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MASSES OF HEAVY TETRAQUARKS WITH HIDDEN CHARM AND BOTTOM

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The relativistic model of heavy tetraquarks is formulated within the diquark–antidiquark picture. The diquark structure is taken into account by the diquark–gluon form factor. New experimental data on charmonium-like states above open charm threshold are discussed. The obtained results indicate that $X(3872)$, $Y(4140)$, $Y(4260)$, $Y(4360)$, $Z_2(4250)$, $Z(4430)$ and $Y(4660)$ could be tetraquark states with hidden charm. Predictions for the masses of bottom counterparts to the charm tetraquark candidates are given.

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Recently, significant experimental progress has been achieved in charmonium spectroscopy. Several new states, such as $X(3872)$, $Y(4140)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z_2(4250)$, $Z(4430)$, etc., were observed [1] which cannot be simply accommodated in the quark–antiquark ($c\bar{c}$) picture. These states and especially the charged ones can be considered as indications of possible existence of exotic multiquark states. The Belle Collaboration [2] observed an enhancement in $e^+e^- \to \Upsilon(1S)\pi^+\pi^-, \Upsilon(2S)\pi^+\pi^-$, and $\Upsilon(3S)\pi^+\pi^-$ production which is not well-described by the conventional $\Upsilon(10860)$ line shape. One of the possible explanations is a bottomonium counterpart to the $Y(4260)$ state which may overlap with the $\Upsilon(5S)$. Here, we consider masses of heavy tetraquarks with hidden charm and bottom in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. We use the diquark–antidiquark picture to reduce a complicated relativistic four-body problem to the successive two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed of light and heavy quarks. At the second step, a heavy tetraquark is considered to be a diquark–antidiquark bound system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark–gluon interaction in terms of the diquark wave functions.

In the adopted approach the quark–quark bound state and diquark–antidiquark bound state are described by the diquark wave function (Ψ_d) and by the tetraquark wave function (Ψ_T), respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [3]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V_{d,T}(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2-m_2^2+m_1^2}{2M}, \quad E_2 = \frac{M^2-m_1^2+m_2^2}{2M}. \tag{3}$$

Here, $M = E_1 + E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks (q and Q) which form the diquark or of diquark (d) and antidiquark ($\bar{d}'$) which form the heavy tetraquark (T), and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V_{d,T}(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark–quark or diquark–antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. The explicit expressions for the corresponding quasipotentials $V_{d,T}(\mathbf{p},\mathbf{q};M)$ can be found in [4,5].

The constituent quark masses $m_c = 1.55$ GeV, $m_b = 4.88$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV have been fixed previously and have values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [3] and the heavy-quark expansion. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ${}^3P_J$ states [3]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

At the first step, we take the previously calculated masses and form factors of the charm and bottom diquarks [4]. The diquark interaction with the gluon field, which takes into account the diquark structure, is expressed through the form factor $F(r)$ entering the vertex of the diquark–gluon interaction [6]. This form factor is determined through the overlap integral of the diquark wave functions.

Table 1. **Masses of heavy-light diquarks (MeV)**

Quark content	Diquark type	Mass	
		$Q = c$	$Q = b$
$[Q, q]$	S	1973	5359
$\{Q, q\}$	A	2036	5381
$[Q, s]$	S	2091	5462
$\{Q, s\}$	A	2158	5482

The values of the masses of heavy-light scalar diquark $[\cdots]$ and axial vector diquark $\{\cdots\}$ ground states are given in Table 1.

At the second step, we calculate the masses of heavy tetraquarks considered as the bound states of a heavy-light diquark and antidiquark. The explicit expression for the diquark–antidiquark interaction is given in [5]. In this picture of heavy tetraquarks both scalar S (antisymmetric in flavour $[Qq]_{S=0} = [Qq]$) and axial vector A (symmetric in flavour $[Qq]_{S=1} = \{Qq\}$) diquarks are considered. As a result, a very rich spectrum of tetraquark states emerges. However, the number of states in the considered diquark–antidiquark picture is significantly less than in the genuine four-quark approach.

In Table 2 we compare our results for the masses of the ground and excited charm diquark–antidiquark bound states with the masses of the recently observed highly-excited charmonium-like states. We assume that the excitations occur only between the bound diquark and antidiquark. Possible excitations of diquarks are not considered. Our calculation of the heavy baryon masses supports such a scheme [6]. In this table we give our predictions only for some of the masses of the orbitally and radially excited states for which possible experimental candidates are observed. From Table 2 we see that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed of the scalar and axial vector diquark and antidiquark in the relative $1S$ state. Belle and BaBar results indicated the possible existence of a second $X(3875)$ particle a few MeV above $X(3872)$. This state could be naturally identified with the second neutral particle predicted by the tetraquark model.

The discovery in the initial state radiation at B factories of $Y(4260)$, $Y(4360)$ and $Y(4660)$ indicates an overpopulation of the expected charmonium 1^{--} states [1]. We find that $Y(4260)$ could be the 1^{--} $1P$ tetraquark state $([cq]_{S=0}[\bar{c}\bar{q}]_{S=0})$ $(S\bar{S})$ which mass is predicted in our model to be close to the mass of $Y(4260)$. Then $Y(4260)$ would decay dominantly into $D\bar{D}$ pairs. The other possible interpretations of $Y(4260)$ are the 1^{--} $1P$ states of $(S\bar{A}-\bar{S}A)/\sqrt{2}$ and $A\bar{A}$ tetraquarks which predicted masses have close values. These additional

Table 2. **Comparison of theoretical predictions for the masses of the ground and excited charm diquark–antidiquark states (in MeV) and possible experimental candidates. Masses of bottom counterparts are also given (in MeV)**

State J^{PC}	Diquark content	Theory $cq\bar{c}\bar{q}$	Theory $cs\bar{c}\bar{s}$	Experiment State	Experiment Mass	Theory $bq\bar{b}\bar{q}$
$1S$						
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871		$X(3872)$ $X(3876)$	3871.4 ± 0.6 (Belle) $3875.2 \pm 0.7^{+0.9}_{-1.8}$ (Belle)	10492
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$		4113	$Y(4140)$	$4143.0 \pm 2.9 \pm 1.2$ (CDF)	
0^{++}	$A\bar{A}$		4110			
2^{++}	$A\bar{A}$	3968		$Y(3940)$	$3943 \pm 11 \pm 13$ (Belle) $3914.3^{+4.1}_{-3.8}$ (BaBar)	10534
$1P$						
1^{--}	$S\bar{S}$	4244		$Y(4260)$	$4259 \pm 8^{+2}_{-6}$ (BaBar) $4247 \pm 12^{+17}_{-32}$ (Belle)	10807
1^{-}	$S\bar{S}$	4244		$Z_2(4250)$	$4248^{+44+180}_{-29-35}$ (Belle)	10807
0^{-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4267				10820
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284		$Y(4260)$	$4284^{+17}_{-16} \pm 4$ (CLEO)	10824
1^{--}	$A\bar{A}$	4277				10827
1^{--}	$A\bar{A}$	4350		$Y(4360)$	$4361 \pm 9 \pm 9$ (Belle) 4324 ± 24 (BaBar)	10850
$2S$						
1^{+}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431		$Z(4430)$	$4433 \pm 4 \pm 2$ (Belle)	10939
0^{+}	$A\bar{A}$	4434				10942
1^{+}	$A\bar{A}$	4461				10951
$2P$						
1^{--}	$S\bar{S}$	4666		$Y(4660)$ $X(4630)$	$4664 \pm 11 \pm 5$ (Belle) 4634^{+8+5}_{-7-8} (Belle)	11122

tetraquark states could be responsible for the mass difference of $Y(4260)$ observed in different decay channels. As we see from Table 2, the recently discovered resonances $Y(4360)$ and $Y(4660)$ in the $e^+e^- \to \pi^+\pi^-\psi'$ cross section can be interpreted as the excited 1^{--} $1P$ $(A\bar{A})$ and $2P$ $(S\bar{S})$ tetraquark states, respectively. The peak $X(4630)$ very recently observed by Belle in $e^+e^- \to \Lambda_c^+\Lambda_c^-$ is consistent with a 1^{--} resonance $Y(4660)$, and therefore has the same interpretation in our model.

The Belle Collaboration reported the observation of a relatively narrow enhancement in the $\pi^+\psi'$ invariant mass distribution in the $B \to K\pi^+\psi'$ decay [1]. This new resonance, $Z^+(4430)$, is unique among other exotic meson candidates, since it is the first state which has a nonzero electric charge. Different theoretical interpretations were suggested [1]. Our calculations indicate that $Z^+(4430)$ can be the 1^+ $2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state. It could be the first radial excitation of the ground state $(S\bar{A} - \bar{S}A)/\sqrt{2}$, which has the same mass as $X(3872)$. The other possible interpretation is the 0^+ $2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state $(A\bar{A})$ which has a very close mass. Measurement of the $Z^+(4430)$ spin will discriminate between these possibilities.

Encouraged by this discovery, the Belle Collaboration performed a study of $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ and observed a double peaked structure in the $\pi^+\chi_{c1}$ invariant mass distribution [1]. These two charged hidden charm peaks, $Z_1(4051)$ and $Z_2(4250)$, are explicitly exotic. We find no tetraquark candidates for the former, $Z_1(4051)$, structure. On the other hand, we see from Table 2 that $Z_2(4250)$ can be interpreted in our model as the charged partner of the 1^- $1P$ state $S\bar{S}$ or as the 0^- $1P$ state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark.

The $Y(4140)$ state, recently observed by the CDF Collaboration in the $J/\psi\phi$ invariant mass distribution from the decay $B^+ \to J/\psi\phi K^+$ [1], could be attributed in our model to the 1^{++} $((S\bar{A} \pm \bar{S}A)/\sqrt{2})$ or 0^{++} $(A\bar{A})$ $cs\bar{c}\bar{s}$ tetraquark state.

In Table 2 we also give masses of the bottom counterparts of the hidden charm tetraquarks. Our model predicts four vector 1^{--} tetraquark states with hidden bottom in the mass range 10807–10850 MeV. Two of these tetraquarks are composed of a scalar diquark and antidiquark $(S\bar{S})$ and scalar diquark and axial vector antidiquark $((S\bar{A} - \bar{S}A)/\sqrt{2})$. Two other 1^{--} states contain an axial vector diquark and antidiquark with the total spin of the diquark and antidiquark $\mathcal{S}$ equal to 0 and 2. Two lighter tetraquarks, with predicted masses 10807 MeV $(S\bar{S})$ and 10827 MeV $(A\bar{A})$, are bottom partners of $Y(4260)$, while the heavier one, with mass 10850 MeV $(A\bar{A})$, is the bottom partner of $Y(4360)$. Therefore, a complicated structure of vector bottomonium states emerges in this mass range, which can be responsible for the anomalous production cross sections for $e^+e^- \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ observed by Belle [2]. Their fit using a single Breit–Wigner resonance shape yielded a peak mass of $(10889.6 \pm 1.8 \pm 1.5)$ MeV. However, a more detailed experimental study is necessary to clarify this question. The bottom counterpart to the vector state $Y(4660)$ has predicted mass 11122 MeV. It is very important to search for the charged or strange states with hidden bottom. Their observation will be a direct proof of the existence of heavy tetraquarks. The masses of the bottom counterparts to charged $Z_2(4250)$ and $Z(4430)$ are predicted at around 10807 and 10939 MeV, respectively.

It is necessary to emphasize that the observation of the bottom counterparts to the new anomalous charmonium-like states is very important since it will allow one to discriminate between different theoretical descriptions of these states. Indeed, theoretical models, such as the hybrid, molecular or diquark–antidiquark pictures, etc., give significantly different results in the bottom sector.

In summary, we calculated the masses of excited heavy tetraquarks with hidden charm and bottom in the diquark–antidiquark picture and the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. It was found that the $X(3872)$, $Y(4140)$, $Z_2(4250)$, $Y(4260)$, $Y(4360)$, $Z(4430)$ and $Y(4660)$ exotic meson candidates can be tetraquark states with hidden charm. The correspondence between bottom and charm tetraquark candidates was discussed.

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