ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2010. Т. 41. ВЫП. 6

LOW-X EVOLUTION EQUATIONS IN MÖBIUS REPRESENTATION

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The Möbius form of the BFKL kernel in the next-to-leading order (NLO) in theories containing fermions and scalars in arbitrary representations of the colour group is presented. The ambiguity of the NLO kernels permits one to get agreement between the BFKL approach and the colour dipole model and to find the quasi-conformal representation of the BFKL kernel.

PACS: 11.10.-z; 11.15.-q

INTRODUCTION

The most common basis for the theoretical description of processes with small ratio $x = Q^2/s$ (Q^2 is a typical virtuality and s is the squared center-of-mass energy) is given in QCD by the Balitsky–Fadin–Kuraev–Lipatov (BFKL) approach [1] based on the gluon Reggeization and applicable for arbitrary colour exchanges. Originally this approach was formulated in the momentum representation, and the kernel of the BFKL equation for the evolution of QCD amplitudes with s was calculated in the space of transverse momenta $\mathbf{q}_1, \mathbf{q}_2$ of two interacting Reggeized gluons (now the kernel is known in the next-to-leading order (NLO) both for the forward scattering [2] and for any momentum and colour transfer [3]). Later it was recognized that for the case of scattering of colourless objects the BFKL equation possesses remarkable properties, which become mostly apparent in the space of conjugate coordinates $\mathbf{r}_1, \mathbf{r}_2$. It was shown [4] that in this case the BFKL equation can be written in the special (Möbius) representation, where the equation is invariant under the conformal (Möbius) transformations of the transverse coordinates. For brevity, we call the BFKL kernel in this representation Möbius kernel, and its form in the coordinate space Möbius (or dipole) form. The Möbius form of the leading order (LO) BFKL kernel coincides with the kernel of the colour dipole model [5] formulated in the coordinate space and is explicitly conformal invariant [6]. Here we present the result of recent investigations of the Möbius form in the NLO.

1. AMBIGUITY OF THE NLO KERNEL

We use the notation of [6, 7], denoting the Reggeon transverse momenta in initial and final t-channel states as $\mathbf{q}_i'$ and $\mathbf{q}_i$ and the corresponding coordinates $\mathbf{r}_i'$ and $\mathbf{r}_i$, $i = 1, 2$. The state normalization is

$$\begin{aligned} \langle \mathbf{q}|\mathbf{q}'\rangle &= \delta(\mathbf{q}-\mathbf{q}'), \\ \langle \mathbf{r}|\mathbf{r}'\rangle &= \delta(\mathbf{r}-\mathbf{r}'), \\ \langle \mathbf{r}|\mathbf{q}\rangle &= \frac{e^{i\mathbf{q}\,\mathbf{r}}}{(2\pi)}. \end{aligned} \tag{1}$$

For brevity, we use $\mathbf{p}_{ij'} = \mathbf{p}_i - \mathbf{p}_j'$.

The s-channel discontinuities of scattering amplitudes for the processes $A + B \to A' + B'$ have the form

$$-4i(2\pi)^2\,\delta(\mathbf{q}_A - \mathbf{q}_B)\ \mathrm{disc}_s\mathcal{A}_{AB}^{A'B'} = \langle A'\bar{A}|\left(\frac{s}{s_0}\right)^{\hat{\mathcal{K}}}\frac{1}{\hat{\mathbf{q}}_1^2\hat{\mathbf{q}}_2^2}|\bar{B}'B\rangle. \tag{2}$$

In this expression s_0 is an appropriate energy scale, $\mathbf{q}_A = p_{A'A}, \mathbf{q}_B = p_{BB'}$, $\hat{\mathcal{K}}$ is the BFKL kernel (note that it differs from the symmetric kernel $\hat{\mathcal{K}}_s$ defined in the momentum space according to [8]: $\hat{\mathcal{K}} = (\hat{\mathbf{q}}_1^2\hat{\mathbf{q}}_2^2)^{-1}\hat{\mathcal{K}}_s$; in the LO it is just $\hat{\mathcal{K}}$ that has the Möbius form which is conformal invariant and coincides with the dipole kernel); $\langle A'\bar{A}|$ and $|\bar{B}'B\rangle$ represent the impact factors. One can see that the discontinuity $\mathrm{disc}_s\mathcal{A}_{AB}^{A'B'}$ in Eq. (2) is invariant under the transformation

$$\hat{\mathcal{K}} \to \hat{U}^{-1}\hat{\mathcal{K}}\hat{U}, \quad \langle A'\bar{A}| \to \langle A'\bar{A}|\hat{U}, \quad \frac{1}{\hat{\mathbf{q}}_1^2\hat{\mathbf{q}}_2^2}|\bar{B}'B\rangle \to \hat{U}^{-1}\frac{1}{\hat{\mathbf{q}}_1^2\hat{\mathbf{q}}_2^2}|\bar{B}'B\rangle. \tag{3}$$

If the kernel is fixed in the LO, transformations with $\hat{U} = 1 - \hat{O}$, where $\hat{O} \sim \alpha_s$, are still possible. Within the NLO accuracy they give

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} - [\hat{\mathcal{K}}^B, \hat{O}], \tag{4}$$

where $\hat{\mathcal{K}}^B$ is the LO value of $\hat{\mathcal{K}}$. Such transformations can be used to simplify the form of the kernel, in particular of its Möbius form. Indeed, it was shown [6, 7] that the last form is simplified by the transformation

$$\hat{\mathcal{K}} \to \hat{K} = \hat{\mathcal{K}} + \frac{\alpha_s}{8\pi}\beta_0\left[\hat{\mathcal{K}}^B, \ln\left(\hat{\mathbf{q}}_1^2\hat{\mathbf{q}}_2^2\right)\right], \tag{5}$$

where β_0 is the first coefficient of the beta-function.

2. MÖBIUS FORM OF $\hat{K}$

In the NLO the Möbius form can be written [6] as follows:

$$\langle \mathbf{r}_1, \mathbf{r}_2|\hat{K}_M|\mathbf{r}_1', \mathbf{r}_2'\rangle = \frac{\alpha_s(\mu^2)N_c}{2\pi^2}\int d\mathbf{r}_0 \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{10}^2\mathbf{r}_{20}^2}\Bigg[\delta(\mathbf{r}_{11'})\delta(\mathbf{r}_{2'0})+\delta(\mathbf{r}_{1'0})\delta(\mathbf{r}_{22'})-$$

$$-\delta(\mathbf{r}_{11'})\delta(r_{22'})\Bigg] + \frac{\alpha_s^2(\mu^2)N_c^2}{4\pi^3}\Bigg[\delta(\mathbf{r}_{11'})g_1(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_2') + \delta(\mathbf{r}_{22'})g_1(\mathbf{r}_2, \mathbf{r}_1; \mathbf{r}_1')+$$

$$+\delta(\mathbf{r}_{11'})\delta(\mathbf{r}_{22'})\int d\mathbf{r}_0\, g_0(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0) + \frac{1}{\pi}g_2(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_1', \mathbf{r}_2')\Bigg]. \quad (6)$$

Here $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, and the whole kernel is symmetric with respect to the substitution $1 \leftrightarrow 2, 1' \leftrightarrow 2'$. The coefficients of $\delta(\mathbf{r}_{11'})\delta(r_{22'})$ in Eq. (6) are written in integral form in order to make explicit the cancellation of the ultraviolet singularities of separate terms. The Möbius kernel (6) is defined with accuracy up to functions independent of $\mathbf{r}_1$ or of $\mathbf{r}_2$, such that after they are added to the kernel the functions $g_{1,2}$ remain zero at $\mathbf{r}_1 = \mathbf{r}_2$ [6, 7]. Therefore, one can add to the kernel only functions which are antisymmetric with respect to the $\mathbf{r}_1 \leftrightarrow \mathbf{r}_2$ substitution. These functions do not change the symmetric part of the kernel, but this is the only part which plays a role because of the symmetry of the impact factors.

In the general case of theory with n_f fermions and n_s scalar particles the direct transfer of the kernel $\hat{K}$ to the Möbius form gives for the functions g_i in Eq. (6)

$$g_0(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0) = 2\pi\zeta(3)\delta(\mathbf{r}_0) - g_1(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0), \quad (7)$$

$$g_1(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_2') = \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{22'}^2\mathbf{r}_{12'}^2}\Bigg[\frac{\beta_0}{2N_c}\Bigg(\ln\left(\frac{\mathbf{r}_{12}^2\mu^2}{4e^{2\psi(1)}}\right)+$$

$$+\frac{\mathbf{r}_{12'}^2 - \mathbf{r}_{22'}^2}{\mathbf{r}_{12}^2}\ln\left(\frac{\mathbf{r}_{22'}^2}{\mathbf{r}_{12'}^2}\right)\Bigg) - \zeta(2) + \frac{67}{18} - \frac{1}{2}\ln\left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{22'}^2}\right)\ln\left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{12'}^2}\right) - \frac{5a_f + 2a_s}{9N_c}\Bigg]+$$

$$+\frac{1}{2\mathbf{r}_{22'}^2}\ln\left(\frac{\mathbf{r}_{12'}^2}{\mathbf{r}_{22'}^2}\right)\ln\left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{12'}^2}\right), \quad (8)$$

$$g_2(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}'_1,\mathbf{r}'_2) = \frac{1}{2\mathbf{r}^4_{1'2'}}\left(\frac{\mathbf{r}^2_{11'}\,\mathbf{r}^2_{22'} - 2\mathbf{r}^2_{12}\,\mathbf{r}^2_{1'2'}}{d}\ln\left(\frac{\mathbf{r}^2_{12'}\,\mathbf{r}^2_{21'}}{\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}}\right) - 1\right)\times$$
$$\times\left(1 - b_f + \frac{b_s}{2}\right) + \left(\frac{(2b_s - 3b_f)}{4\mathbf{r}^2_{1'2'}}\frac{\mathbf{r}^2_{12}}{d} + \frac{1}{4\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}}\left(\frac{\mathbf{r}^4_{12}}{d} - \frac{\mathbf{r}^2_{12}}{\mathbf{r}^2_{1'2'}}\right)\right)\times$$
$$\times\ln\left(\frac{\mathbf{r}^2_{12'}\,\mathbf{r}^2_{21'}}{\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}}\right) + \frac{\mathbf{r}^2_{12}\ln\left(\mathbf{r}^2_{11'}/\mathbf{r}^2_{1'2'}\right)}{2\mathbf{r}^2_{11'}\mathbf{r}^2_{12'}\mathbf{r}^2_{22'}} + \frac{\ln\left(\mathbf{r}^2_{12}\mathbf{r}^2_{1'2'}/\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}\right)}{2\mathbf{r}^2_{12'}\mathbf{r}^2_{21'}}\times$$
$$\times\left(\mathbf{r}^2_{12}/2\mathbf{r}^2_{1'2'} + \frac{1}{2} - \mathbf{r}^2_{22'}/\mathbf{r}^2_{1'2'}\right) + \frac{\mathbf{r}^2_{21'}\ln\left(\mathbf{r}^2_{21'}\mathbf{r}^2_{1'2'}/\mathbf{r}^2_{12}\mathbf{r}^2_{11'}\right)}{2\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}\mathbf{r}^2_{1'2'}}+$$
$$+\frac{\ln\left(\mathbf{r}^2_{12}/\mathbf{r}^2_{1'2'}\right)}{4\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}} + \frac{\ln\left(\mathbf{r}^2_{22'}/\mathbf{r}^2_{12}\right)}{2\mathbf{r}^2_{11'}\mathbf{r}^2_{12'}} + \frac{\mathbf{r}^2_{12}\ln\left(\mathbf{r}^2_{12}\mathbf{r}^2_{1'2'}/\mathbf{r}^2_{12'}\mathbf{r}^2_{21'}\right)}{4\mathbf{r}^2_{11'}\mathbf{r}^2_{22'}\mathbf{r}^2_{1'2'}}+$$
$$+\frac{\ln\left(\mathbf{r}^2_{12}\mathbf{r}^2_{1'2'}/\mathbf{r}^2_{12'}\mathbf{r}^2_{22'}\right)}{2\mathbf{r}^2_{11'}\mathbf{r}^2_{1'2'}} + \frac{\ln\left(\mathbf{r}^2_{12}\mathbf{r}^2_{11'}/\mathbf{r}^2_{22'}\mathbf{r}^2_{1'2'}\right)}{2\mathbf{r}^2_{12'}\mathbf{r}^2_{1'2'}} + (1\leftrightarrow 2, 1'\leftrightarrow 2'). \quad (9)$$

Here $\beta_0 = 11N_c/3 - 2a_f/3 - a_s/6$, $a_f = 2\kappa_f n_f T_f, a_s = 2\kappa_s n_s T_s$, T_f and T_s are the colour group generators for fermions and scalars, respectively, and are defined by the relations

$$\mathrm{Tr}\left(T^a_f T^b_f\right) = T_f\delta^{ab}, \quad \mathrm{Tr}\left(T^a_s T^b_s\right) = T_s\delta^{ab}, \quad (10)$$

κ_f (κ_s) is equal to $1/2$ for Majorana fermions (neutral scalars) in self-conjugated representations and to 1 otherwise,

$$d = \mathbf{r}^2_{12'}\mathbf{r}^2_{21'} - \mathbf{r}^2_{11'}\mathbf{r}^2_{22'}, \quad b_j = \frac{4n_j\kappa_j}{N_c^2 - 1}\mathrm{Tr}\left(\frac{C_j^2}{N_c^2} - \frac{C_j}{2N_c}\right), \quad C_j = T^a_j T^a_j, \; (11)$$

$j = f, s$. The gluon part of the functions g_i was calculated in [7]; the quark and scalar parts can be obtained by a simple colour algebra from the results of [6, 9] and [10], respectively. The quark part completely agrees with the corresponding part calculated in the colour dipole model [11]. However, the result for the gluon part obtained in this model [12] strongly differs from the one presented above. In both results the conformal invariance is violated not only by the terms related to renormalization (proportional to β_0). The problem of finding transformations of the kind (4) matching the kernels and bringing them to a quasi-conformal shape (where the conformal invariance is broken only by terms proportional to β_0) was formulated in [10]. For the colour dipole kernel the last problem was solved in [13]. In the BFKL framework both problems were solved in [14].

3. TRANSFORMATION TO THE QUASI-CONFORMAL SHAPE

It was shown in [14] that after the application to the kernel $\hat{K}$ of the transformation (4) with $\hat{O}$ defined as

$$\langle \mathbf{q}_1, \mathbf{q}_2 | \hat{O} | \mathbf{q}'_1, \mathbf{q}'_2 \rangle = \frac{\alpha_s N_c}{2\pi^2} \left[-\delta(\mathbf{q}_{11'} + \mathbf{q}_{22'}) \left(\frac{\mathbf{k}}{\mathbf{k}^2} - \frac{\mathbf{q}_1}{\mathbf{q}_1^2} \right) \left(\frac{\mathbf{k}}{\mathbf{k}^2} + \frac{\mathbf{q}_2}{\mathbf{q}_2^2} \right) \ln \mathbf{k}^2 + \right.$$

$$\left. + \delta(\mathbf{q}_{22'}) \delta(\mathbf{q}_{11'}) \int d^2 l \left(\frac{1}{\mathbf{l}^2} - \frac{\mathbf{l}(\mathbf{l} - \mathbf{q}_1)}{2\mathbf{l}^2 (\mathbf{l} - \mathbf{q}_1)^2} - \frac{\mathbf{l}(\mathbf{l} - \mathbf{q}_2)}{2\mathbf{l}^2 (\mathbf{l} - \mathbf{q}_2)^2} \right) \ln \mathbf{l}^2 \right], \quad (12)$$

where $\mathbf{k} = \mathbf{q}_1 - \mathbf{q}'_1$, one obtains for the Möbius form of the transformed kernel what follows:

$$g_0^T(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0) = g_0(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0) = -g_1^T(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_0) + 2\pi\zeta(3)\delta(\mathbf{r}_0), \quad (13)$$

$$g_1^T(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_2) = \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{22'}^2 \mathbf{r}_{12'}^2} \left[\frac{\beta_0}{2N_c} \left(\ln \left(\frac{\mathbf{r}_{12}^2 \mu^2}{4e^{2\psi(1)}} \right) + \frac{\mathbf{r}_{12'}^2 - \mathbf{r}_{22'}^2}{\mathbf{r}_{12}^2} \ln \left(\frac{\mathbf{r}_{22'}^2}{\mathbf{r}_{12'}^2} \right) \right) - \right.$$

$$\left. -\zeta(2) + \frac{67}{18} - \ln \left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{22'}^2} \right) \ln \left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{12'}^2} \right) - \frac{5a_f + 2a_s}{9N_c} \right], \quad (14)$$

and

$$g_2^T(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \frac{1}{\mathbf{r}_{1'2'}^4} \left(\frac{\mathbf{r}_{11'}^2 \mathbf{r}_{22'}^2 + \mathbf{r}_{12'}^2 \mathbf{r}_{21'}^2 - 4\mathbf{r}_{12}^2 \mathbf{r}_{1'2'}^2}{2d} \times \right.$$

$$\left. \times \ln \left(\frac{\mathbf{r}_{12'}^2 \mathbf{r}_{21'}^2}{\mathbf{r}_{11'}^2 \mathbf{r}_{22'}^2} \right) - 1 \right) \left(1 - b_f + \frac{b_s}{2} \right) + \left[\frac{1}{4\mathbf{r}_{11'}^2 \mathbf{r}_{22'}^2} \left(\frac{\mathbf{r}_{12}^4}{d} - \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{1'2'}^2} \right) + \right.$$

$$\left. + \frac{1}{4\mathbf{r}_{12'}^2 \mathbf{r}_{21'}^2} \left(\frac{\mathbf{r}_{12}^4}{d} + \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{1'2'}^2} \right) + \frac{(2b_s - 3b_f)}{2\mathbf{r}_{1'2'}^2} \frac{\mathbf{r}_{12}^2}{d} \right] \ln \left(\frac{\mathbf{r}_{12'}^2 \mathbf{r}_{21'}^2}{\mathbf{r}_{11'}^2 \mathbf{r}_{22'}^2} \right) \quad (15)$$

plus terms antisymmetric with respect to the exchange $\mathbf{r}_1 \leftrightarrow \mathbf{r}_2$, which can be omitted due to the symmetry of impact factors. Up to the difference in the renormalization scales (which is discussed in detail in [15]) and with account of the correction of the result of [12] made in [13], the gluon part of Eqs. (13)–(15) coincides with this result. Thus, the discrepancy between the BFKL and the colour dipole approaches is completely removed.

Moreover, it was shown in [14] that using the transformation (4), one can come to the quasi-conformal kernel. Specifically, the quasi-conformal kernel $\hat{\mathcal{K}}^{\mathrm{QC}}$ can be defined by the relation

$$\hat{\mathcal{K}}^{\mathrm{QC}} = \hat{\mathcal{K}} + \frac{\alpha_s}{8\pi} \beta_0 \left[\hat{\mathcal{K}}^B, \ln \left(\hat{\mathbf{q}}_1^2 \hat{\mathbf{q}}_2^2 \right) \right] - [\hat{\mathcal{K}}^B, \hat{O} + \hat{O}_1 \Big], \quad (16)$$

where the operator $\hat{O}$ is given in Eq. (12) and $\hat{O}_1$ is defined through

$$\langle \mathbf{r}_1\mathbf{r}_2|\hat{O}_{1M}|\mathbf{r}'_1\mathbf{r}'_2\rangle = \frac{\alpha_s N_c}{4\pi^2}\int d\mathbf{r}_0 \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{10}^2\mathbf{r}_{20}^2}\ln\left(\frac{\mathbf{r}_{12}^2}{\mathbf{r}_{10}^2\mathbf{r}_{20}^2}\right)\times$$
$$\times\left[\delta(\mathbf{r}_{11'})\delta(\mathbf{r}_{2'0}) + \delta(\mathbf{r}_{1'0})\delta(\mathbf{r}_{22'}) - \delta(\mathbf{r}_{11'})\delta(r_{22'})\right]. \quad (17)$$

For the Möbius form of $\hat{\mathcal{K}}^{\rm QC}$ one has

$$g_0^{\rm QC}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_0) = 6\pi\zeta\,(3)\,\delta\,(\mathbf{r}_0) - g_1^{\rm QC}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_0), \quad (18)$$

$$g_1^{\rm QC}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}'_2) = \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{22'}^2\mathbf{r}_{12'}^2}\left[\frac{\beta_0}{2N_c}\left(\ln\left(\frac{\mathbf{r}_{12}^2\mu^2}{4e^{2\psi(1)}}\right) + \frac{\mathbf{r}_{12'}^2-\mathbf{r}_{22'}^2}{\mathbf{r}_{12}^2}\ln\left(\frac{\mathbf{r}_{22'}^2}{\mathbf{r}_{12'}^2}\right)\right)+\right.$$
$$\left.+\frac{67}{18} - \zeta(2) - \frac{5a_f+2a_s}{9}\right], \quad \beta_0 = \frac{11N_c}{3} - \frac{2a_f}{3} - \frac{a_s}{6}, \quad (19)$$

$$g_2^{\rm QC}(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}'_1,\mathbf{r}'_2) = \frac{1}{\mathbf{r}_{1'2'}^4}\left(\frac{\mathbf{r}_{11'}^2\,\mathbf{r}_{22'}^2 - 2\mathbf{r}_{12}^2\,\mathbf{r}_{1'2'}^2}{d}\ln\left(\frac{\mathbf{r}_{12'}^2\,\mathbf{r}_{21'}^2}{\mathbf{r}_{11'}^2\mathbf{r}_{22'}^2}\right) - 1\right)\times$$
$$\times\left(1 - b_f + \frac{b_s}{2}\right) + \left(\frac{(2b_s-3b_f)}{2\mathbf{r}_{1'2'}^2}\frac{\mathbf{r}_{12}^2}{d} + \frac{1}{2\mathbf{r}_{11'}^2\,\mathbf{r}_{22'}^2}\left(\frac{\mathbf{r}_{12}^4}{d} - \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{1'2'}^2}\right)\right)\times$$
$$\times\ln\left(\frac{\mathbf{r}_{12'}^2\,\mathbf{r}_{21'}^2}{\mathbf{r}_{11'}^2\mathbf{r}_{22'}^2}\right) + \frac{\mathbf{r}_{12}^2}{\mathbf{r}_{11'}^2\mathbf{r}_{22'}^2\mathbf{r}_{1'2'}^2}\ln\left(\frac{\mathbf{r}_{12}^2\mathbf{r}_{1'2'}^2}{\mathbf{r}_{12'}^2\,\mathbf{r}_{21'}^2}\right), \; d = \mathbf{r}_{12'}^2\mathbf{r}_{21'}^2 - \mathbf{r}_{11'}^2\mathbf{r}_{22'}^2. \quad (20)$$

Here the conformal invariance is violated only by terms proportional to β_0. N-extended supersymmetric Yang–Mills theories ($N = 1, 2, 4$) contain, besides the Yang–Mills fields (gluons), $n_M = N$ Majorana spinors (gluinos) and $n_S = 2(N-1)$ scalars (for our purposes there is no difference between scalars and pseudoscalars). All particles are in the adjoint representation of the colour group. Therefore, in such theories we have $a_f = n_M N_c, a_s = n_S N_c, b_f = n_M, b_s = n_S$. In the case of $N = 4$ one has $\beta_0 = 0$ and the Möbius form of $\hat{\mathcal{K}}^{\rm QC}$ is conformal invariant.

Acknowledgements. Work is supported in part by the RFBR grant 10-02-01238, in part by the RFBR–MSTI grant 06-02-72041, in part by the INTAS grant 05-1000008-8328 and in part by Ministero Italiano dell'Istruzione, dell'Università e della Ricerca.

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