of the total Hamiltonian we have

$$Ha_H = \lambda_H a_H; \quad Ha_L = \lambda_L a_L. \tag{82}$$

Here

$$\lambda_H = m_H - \frac{i}{2}\,\Gamma_H; \quad \lambda_L = m_L - \frac{i}{2}\,\Gamma_L, \tag{83}$$

where m_H, m_L and Γ_H, Γ_L are masses and total widths of B_H^0 and B_L^0 mesons.

From (82) for the states of B_H^0 and B_L^0 mesons we have

$$|B_H^0\rangle = N\left(|B_d^0\rangle + \frac{q}{p}\,|\bar{B}_d^0\rangle\right); \quad |B_L^0\rangle = N\left(|B_d^0\rangle - \frac{q}{p}\,|\bar{B}_d^0\rangle\right). \tag{84}$$

Here $N = 1/\sqrt{1+|q/p|^2}$, $q = \sqrt{H_{\bar{B}_d B_d}}$, and $p = \sqrt{H_{B_d \bar{B}_d}}$. The parameter $\frac{q}{p}$ characterizes CP violation of the weak interaction (if CP is conserved $\frac{q}{p} = 1$).

For the states of B_d^0 and $\bar{B}_d^0$ mesons, produced in the decays of $\Upsilon(4S)$ and in other strong processes, from (84) we have

$$|B_d^0\rangle = \frac{1}{2N}\left(|B_H^0\rangle + |B_L^0\rangle\right); \; |\bar{B}_d^0\rangle = \frac{1}{2N}\,\frac{p}{q}\left(|B_H^0\rangle - |B_L^0\rangle\right). \tag{85}$$

Thus, in the strong interaction, *coherent superpositions* of states of B_H^0 and B_L^0 mesons, particles with definite masses and widths, are produced.

Let us compare relations (85) with neutrino relations (31) and (32). Flavor neutrinos and antineutrinos ν_l and $\bar{\nu}_l$ are produced in weak processes in which at neutrino energies many orders of magnitudes larger than neutrino masses the lepton flavor numbers L_e, L_μ, and L_τ are effectively conserved. The states of these neutrinos are coherent superpositions of the states of neutrinos with definite masses. Flavor B_d^0 and $\bar{B}_d^0$ mesons are produced because quark flavor is conserved in strong interaction. Their states are coherent superposition (85).

There exists also an important difference between mixing of states of neutral mesons and mixing of neutrino states. Neutrino mixing is determined by the PMNS mixing matrix. Even in the case of the two neutrinos mixing angle is an arbitrary parameter. In the case of neutral mesons mixing is maximal (independently of the values of CKM mixing angles). This is connected with CPT invariance of the strong interaction and the fact that B_d^0 and $\bar{B}_d^0$ are transformed into each other under CP transformation. CPT invariance does not put any constraints on neutrino mixing angles.

We will consider now time evolution of the states of B_d^0 and $\bar{B}_d^0$ mesons. From (80), (82), and (85) we find

$$|B_d^0\rangle_t = \frac{1}{2N}\left(|B_H^0\rangle\,\mathrm{e}^{-i\lambda_H t} + |\bar{B}_L^0\rangle\,\mathrm{e}^{-i\lambda_L t}\right) \tag{86}$$

and

$$|\bar{B}_d^0\rangle_t = \frac{1}{2N}\,\frac{p}{q}\left(|B_H^0\rangle\,\mathrm{e}^{-i\lambda_H t} - |\bar{B}_L^0\rangle\,\mathrm{e}^{-i\lambda_L t}\right), \tag{87}$$

where t is the proper time.

Neutral B mesons are detected through the observation of the decays of B_d^0 and $\bar{B}_d^0$, which are determined by transitions $\bar{b} \to \bar{c}(\bar{u}) + W^+$ and $b \to c(u) + W^-$. For lepton decays we have

$$B_d^0 \to l^+ + \nu_l + X; \quad \bar{B}_d^0 \to l^- + \bar{\nu}_l + X.$$

Thus, the sign of the charged lepton determines the type of the neutral B meson. From (86) and (87) we find

$$|B_d^0\rangle_t = g_+(t)|B_d^0\rangle + \frac{q}{p}\, g_-(t)|\bar{B}^0\rangle \tag{88}$$

and

$$|\bar{B}_d^0\rangle_t = g_-(t)\,\frac{p}{q}\,|B_d^0\rangle + g_+(t)|\bar{B}^0\rangle, \tag{89}$$

here

$$g_\pm(t) = \frac{1}{2}\,(\mathrm{e}^{-i\lambda_H t} \pm \mathrm{e}^{-i\lambda_L t}). \tag{90}$$

Equations (88) and (89) are analogous to Eqs. (45) and (46) in the neutrino case. For the transition probabilities we find the following expression:

$$P(B_d^0 \to \bar{B}_d^0) = P(\bar{B}_d^0 \to B_d^0) = \frac{1}{2}\,\mathrm{e}^{-\Gamma t}(1 - \cos\Delta m_B t), \tag{91}$$

where

$$\Delta m_B = m_H - m_L \tag{92}$$

is the difference of masses of B_H^0 and B_L^0 mesons. We took into account in (91) that $\Gamma_H \simeq \Gamma_L = \Gamma$ and $|p/q| \simeq 1$ (see [50]).

The probability of B^0 ($\bar{B}^0$) to survive is given by the expression

$$P(B_d^0 \to B_d^0) = P(\bar{B}_d^0 \to \bar{B}_d^0) = \frac{1}{2}\,\mathrm{e}^{-\Gamma t}(1 + \cos\Delta m_B t). \tag{93}$$

Equations (91) and (93) are analogous to equations (67) and (68) in the neutrino case.

The main differences between neutrino oscillations and $B_d^0 \leftrightarrows \bar{B}_d^0$ oscillations are the following:

• In the B-mesons case mixing is maximal. In the neutrino case mixing angles are parameters. Investigation of neutrino oscillations is the only possible source of information about the elements of the PMNS mixing matrix. Modulus of elements of the quark CKM mixing matrix can be determined from the investigation of different weak decays and neutrino reactions. Information about CP phase can be inferred from the measurement of CP-odd asymmetry in $B_d^0(\bar{B}_d^0) \to J/\psi + K_S$ and other decays (see [48]).

• Neutral B mesons are unstable particles. Lifetime is given by $\simeq 1/\Gamma$ and time-energy uncertainty relation

$$\Delta m_B\,\frac{1}{\Gamma} \geqslant 1 \tag{94}$$

is the condition of the observation of $B_d^0 \leftrightarrows \bar{B}_d^0$ oscillations.

Neutrinos are stable particles. The time $t \simeq L$ in the time-energy uncertainty relation (49) is determined by experimental conditions (intensity of a neutrino beam, size of a detector, background conditions, etc.).

The formalism of $B_d^0 \leftrightarrows \bar{B}_d^0$ oscillations, which we shortly discussed here (and similar formalism of $K^0 \leftrightarrows \bar{K}^0$ oscillations) is based on the evolution equation (80), mixed flavor states, nonstationary states $|B_d^0\rangle_t$ and $|\bar{B}_d^0\rangle_t$, given by (88) and (89). This formalism describes a lot of precise experimental data. From our point of view correct formalism of neutrino oscillations also must be based on the Schrödinger evolution equation, mixed flavor neutrino states and time-energy uncertainty relation.

CONCLUSION

Observation of neutrino oscillations in SK, SNO, KamLAND and other neutrino experiments [1–8] is an important recent discovery in particle physics. Investigation of this new phenomenon allowed one to determine such fundamental parameters as neutrino mass squared differences and neutrino mixing angles. It is a common opinion that generation of neutrino masses, which are many orders of magnitude smaller than masses of leptons and quarks, requires a new physics and a new beyond the Standard Model mechanism of the mass generation.

Taking into account importance of neutrino oscillations for particle physics we think that the understanding of the physical basis of this new phenomenon is an important issue. In literature there exist different interpretations of the basics of neutrino oscillations (see [9, 42]). We present here the following point of view:

• Neutrino masses are many orders of magnitude smaller than energies of neutrinos in reactor, solar, atmospheric and accelerator neutrino experiments. This means that neutrino masses can be neglected in matrix elements of neutrino-production and neutrino-detection processes. Therefore in CC weak processes together with l^+ (l^-) flavor left-handed neutrinos ν_l (flavor right-handed antineutrinos $\bar{\nu}_l$) are produced which states are described by coherent superpositions of states of neutrinos with definite masses. The possibility of neglecting neutrino masses in production and detection processes signifies that in these processes flavor lepton numbers are effectively conserved.

• Flavor neutrinos and antineutrinos at time t after production are described by nonstationary states (coherent superposition of stationary states). For such states, time-energy uncertainty relation is valid. This means that neutrino oscillations with finite oscillation time require uncertainty of energy.

• The presented here nonstationary formalism of neutrino oscillations is the same as the well-known formalism of $K^0 \leftrightarrows \bar{K}^0$, $B_d^0 \leftrightarrows \bar{B}_d^0$, etc., oscillations, which is confirmed by numerous high-precision experiments.

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