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THREE-LOOP RUNNING OF THE STRONG COUPLING CONSTANT AND THE MASS OF THE b QUARK IN THE SUPERSYMMETRIC QCD

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Supersymmetric extension of the QCD (SQCD) is considered, and the scale dependence of some important parameters, i.e., strong coupling α_s and b-quark running mass m_b, is studied with the help of three-loop renormalization group equations. Five-flavour QCD is considered as a low-energy effective theory of SQCD, and two-loop decoupling (threshold) effects are taken into account. Variations in running α_s and m_b at the GUT scale $\mu_{\rm GUT} = 10^{16}$ GeV due to uncertainties in the initial experimental data and the ambiguities in the decoupling scale are analyzed for a particular point of the parameter space. Comparison with known results is given.

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INTRODUCTION

Minimal supersymmetric extension of the Standard Model (so-called MSSM) predicts a lot of superparticles that have the same quantum numbers (except spin) as ordinary particles known from experiment. Among these particles one can distinguish color scalars (squarks) and fermions (gluino). Neglecting all the electroweak interactions, the dynamics of these new particles, together with that of quarks and gluons, can be described by supersymmetric QCD.

As in the MSSM, the predictive power of SQCD is spoiled by unknown supersymmetry-breaking parameters that define mass splitting between ordinary particles and corresponding superpartners. In order to deal with the problem, one usually looks for constraints, both experimental and theoretical, that allow one to reduce the number of independent model parameters.

By means of so-called decoupling procedure, one can express running $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [1] and $m_b(m_b) = (4.164 \pm 0.025)$ GeV [2] defined in $\overline{\rm MS}$-scheme in five-flavour QCD in terms of SQCD parameters defined in $\overline{\rm DR}$-scheme [3] and use corresponding known values as constraints on the whole set of SQCD parameters.

Among theoretical restrictions on possible models, one can consider those that are related to coupling constant unification at high energies, e.g., to gauge or Yukawa unification. For imposing such constraints, one needs to know the scale dependence of corresponding couplings that is given by renormalization group equations (RGEs) and boundary conditions for these equations at low energies. It is the role of decoupling constants to define such a boundary condition at some renormalization (=decoupling) scale (see Eq. (3) below).

In formal perturbation theory it can be proven that L-loop running requires implementation of $(L-1)$-loop matching (see, e.g., [4]). Since in this talk I consider three-loop running based on RGEs given in [5,6], two-loop decoupling relations obtained in [7,8] are used. Some details on running procedure are given in the next section.

1. RUNNING STRATEGY

In order to obtain the values of $\alpha_s(\mu_{\rm GUT})$ and $m_b(\mu_{\rm GUT})$ with three-loop precision, I made use of the following strategy.

First of all, by three-loop RGE defined in the effective five-flavour QCD

$$\frac{d\alpha_s}{dt} = -\alpha_s \left[\frac{23}{3}\left(\frac{\alpha_s}{4\pi}\right) + \frac{116}{3}\left(\frac{\alpha_s}{4\pi}\right)^2 + \frac{9769}{54}\left(\frac{\alpha_s}{4\pi}\right)^3 \right], \quad t = \ln\mu^2 \qquad (1)$$

and a given experimental value of $\alpha_s(M_Z)$, one obtains $\alpha_s(\mu_b)$ at the scale μ_b at which b-quark mass is defined $\mu_b = m_b(\mu_b) = (4.164 \pm 0.025)$ GeV [2]. Then with the calculated value of $\alpha_s(\mu_b)$ and known $m_b(\mu_b)$, three-loop RGEs

$$\frac{dm_b}{dt} = -m_b \left[4\left(\frac{\alpha_s}{4\pi}\right) + \frac{506}{9}\left(\frac{\alpha_s}{4\pi}\right)^2 + \left(\frac{64429}{81} - \frac{800}{3}\zeta(3)\right)\left(\frac{\alpha_s}{4\pi}\right)^3 \right] \qquad (2)$$

and (1) are solved numerically to find the values of running $\overline{\rm MS}$-parameters $\alpha_s(\mu_{\rm dec})$ and $m_b(\mu_{\rm dec})$ at chosen decoupling scale $\mu_{\rm dec} \sim 1$ TeV. After that one applies the decoupling relations

$$\begin{aligned} \alpha_s^{\overline{\rm DR}}(\mu_{\rm dec}) &= \alpha_s^{\overline{\rm MS}}(\mu_{\rm dec}) \times \tilde{\zeta}^{(2)}_{\alpha_s}\left(\alpha_s^{\overline{\rm MS}}, M_t, M_{\rm SUSY}, \mu_{\rm dec}\right), \\ m_b^{\overline{\rm DR}}(\mu_{\rm dec}) &= m_b^{\overline{\rm MS}}(\mu_{\rm dec}) \times \tilde{\zeta}^{(2)}_{m_b}\left(\alpha_s^{\overline{\rm MS}}, M_t, M_{\rm SUSY}, \mu_{\rm dec}\right) \end{aligned} \qquad (3)$$

to convert $\overline{\rm MS}$ QCD running parameters at $\mu_{\rm dec}$ to that of SQCD defined in $\overline{\rm DR}$-scheme*. Here M_t corresponds to t-quark pole mass and $M_{\rm SUSY}$ collectively denotes (unknown) values of superparticle pole masses. The latter are fixed if one considers a particular MSSM scenario. The full expressions for two-loop

*Change in renormalization scheme $\overline{\rm DR} \to \overline{\rm MS}$ can also be treated as decoupling of so-called ε-scalars [7].

decoupling constants $\tilde{\zeta}^{(2)}$ are huge and can be used only in numerical calculations. However, in the limit when all the particles heavier than the b quark have the same mass M, they are given by the following expressions with $\mu \equiv \mu_{\rm dec}$ and $a_{b,t}$ being off-diagonal mixing matrix elements for bottom and top squarks expressed in terms of soft trilinear couplings $A_{b,t}$, the ratio of Higgs vacuum expectation value $\tan\beta$, and Higgs superfield mixing parameter $\bar{\mu}$:

$$\begin{aligned}
\zeta_{\alpha_s}^{(2)} &= 1 - \frac{\alpha_s}{4\pi}\left(1 - \frac{14}{3}\log\frac{M}{\mu}\right)[S_2 = 4/(9\sqrt{3})\mathrm{Cl}_2(\pi/3)] + \\
&+ \frac{\alpha_s^2}{(4\pi)^2}\left(-\frac{2179}{54} - \frac{44\pi}{9\sqrt{3}} + \frac{a_t}{M}\left(\frac{64}{81} + \frac{184\pi}{27\sqrt{3}} - \frac{380}{9}S_2\right) - \right. \\
&\left. - \frac{68}{3}S_2 + \frac{130}{3}\log\frac{M}{\mu} + \frac{196}{9}\log^2\frac{M}{\mu}\right), \\
\zeta_{m_b}^{(2)} &= 1 + \frac{\alpha_s}{3\pi}\left(1 - \frac{a_b}{M} + \log\frac{M}{\mu}\right) + \\
&+ \frac{\alpha_s^2}{(4\pi)^2}\left(\frac{61}{6} - \frac{8\pi}{9\sqrt{3}} + \frac{10}{3}S_2 - \frac{a_t}{M}\left(\frac{80}{81} - \frac{16\pi}{27\sqrt{3}} - \frac{16}{9}S_2\right) - \right. \\
&- \frac{a_b}{M}\left(\frac{512}{27} - \frac{4\pi}{9\sqrt{3}} + \frac{32}{3}S_2\right) + \frac{a_b}{M}\frac{a_t}{M}\left(\frac{88}{81} - \frac{8\pi}{27\sqrt{3}} - \frac{32}{9}S_2\right) + \\
&\left. + \left(\frac{442}{9} + \frac{20}{9}\frac{a_b}{M}\right)\log\frac{M}{\mu} + \frac{74}{9}\log^2\frac{M}{\mu}\right),
\end{aligned} \tag{4}$$

$$a_{b,t} = A_{b_t} - \bar{\mu}\{\tan\beta, \cot\beta\}. \tag{5}$$

Finally, within SQCD

$$\begin{aligned}
\frac{d\alpha_s}{dt} &= -\alpha_s\left[3\left(\frac{\alpha_s}{4\pi}\right) - 14\left(\frac{\alpha_s}{4\pi}\right)^2 - \frac{347}{3}\left(\frac{\alpha_s}{4\pi}\right)^3\right], \quad t = \ln\mu^2, \\
\frac{dm_b}{dt} &= -m_b\left[\frac{8}{3}\left(\frac{\alpha_s}{4\pi}\right) + \frac{8}{9}\left(\frac{\alpha_s}{4\pi}\right)^2 - \left(\frac{2720}{27} + 320\zeta(3)\right)\left(\frac{\alpha_s}{4\pi}\right)^3\right]
\end{aligned} \tag{6}$$

are solved to obtain $\alpha_s(\mu_{\rm GUT})$ and $m_b(\mu_{\rm GUT})$ at GUT scale.

2. RESULTS

Here I present some results obtained with the help of a C++ program* that implements the mentioned strategy to evaluate running α_s and m_b at any scale within the supersymmetric QCD. It is worth mentioning that the program can

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*It is available from the author.

use either full two-loop decoupling corrections that depend on the pole masses of superparticles and squark off-diagonal terms (5) or simple threshold effects with common scale M in a form of Eq. (4).

For definiteness I use so-called SPS1a point in mSUGRA parameter space [9] that can be characterized by the following set of parameters: $M_{\tilde{t}_1} = 366.5$ GeV, $M_{\tilde{t}_2} = 575.5$ GeV, $M_{\tilde{b}_1} = 506.3$ GeV, $M_{\tilde{b}_2} = 545.7$ GeV, $M_{\tilde{g}} = 607.1$ GeV. Since all the squarks of first two generations have the same mass within considered approximation, it is chosen to be equal to $M_{\rm SUSY} = 462$ GeV. There is a peculiarity in the chosen renormalization prescription for a_q $(q = t, b)$ (for SPS1, $a_t = -961$ GeV, $a_b = -425$ GeV have been chosen). They do not depend on renormalization scale and are related to bare quantities by the following formula:

$$a_{q,\rm bare} = a_q + \left(\frac{\alpha_s}{4\pi}\right)\frac{C_F}{8}M_{\tilde{g}}\left[\frac{2}{\varepsilon} + [B_0(M_{\tilde{q}_1}, M_q, M_{\tilde{g}}) + (1 \to 2)]\right]. \quad (7)$$

Here $C_F = 4/3$, ε is a regularization parameter that is related to space-time dimension $d = 4 - 2\varepsilon$, and $B_0(p, m_1, m_2)$ is a Passarino–Veltman master integral for two-point function that implicitly depends on the renormalization scale.

This set of parameters allows me to make a comparison with known results of [10]. A good agreement (see the table) was found in spite of the fact that different treatment of squark mixing had been employed. From the table it is clear that theoretical ambiguities due to variation of the decoupling scale in the range from 100 GeV to 1 TeV are comparable with the uncertainties due to current error in low-energy input parameters.

In Figures 1 and 2 the dependence on the decoupling scale $\mu_{\rm dec}$ is presented for $\alpha_s(\mu_{\rm GUT})$ and $m_b(\mu_{\rm GUT})$ at GUT scale $\mu_{\rm GUT} = 10^{16}$ GeV. One can see that the three-loop result that is based on the three-loop RGEs and two-loop decoupling relations has reduced $\mu_{\rm dec}$ dependence in comparison with one- and two-loop results.

A comparison of the obtained values of α_s and m_b at GUT scale $\mu_{\rm GUT} = 10^{16}$ GeV for SPS1a point. Decoupling scale is $\mu_{\rm dec} = 600$ GeV. Variations in the result due to uncertainties in the initial data $\delta\alpha_s = \pm 0.0007$ and $\delta m_b = \pm 0.025$ GeV are shown together with ambiguity due to decoupling scale $\mu_{\rm dec}$ variation from 100 GeV to 1 TeV

Bauer, Mihaila, Salomon [10]				
$\alpha_s(\mu_{\rm GUT})$	0.0405	$\pm 0.0001\|_{\delta\,\alpha_s(M_Z)}$		$\pm 0.0001\|_{\rm th}$
$m_b(\mu_{\rm GUT})$	1.016	$\pm 0.011\|_{\delta\,\alpha_s(M_Z)}$	$\pm 0.007\|_{\delta\,m_b(m_b)}$	$\pm 0.005\|_{\rm th}$
Bednyakov				
$\alpha_s(\mu_{\rm GUT})$	0.0405	$\pm 0.0001\|_{\delta\,\alpha_s(M_Z)}$		$\pm 0.0002\|_{\rm th}$
$m_b(\mu_{\rm GUT})$	1.016	$\pm 0.008\|_{\delta\,\alpha_s(M_Z)}$	$\pm 0.013\|_{\delta\,m_b(m_b)}$	$\pm 0.002\|_{\rm th}$

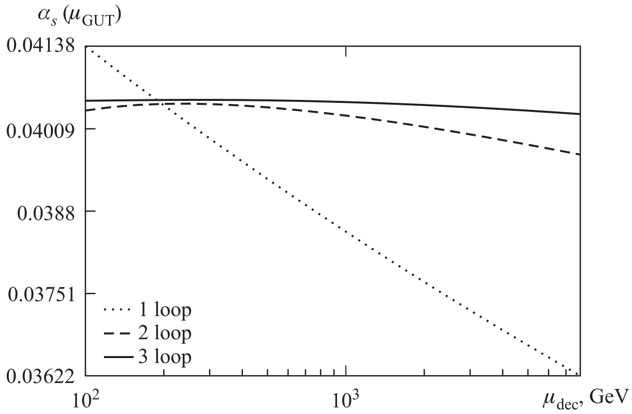

Fig. 1. The dependence of running $\alpha_s(\mu_{\rm GUT})$ at the GUT scale $\mu_{\rm GUT} = 10^{16}$ GeV on decoupling scale $\mu_{\rm dec}$ for SPS1a point [9]. Dotted, dashed, and solid lines indicate that one-, two-, and three-loop RGEs, together with appropriate matching conditions, are used during the calculation of $\alpha_s(\mu_{\rm GUT})$

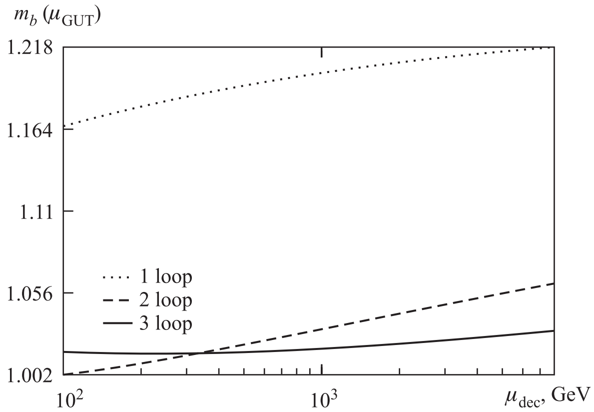

Fig. 2. The dependence of running $m_b(\mu_{\rm GUT})$ at GUT scale $\mu_{\rm GUT} = 10^{16}$ GeV on the decoupling scale $\mu_{\rm dec}$ for SPS1a point

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