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THE VACUUM STRUCTURE AND SPECIAL RELATIVITY REVISITED: A FIELD THEORY NO-GEOMETRY APPROACH WITHIN THE LAGRANGIAN AND HAMILTONIAN FORMALISMS

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The work is devoted to studying the vacuum structure, special relativity, electrodynamics of interacting charged point particles and quantum mechanics, and is a continuation of [6, 7]. Based on the vacuum field theory no-geometry approach, the Lagrangian and Hamiltonian reformulation of some alternative classical electrodynamics models is devised. The Dirac-type quantization procedure, based on the canonical Hamiltonian formulation, is developed for some alternative electrodynamics models. Within an approach developed a possibility of the combined description both of electrodynamics and gravity is analyzed.

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1. INTRODUCTION

The classical relativistic electrodynamics of a freely moving charged point particle in the Minkowski space-time $\mathbb{M}^4 := \mathbb{E}^3 \times \mathbb{R}$ is, as well known, based [5, 9, 11, 31] on the Lagrangian formalism assigning to it the following Lagrangian function:

$$\mathcal{L} := -m_0(1 - u^2)^{1/2}, \tag{1.1}$$

where $m_0 \in \mathbb{R}$ is the so-called particle rest mass and $u \in \mathbb{E}^3$ is its spatial velocity in the Euclidean space $\mathbb{E}^3$, expressed here and throughout further in the light

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speed units (that is the light speed c units). The least action Fermat principle in the form

$$\delta S = 0, \quad S := -\int_{t_1}^{t_2} m_0(1-u^2)^{1/2} dt \tag{1.2}$$

for any fixed temporal interval $[t_1, t_2] \subset \mathbb{R}$ gives rise to the well-known relativistic relationships for the mass of the particle

$$m = m_0(1-u^2)^{-1/2}, \tag{1.3}$$

the momentum of the particle

$$p := mu = m_0 u(1-u^2)^{-1/2} \tag{1.4}$$

and the energy of the particle

$$\mathcal{E}_0 = m = m_0(1-u^2)^{-1/2}. \tag{1.5}$$

The origin of Lagrangian (1.1), owing to the reasonings from [11, 31], can be extracted from the action expression

$$S := -\int_{t_1}^{t_2} m_0(1-u^2)^{1/2} dt = -\int_{\tau_1}^{\tau_2} m_0\, d\tau, \tag{1.6}$$

on the suitable temporal interval $[\tau_1, \tau_2] \subset \mathbb{R}$, where, by definition,

$$d\tau := dt(1-u^2)^{1/2} \tag{1.7}$$

and $\tau \in \mathbb{R}$ is the so-called proper temporal parameter assigned to a freely moving particle with respect to the «rest» reference system $\mathcal{K}_r$. The action (1.6) looks from the dynamical point of view sightly controversial, since it is physically defined with respect to the «rest» reference system $\mathcal{K}_r$, giving rise to the constant action $S = -m_0(\tau_2 - \tau_1)$, as limits of integrations $\tau_1 < \tau_2 \in \mathbb{R}$ were taken to be fixed from the very beginning. Moreover, considering this particle as charged with a charge $q \in \mathbb{R}$ and moving in the Minkowski space-time $\mathbb{M}^4$ under action of an electromagnetic field $(\varphi, A) \in \mathbb{R} \times \mathbb{E}^3$, the corresponding classical (relativistic) action functional is chosen (see [5, 9, 11, 31]) as follows:

$$S := \int_{\tau_1}^{\tau_2} [-m_0 d\tau + q\langle A, \dot{r}\rangle d\tau - q\varphi(1-u^2)^{-1/2} d\tau], \tag{1.8}$$

with respect to the so-called «rest» reference system, parameterized by the Euclidean space-time variables $(r, \tau) \in \mathbb{E}^4$, where as before, $\langle \cdot, \cdot \rangle$ is the standard scalar product in the related Euclidean subspace $\mathbb{E}^3$ and there is denoted $\dot{r} := dr/d\tau$ in contrast to the definition $u := dr/dt$. The action (1.8) can be rewritten, with respect to the moving with velocity vector $u \in \mathbb{E}^3$ reference system, as

$$S = \int_{t_1}^{t_2} \mathcal{L} dt, \quad \mathcal{L} := -m_0(1-u^2)^{1/2} + q\langle A, u\rangle - q\varphi, \tag{1.9}$$

on the suitable temporal interval $[t_1, t_2] \subset \mathbb{R}$, giving rise to the following [5, 9, 11, 31] dynamical expressions:

$$P = p + qA, \quad p = mu, \tag{1.10}$$

for the particle momentum and

$$\mathcal{E}_0 = [m_0^2 + (P - qA)^2]^{1/2} + q\varphi \tag{1.11}$$

for the particle energy, where, by definition, $P \in \mathbb{E}^3$ means the common momentum of the particle and the ambient electromagnetic field at a space-time point $(r, t) \in \mathbb{M}^4$.

The obtained expression (1.11) for the particle energy $\mathcal{E}_0$ also looks slightly controversial, since the potential energy $q\varphi$, entering additively, has no impact onto the particle mass $m = m_0(1-u^2)^{-1/2}$. As it was already mentioned [14] by L. Brillouin, the fact that the potential energy has no impact on the particle mass says us that «... any possibility of existing the particle mass related with an external potential energy, is completely excluded». This and some other special relativity theory and electrodynamics problems, as is well known, stimulated many other prominent physicists of the past [4, 14, 19, 31, 33] and the present [18, 20–24, 27–30, 32, 34–37, 40, 41] to make significant efforts aiming to develop alternative relativity theories based on completely different space-time and matter structure principles.

There is also another controversial inference from the action expression (1.9). As one can easily show [5, 9, 11, 31], the corresponding dynamical equation for the Lorentz force is given as follows:

$$\frac{dp}{dt} = F := qE + qu \times B, \tag{1.12}$$

where the operation «$\times$» denotes, as before, the standard vector product and we put, by definition,

$$E := -\frac{\partial A}{\partial t} - \nabla\varphi \tag{1.13}$$

for the related electric field and

$$B := \nabla \times A \tag{1.14}$$

for the related magnetic field, acting on the charged point particle q; the operation «∇» here is, as before, the standard gradient. The obtained expression (1.12) means, in particular, that the Lorentz force F depends linearly on the particle velocity vector $u \in \mathbb{E}^3$, giving rise to its strong dependence on the reference system with respect to which the charged particle q moves. Namely, the attempts to reconcile this and some related controversies [4, 14, 18, 26] forced A. Einstein to devise his special relativity theory and proceed further to creating his general relativity theory trying to explain the gravity by means of a geometrization of space-time and matter in the Universe. Here we must mention that the classical Lagrangian function $\mathcal{L}$ in (1.9) is written by means of the mixed combinations of terms expressed by means of both the Euclidean «rest» reference system variables $(r, \tau) \in \mathbb{E}^4$ and the arbitrarily chosen reference system variables $(r, t) \in \mathbb{M}^4$.

These problems were recently analyzed from a completely another «no-geometry» point of view in [6, 7, 18], where new dynamical equations were derived, being free of controversy mentioned above. Moreover, the devised approach allowed one to avoid the introduction of the well-known Lorentz transformations of the space-time reference systems with respect to which the action functional (1.9) is invariant. From this point of view there are very interesting reasonings of work [22], in which there are reanalyzed Galilean invariant Lagrangians, possessing the intrinsic Poincare–Lorentz group symmetry. Below we will reanalyzed the results obtained in [6, 7] from the classical Lagrangian and Hamiltonian formalisms, what will shed a new light on the related physical backgrounds of the vacuum field theory approach to common studying electromagnetic and gravitational effects.

2. THE VACUUM FIELD THEORY ELECTRODYNAMICS EQUATIONS: LAGRANGIAN ANALYSIS

2.1. A Freely Moving Point Particle — an Alternative Electrodynamical Model. Within the vacuum field theory approach to common describing the electromagnetism and gravity, devised in [6, 7], the main vacuum potential field function $\bar{W} : \mathbb{M}^4 \to \mathbb{R}$, related to a charged point particle q, satisfies in the case of the rested external charged point objects the following [6] dynamical equation:

$$\frac{d}{dt}(-\bar{W}u) = -\nabla \bar{W}, \tag{2.1}$$

where, as above, $u := dr/dt$ is the particle velocity with respect to some reference system.

To analyze the dynamical equation (2.1) from the Lagrangian point of view, we will write the corresponding action functional as

$$S := -\int_{t_1}^{t_2} \bar{W} dt = -\int_{\tau_1}^{\tau_2} \bar{W}(1+\dot{r}^2)^{1/2}\, d\tau, \tag{2.2}$$

expressed with respect to the «rest» reference system $\mathcal{K}_r$. Having fixed proper temporal parameters $\tau_1 < \tau_2 \in \mathbb{R}$, from the least action condition $\delta S = 0$ one easily finds that

$$\begin{aligned} p &:= \frac{\partial \mathcal{L}}{\partial \dot{r}} = -\bar{W}\dot{r}(1+\dot{r}^2)^{-1/2} = -\bar{W}u, \\ \dot{p} &:= \frac{dp}{d\tau} = \frac{\partial \mathcal{L}}{\partial r} = -\nabla \bar{W}(1+\dot{r}^2)^{1/2}, \end{aligned} \tag{2.3}$$

where, owing to (2.2), the corresponding Lagrangian function

$$\mathcal{L} := -\bar{W}(1+\dot{r}^2)^{1/2}. \tag{2.4}$$

Recalling now the definition of the particle mass

$$m := -\bar{W} \tag{2.5}$$

and the relationships

$$d\tau = dt(1-u^2)^{1/2}, \quad \dot{r}\, d\tau = u\, dt, \tag{2.6}$$

from (2.3) we easily obtain exactly the dynamical equation (2.1). Moreover, one easily obtains that the dynamical mass, defined by means of expression (2.5), is given as

$$m = m_0(1-u^2)^{-1/2},$$

coinciding with result (1.3) of the preceding section. Thereby, based on the above-obtained results, one can formulate the following proposition.

Proposition 2.1. *The alternative freely moving point particle electrodynamical model (2.1) allows the least action formulation (2.2) with respect to the «rest» reference system variables, where the Lagrangian function is given by expression (2.4). Its electrodynamics is completely equivalent to that of a classical relativistic freely moving point particle, described in Sec. 2.*

2.2. A Moving Charged Point Particle — an Alternative Electrodynamical Model. Proceed now to the case when our charged point particle q moves in the space-time with velocity vector $u \in \mathbb{E}^3$ and interacts with another external charged point particle, moving with velocity vector $u_f \in \mathbb{E}^3$ subject to some common reference system $\mathcal{K}$. As was shown in [6,7], the corresponding dynamical equation on the vacuum potential field function $\bar{W} : \mathbb{M}^4 \rightarrow \mathbb{R}$ is given as

$$\frac{d}{dt}[-\bar{W}(u-u_f)] = -\nabla \bar{W}. \tag{2.7}$$

As the external charged particle moves in the space-time, it generates the related magnetic field $B := \nabla \times A$, whose magnetic vector potential $A : \mathbb{M}^4 \rightarrow \mathbb{E}^3$ is defined, owing to the results of [6,7,18], as

$$qA := \bar{W} u_f. \tag{2.8}$$

Since, owing to (2.3), the particle momentum $p = -\bar{W}u$, Eq. (2.7) can be equivalently rewritten as

$$\frac{d}{dt}(p + qA) = -\nabla \bar{W}. \tag{2.9}$$

To represent the dynamical equation (2.9) within the classical Lagrangian formalism, we start from the following action functional naturally generalizing functional (2.2):

$$S := -\int_{\tau_1}^{\tau_2} \bar{W}(1 + |\dot{r} - \dot{\xi}|^2)^{1/2}\, d\tau, \tag{2.10}$$

where we denoted by $\dot{\xi} = u_f dt/d\tau$, $d\tau = dt(1 - |u-u_f|^2)^{1/2}$, which take into account the relative velocity of our charged point particle q with respect to the reference system $\mathcal{K}'$, moving with velocity vector $u_f \in \mathbb{E}^3$ subject to the reference system $\mathcal{K}$. In this case, evidently, our charged point particle q moves with velocity vector $u - u_f \in \mathbb{E}^3$ subject to the reference system $\mathcal{K}'$, and the external charged particle is, respectively, in rest.

Compute now the least action variational condition $\delta S = 0$, taking into account that, owing to (2.10), the corresponding Lagrangian function is given as

$$\mathcal{L} := -\bar{W}(1 + |\dot{r} - \dot{\xi}|^2)^{1/2}. \tag{2.11}$$

Thereby, the common particles momentum

$$\begin{aligned} P := \frac{\partial \mathcal{L}}{\partial \dot{r}} &= -\bar{W}(\dot{r} - \dot{\xi})(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} = \\ &= -\bar{W}\dot{r}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} + \bar{W}\dot{\xi}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} = \\ &= mu + qA := p + qA \end{aligned} \tag{2.12}$$

and the dynamical equation is given as

$$\frac{d}{d\tau}(p+qA) = -\nabla\bar{W}(1+|\dot{r}-\dot{\xi}|^2)^{1/2}. \tag{2.13}$$

Taking into account that $d\tau = dt(1-|u-u_f|^2)^{1/2}$ and $(1+|\dot{r}-\dot{\xi}|^2)^{1/2} = (1-|u-u_f|^2)^{-1/2}$, we obtain finally from (2.13) exactly the dynamical equation (2.9). Thus, we can formulate our result as the next proposition.

Proposition 2.2. *The alternative classical relativistic electrodynamical model (2.7) allows the least action formulation (2.10) with respect to the «rest» reference system variables, where the Lagrangian function is given by expression (2.11).*

2.3. A Moving Charged Point Particle — a Dual to the Classical Alternative Electrodynamical Model. It is easy to observe that the action functional (2.10) is written taking into account the classical Galilean transformations of reference systems. If we now consider the action functional (2.2) for a charged point particle, moving with respect to the reference system $\mathcal{K}_r$, and take into account its interaction with an external magnetic field, generated by the vector potential $A:\mathbb{M}^4\to\mathbb{E}^3$, it can be naturally generalized as

$$S := \int_{t_1}^{t_2}(-\bar{W}dt + q\langle A, dr\rangle) = \int_{\tau_1}^{\tau_2}[-\bar{W}(1+\dot{r}^2)^{1/2} + q\langle A,\dot{r}\rangle]d\tau, \tag{2.14}$$

where we accepted that $d\tau = dt(1-u^2)^{1/2}$.

Thus, the corresponding common particle-field momentum looks as follows:

$$P := \frac{\partial\mathcal{L}}{\partial\dot{r}} = -\bar{W}\dot{r}(1+\dot{r}^2)^{-1/2} + qA = mu + qA := p + qA, \tag{2.15}$$

satisfying the equation

$$\begin{aligned}\dot{P} := \frac{dP}{d\tau} = \frac{\partial\mathcal{L}}{\partial r} = -\nabla\bar{W}(1+\dot{r}^2)^{1/2} + q\nabla\langle A,\dot{r}\rangle = \\ = -\nabla\bar{W}(1-u^2)^{-1/2} + q\nabla\langle A,u\rangle(1-u^2)^{-1/2},\end{aligned} \tag{2.16}$$

where

$$\mathcal{L} := -\bar{W}(1+\dot{r}^2)^{1/2} + q\langle A,\dot{r}\rangle \tag{2.17}$$

is the corresponding Lagrangian function. Taking now into account that $d\tau = dt(1-u^2)^{1/2}$, one easily finds from (2.16) that

$$\frac{dP}{dt} = -\nabla\bar{W} + q\nabla\langle A,u\rangle. \tag{2.18}$$

Upon substituting (2.15) into (2.18) and making use of the well-known [11] identity

$$\nabla\langle a,b\rangle = \langle a,\nabla\rangle b + \langle b,\nabla\rangle a + b\times(\nabla\times a) + a\times(\nabla\times b), \tag{2.19}$$

where $a, b \in \mathbb{E}^3$ are arbitrary vector functions, we obtain finally the classical expression for the Lorentz force F, acting on the moving charged point particle q:

$$\frac{dp}{dt} := F = qE + qu\times B, \tag{2.20}$$

where, by definition,

$$E := -\nabla\bar{W}q^{-1} - \frac{\partial A}{\partial t} \tag{2.21}$$

is the corresponding electric field and

$$B := \nabla\times A \tag{2.22}$$

is the corresponding magnetic field.

The result obtained we formulate as the next proposition.

Proposition 2.3. *The classical relativistic Lorentz force (2.20) allows the least action formulation (2.14) with respect to the «rest» reference system variables, where Lagrangian function is given by expression (2.17). Its electrodynamics, described by the Lorentz force (2.20) is completely equivalent to the classical relativistic moving point particle electrodynamics, described by means of the Lorentz force (1.12) in Sec. 2.*

Concerning the previously obtained dynamical equation (2.13), we can easily observe that it can be equivalently rewritten as follows:

$$\frac{dp}{dt} = \left(-\nabla\bar{W} - \frac{qdA}{dt} + q\nabla\langle A,u\rangle\right) - q\nabla\langle A,u\rangle. \tag{2.23}$$

The latter, owing to (2.18) and (2.20), takes finally the following Lorentz type force in the form:

$$\frac{dp}{dt} = qE + qu\times B - q\nabla\langle A,u\rangle, \tag{2.24}$$

before found in [6, 7, 18].

Expressions (2.20) and (2.24) are equal to each other up to the gradient term $F_c := -q\nabla\langle A,u\rangle$, which allows one to reconcile the Lorentz forces acting on a charged moving particle q with respect to different reference systems. This fact is important for our vacuum field theory approach since it needs to use no special geometry and makes it possible to analyze both electromagnetic and gravitational fields simultaneously, based on a new definition of the dynamical mass by means of expression (2.5).

3. THE VACUUM FIELD THEORY ELECTRODYNAMICS EQUATIONS: HAMILTONIAN ANALYSIS

It is well known [1, 2, 8, 9, 17] that any Lagrangian theory allows the equivalent canonical Hamiltonian representation via the classical Legendrian transformation. As we have already formulated above our vacuum field theory of a moving charged particle q in the Lagrangian form, we proceed now to its Hamiltonian analysis making use of the action functionals (2.2), (2.11) and (2.14).

Take, first, the Lagrangian function (2.4) and the momentum expression (2.3) for defining the corresponding Hamiltonian function

$$H := \langle p, \dot r\rangle - \mathcal{L} = -\langle p, p\rangle \bar W^{-1}\left(1-\frac{p^2}{\bar W^2}\right)^{-1/2} + \bar W\left(1-\frac{p^2}{\bar W^2}\right)^{-1/2} = \\ = -p^2\bar W^{-1}\left(1-\frac{p^2}{\bar W^2}\right)^{-1/2} + \bar W^2\bar W^{-1}\left(1-\frac{p^2}{\bar W^2}\right)^{-1/2} = \\ = -(\bar W^2-p^2)(\bar W^2-p^2)^{-1/2} = -(\bar W^2-p^2)^{1/2}. \quad (3.1)$$

As a result, we easily obtain [1, 2, 8, 9] that the Hamiltonian function (3.1) is a conservation law of the dynamical field equation (2.1), that is for all $\tau, t \in \mathbb{R}$

$$\frac{dH}{dt} = 0 = \frac{dH}{d\tau}, \quad (3.2)$$

which naturally allows one to interpret it as the energy expression. Thus, we can write that the particle energy

$$\mathcal{E} = (\bar W^2 - p^2)^{1/2}. \quad (3.3)$$

The suitable Hamiltonian equations, equivalent to the vacuum field equation (2.1), look as follows:

$$\begin{aligned} \dot r &:= \frac{dr}{d\tau} = \frac{\partial H}{\partial p} = p(\bar W^2 - p^2)^{-1/2}, \\ \dot p &:= \frac{dp}{d\tau} = -\frac{\partial H}{\partial r} = \bar W\nabla\bar W(\bar W^2-p^2)^{-1/2}. \end{aligned} \quad (3.4)$$

Thereby, based on the above-obtained results, one can formulate the following proposition.

Proposition 3.1. *The alternative freely moving point particle electrodynamical model (2.1) allows the canonical Hamiltonian formulation (3.4) with respect to the «rest» reference system variables, where the Hamiltonian function is given by expression (3.1). Its electrodynamics is completely equivalent to the classical relativistic freely moving point particle electrodynamics, described in Sec. 2.*

Based now on the Lagrangian expression (2.11) one can construct, the same way as above, the Hamiltonian function for the dynamical field equation (2.9), describing the motion of charged particle q in external electromagnetic field in the canonical Hamiltonian form:

$$\dot{r} := \frac{dr}{d\tau} = \frac{\partial H}{\partial P}, \quad \dot{P} := \frac{dP}{d\tau} = -\frac{\partial H}{\partial r}, \tag{3.5}$$

where

$$\begin{aligned} H &:= \langle P, \dot{r}\rangle - \mathcal{L} = \\ &= \left\langle P, \dot{\xi} - P\bar{W}^{-1}\left(1 - \frac{P^2}{\bar{W}^2}\right)^{-1/2}\right\rangle + \bar{W}[\bar{W}^2(\bar{W}^2 - P^2)^{-1}]^{1/2} = \\ &= \langle P, \dot{\xi}\rangle + P^2(\bar{W}^2 - P^2)^{-1/2} - \bar{W}^2(\bar{W}^2 - P^2)^{-1/2} = \\ &= -(\bar{W}^2 - P^2)(\bar{W}^2 - P^2)^{-1/2} + \langle P, \dot{\xi}\rangle = \\ &= -(\bar{W}^2 - P^2)^{1/2} - q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2}. \end{aligned} \tag{3.6}$$

Here we took into account that, owing to definitions (2.8) and (2.12),

$$\begin{aligned} qA &:= \bar{W}u_f = \bar{W}\frac{d\xi}{dt} = \bar{W}\frac{d\xi}{d\tau}\frac{d\tau}{dt} = \bar{W}\dot{\xi}(1 - |u - v|^2)^{1/2} = \\ &= \bar{W}\dot{\xi}(1 + |\dot{r} - \dot{\xi}|^2)^{-1/2} = -\bar{W}\dot{\xi}(\bar{W}^2 - P^2)^{1/2}\bar{W}^{-1} = \\ &= -\dot{\xi}(\bar{W}^2 - P^2)^{1/2}, \end{aligned} \tag{3.7}$$

or

$$\dot{\xi} = -qA(\bar{W}^2 - P^2)^{-1/2}, \tag{3.8}$$

where $A : \mathbb{M}^4 \to \mathbb{R}^3$ is the related magnetic vector potential, generated by the moving external charged particle.

Thereby, we can state that the Hamiltonian function (3.6) satisfies the energy conservation conditions

$$\frac{dH}{dt} = 0 = \frac{dH}{d\tau}, \tag{3.9}$$

for all $\tau, t \in \mathbb{R}$, that is the suitable energy expression

$$\mathcal{E} = (\bar{W}^2 - P^2)^{1/2} + q\langle A, P\rangle(\bar{W}^2 - P^2)^{-1/2} \tag{3.10}$$

holds. The result (3.10) essentially differs from that obtained in [11], which makes use of the well-known Einsteinian Lagrangian for a moving charged point particle q in external electromagnetic field. Thereby, our result can be formulated as follows.

Proposition 3.2. *The alternative classical relativistic electrodynamical model (2.7) allows the Hamiltonian formulation (3.5) with respect to the «rest» reference system variables, where the Hamiltonian function is given by expression (3.6).*

To make this difference more clear, we will analyze below the Lorentz force (2.20) from the Hamiltonian point of view based on the Lagrangian function (2.17). Thus, we obtain that the corresponding Hamiltonian function

$$\begin{aligned} H &:= \langle P, \dot{r}\rangle - \mathcal{L} = \\ &= \langle P, \dot{r}\rangle + \bar{W}(1+\dot{r}^2)^{1/2} - q\langle A, \dot{r}\rangle = \langle P - qA, \dot{r}\rangle + \bar{W}(1+\dot{r}^2)^{1/2} = \\ &= -\langle p, p\rangle \bar{W}^{-1}\left(1 - \frac{p^2}{\bar{W}^2}\right)^{-1/2} + \bar{W}\left(1 - \frac{p^2}{\bar{W}^2}\right)^{-1/2} = \\ &= -(\bar{W}^2 - p^2)(\bar{W}^2 - p^2)^{-1/2} = -(\bar{W}^2 - p^2)^{1/2}. \end{aligned} \tag{3.11}$$

Since $p = P - qA$, expression (3.11) takes the final «*no interaction*» [11, 31, 38, 39] form

$$H = -[\bar{W}^2 - (P - qA)^2]^{1/2}, \tag{3.12}$$

being conservative with respect to the evolution equations (2.15) and (2.16), that is

$$\frac{dH}{dt} = 0 = \frac{dH}{d\tau} \tag{3.13}$$

for all $\tau, t \in \mathbb{R}$. The latter are simultaneously equivalent to the following Hamiltonian system:

$$\begin{aligned} \dot{r} &= \frac{\partial H}{\partial P} = (P - qA)[\bar{W}^2 - (P - qA)^2]^{-1/2}, \\ \dot{P} &= -\frac{\partial H}{\partial r} = (\bar{W}\nabla\bar{W} - \nabla\langle qA, (P - qA)\rangle)[\bar{W}^2 - (P - qA)^2]^{-1/2}, \end{aligned} \tag{3.14}$$

that can be easily checked by direct calculations. Really, the first equation

$$\begin{aligned} \dot{r} &= (P - qA)[\bar{W}^2 - (P - qA)^2]^{-1/2} = p(\bar{W}^2 - p^2)^{-1/2} = \\ &= mu(\bar{W}^2 - p^2)^{-1/2} = -\bar{W}u(\bar{W}^2 - p^2)^{-1/2} = u(1 - u^2)^{-1/2}, \end{aligned} \tag{3.15}$$

holds, owing to the condition $d\tau = dt(1 - u^2)^{1/2}$ and definitions $p := mu$, $m = -\bar{W}$, postulated from the very beginning. Similarly, we obtain that

$$\begin{aligned} \dot{P} &= -\nabla\bar{W}(1 - p^2/\bar{W}^2)^{-1/2} + \nabla\langle qA, u\rangle(1 - p^2/\bar{W}^2)^{-1/2} = \\ &= -\nabla\bar{W}(1 - u^2)^{-1/2} + \nabla\langle qA, u\rangle(1 - u^2)^{-1/2}, \end{aligned} \tag{3.16}$$

exactly coinciding with Eq. (2.18) subject to the evolution parameter $t \in \mathbb{R}$. Our result we now formulate as the next proposition.

Proposition 3.3. *The dual to the classical relativistic electrodynamical model (2.20) allows the Hamiltonian formulation (3.14) with respect to the «rest» reference system variables, where the Hamiltonian function is given by expression (3.12).*

4. CONCLUSION

Thereby, we can claim that all of dynamical field equations discussed above are canonical Hamiltonian systems with respect to the corresponding proper «rest» reference systems, parameterized by suitable time parameters $\tau \in \mathbb{R}$. Owing to the passing to the basic reference system $\mathcal{K}$ with the time parameter $t \in \mathbb{R}$, the related Hamiltonian structure is naturally lost, giving rise to a new interpretation of the real particle motion as such having the absolute sense only with respect to the proper «rest» reference system and being completely relative with respect to all other reference systems. Concerning the Hamiltonian expressions (3.1), (3.6) and (3.12) obtained above, one observes that all of them depend strongly on the vacuum potential field function $\bar{W} : \mathbb{M}^4 \rightarrow \mathbb{R}$, thereby dissolving the mass problem of the classical energy expression, before pointed out [14] by L. Brillouin. It is necessary here to mention that subject to the canonical Dirac-type quantization procedure it can be applied only to the corresponding dynamical field systems considered with respect to their proper «rest» reference systems.

Remark 4.1. *Some comments can be also made concerning the classical relativity principle. Namely, we have obtained our results completely without using the Lorentz transformations of reference systems but only the natural notion of the «rest» reference system and its suitable parametrization with respect to any other moving reference systems. It looks reasonable since, in reality, the true state changes of a moving charged particle q are exactly realized only with respect to its proper «rest» reference system. Thereby, the only question, still here left open, is that about the physical justification of the corresponding relationship between time parameters of moving and «rest» reference systems.*

This relationship, being accepted throughout this work, looks as

$$d\tau = dt(1 - u^2)^{1/2}, \tag{4.1}$$

where $u := dr/dt \in \mathbb{E}^3$ is the velocity vector with which the «rest» reference system $\mathcal{K}_r$ moves with respect to other arbitrarily chosen reference system $\mathcal{K}$. The expression (4.1) means, in particular, that there holds the equality

$$dt^2 - dr^2 = d\tau^2, \tag{4.2}$$

which exactly coincides with the classical infinitesimal Lorentz invariant. Its appearance is, evidently, not casual here, since all our dynamical vacuum field equations were successively derived [6, 7] from the governing equations on the vacuum potential field function $W : \mathbb{M}^4 \to \mathbb{R}$ in the form

$$\frac{\partial^2 W}{\partial t^2} - \nabla^2 W = \rho, \quad \frac{\partial W}{\partial t} + \nabla(vW) = 0, \quad \frac{\partial \rho}{\partial t} + \nabla(v\rho) = 0, \tag{4.3}$$

being $a\ priori$ Lorentz invariant, where we denoted by $\rho \in \mathbb{R}$ the charge density and by $v := dr/dt$ the suitable local velocity of the vacuum field potential changes. Thereby, the dynamical infinitesimal Lorentz invariant (4.2) reflects this intrinsic structure of equations (4.3). Being rewritten in the following nonstandard Euclidean form:

$$dt^2 = d\tau^2 + dr^2, \tag{4.4}$$

it gives rise to a completely other time relationship between reference systems $\mathcal{K}$ and $\mathcal{K}_r$:

$$dt = d\tau(1 + \dot{r}^2)^{1/2}, \tag{4.5}$$

where, as earlier, we denoted by $\dot{r} := dr/d\tau$ the related particle velocity with respect to the «rest» reference system. Thus, we observe that all our Lagrangian analysis completed in Sec. 2 is based on the corresponding functional expressions written in these «Euclidean» space-time coordinates and with respect to which the least action principle was applied. So, we see that there exist two alternatives — the first is to apply the least action principle to the corresponding Lagrangian functions expressed in the Minkowski-type space-time variables with respect to an arbitrarily chosen reference system $\mathcal{K}$, and the second is to apply the least action principle to the corresponding Lagrangian functions expressed in the space-time Euclidean-type variables with respect to the «rest» reference system $\mathcal{K}_r$.

As a slightly amusing but exciting inference, following from our analysis in this work, is the fact that all of classical special relativity results, related to electrodynamics of charged point particles, can be obtained one-to-one making use of our new definitions of the dynamical particle mass and the least action principle with respect to the associated Euclidean-type space-time variables parameterizing the «rest» reference system.

An additional remark is here needed concerning the quantization procedure of proposed electrodynamics models. If the dynamical vacuum field equations are expressed in the canonical Hamiltonian form, only technical problems left to quantize them and obtain the corresponding Schrödinger-type evolution equations in suitable Hilbert spaces of quantum states. There exists still another important inference from the approach devised in this work, consisting in complete lost of the essence of the well-known Einsteinian equivalence principle [4, 5, 11, 26, 31], becoming superfluous for our vacuum field theory of electromagnetism and gravity.

Based on the canonical Hamiltonian formalism devised in this work, concerning the alternative charged point particle electrodynamics models, we succeeded in treating their Dirac-type quantization. The obtained results were compared with classical ones, but the physically motivated choice of a true model is left for the future studies. Another important aspect of the developed vacuum field theory no-geometry approach to combining the electrodynamics with the gravity consists in singling out the decisive role of the related «rest» reference system $\mathcal{K}_r$. Namely, with respect to the «rest» reference system evolution parameter $\tau \in \mathbb{R}$ all of our electrodynamics models allow both the Lagrangian and Hamiltonian formulations suitable for the canonical quantization. The physical nature of this fact remains, by now, not enough understood. There is, by now [11, 26, 29–31], no physically reasonable explanation of this decisive role of the «rest» reference system, except for the very interesting reasonings by R. Feynman who argued in [5] that the relativistic expression for the classical Lorentz force (1.12) has physical sense only with respect to the «rest» reference system variables $(r, \tau) \in \mathbb{E}^4$. In the sequel of our work we plan to analyze the quantization scheme in more detail and make a step toward the vacuum quantum field theory of infinite many-particle systems.

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