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BOGOLIUBOV KINETIC EQUATIONS AND DIELECTRIC FUNCTION WITH EXCHANGE INTERACTION

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Starting with the quantum BBGKY hierarchy for the statistical operators, we have obtained the quantum kinetic equation including the dynamical screening of the interaction potential, which exactly takes into account the exchange scattering in the plasma.

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INTRODUCTION

The famous Bogoliubov paper on «Problems of Dynamic Theory in Statistical Physics» [1] is a basis for kinetic theory conceptions of gases, fluids and plasma. In the work of Bogoliubov and Gurov [2] the kinetic equation for the charged particles statistical operator was obtained from a chain of equations for the density matrix. In works [3–6], a quantum kinetic equation was obtained that differs from Bogoliubov–Gurov equation in taking into account the medium polarization more properly. The corresponding classical equations were derived earlier by Balescu and Lenard [7, 8]. In the quantum kinetic equation for weakly coupled polarizable plasmas derived by Balescu and Guernsey [5, 6], the exchange interaction of particles was retained only in the distribution functions. But it is also necessary to take into account the exchange interaction in the scattering amplitude and in the dielectric function. Starting from the quantum BBGKY hierarchy for the distribution function we have solved, in the so-called plasma approximation, the equation for the quantum pair correlation function. The solution to this equation can be expressed in terms of the resolvent of the linearized Hartree–Fock equation. As a result, we obtain a quantum kinetic equation, which takes into account the dynamical screening of the interaction potential and the exchange interaction in a nontrivial way. In particular, this equation contains the dielectric function, which exactly describes the exchange scattering in plasma.

1. BOGOLIUBOV QUANTUM KINETIC EQUATIONS

The quantum hierarchy for a multicomponent plasma in the operator techniques takes the form

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b \mathrm{Sp}_{(2)}[U_{ab}(12), f_{ab}(12)], \tag{1}$$

$$\frac{\partial}{\partial t} f_{ab}(12) = [H_{ab}(12), f_{ab}(12)] + \sum_c \mathrm{Sp}_{(3)}[U_{ac}(13) + U_{bc}(23), f_{abc}(123)], \tag{2}$$

where $f_a(1)$ and $f_{ab}(12)$ are one- and two-particle density matrices; $[A, B]$ is the commutator of operators; $H_a(1) = \dfrac{p^2(1)}{2m_a}$ is the kinetic energy, $H_{ab}(12) = \dfrac{p^2(1)}{2m_a} + \dfrac{p^2(2)}{2m_b} + U_{ab}(12)$ is the two-particle Hamiltonian, and $U_{ab}(12)$ is the two-particle interaction potential. Let us introduce the new operators [9]:

$$f_{ab}(12) = \gamma_{ab}(12) f'_{ab}(12), \quad f_{abc}(123) = \gamma_{abc}(123) f'_{abc}(123), \tag{3}$$

where the symmetrization operators are

$$\gamma_{ab}(12) = 1 + \delta_{ab}\eta_a P(12), \quad \gamma_{abc}(123) = \gamma_{ab}(12)\{1 + \delta_{ac}\eta_a P(13) + \delta_{bc}\eta_b P(23)\}, \tag{4}$$

$\eta_a = 1(\text{Bose}), -1(\text{Fermi})$; $P(12)$ is the permutation operator. Therefore,

$$\frac{\partial}{\partial t} f_a(1) = [H_a(1), f_a(1)] + \sum_b \mathrm{Sp}_{(2)}[U_{ab}(12), \gamma_{ab}(12) f'_{ab}(12)], \tag{5}$$

$$\frac{\partial}{\partial t} f'_{ab}(12) = [H_{ab}(12), f'_{ab}(12)] + \\ + \sum_c \mathrm{Sp}_{(3)}[U_{ac}(13) + U_{bc}(23), (1 + \delta_{ac}\eta_a P(13) + \delta_{bc}\eta_b P(23)) f'_{abc}(123)]. \tag{6}$$

The symmetrization operators (4) are convenient in that they give the possibility to partially transmit the permutation operator $P(12)$ from the density matrix to the interaction potentials. The density matrices $f_{ab}(12)$, etc., possess the quantum symmetry properties: $P(12) f_{ab}(12) = f_{ab}(12) P(12)$, etc., whereas the density matrices $f'_{ab}(12)$, etc., possess only the classical symmetry properties: $P(12) f'_{ab}(12) P(12) = f'_{ab}(12)$, etc. For the classically symmetric density matrices the usual conditions for disentanglement of equations hold, which are the same as those in the classical statistics. Specifically, in the plasma approximation [10], when the triple correlation function is neglected:

$$f'_{ab}(12) = f_a(1) f_b(2) + g'_{ab}(12), \tag{7}$$

$$f'_{abc}(123) = f_a(1)f_b(2)f_c(3) + g'_{ab}(12)f_c(3) + g'_{ac}(13)f_b(2) + g'_{bc}(23)f_a(1), \quad (8)$$

where $g'_{ab}(12)$ is the pair correlation function. By substituting Eqs. (3), (7) and (8) into Eqs. (1) and (2) one obtains a closed set of equations for the one-particle and two-particle statistical operators

$$\frac{\partial}{\partial t} f_a(1) = [H'_a(1), f_a(1)] + \sum_b \mathrm{Sp}_{(2)}[U'_{ab}(12), \gamma_{ab}(12)g'_{ab}(12)], \quad (9)$$

$$\frac{\partial}{\partial t} g'_{ab}(12) = [H'_a(1) + H'_b(1), g'_{ab}(12)] + A'_{ab}(12)+$$
$$+ \sum_c \mathrm{Sp}_{(3)}\{[U'_{bc}(23), f_b(2)g'_{ac}(13)] + [U'_{ac}(13), f_a(1)g'_{bc}(23)]\}, \quad (10)$$

where

$$H'_a(1) = \frac{p^2(1)}{2m_a} + U_a^H(1) + U_a^F(1), \quad (11)$$

$$i\hbar A'_{ab}(12) = [1 + \eta_a f_a(1)][1 + \eta_b f_b(2)]U_{ab}(12)f_a(1)f_b(2)-$$
$$- f_a(1)f_b(2)U_{ab}(12)[1 + \eta_a f_a(1)][1 + \eta_b f_b(2)], \quad (12)$$

$U_a^H(1) = \sum_b \mathrm{Sp}_{(2)}[U_{ab}(12), f_b(2)]$ is the Hartree field, i.e., mean self-consistent field and $U_a^F(1) = \sum_b \mathrm{Sp}_{(2)}\delta_{ab}\eta_a P(12)[U_{ab}(12), f_b(2)]$ is the Fock field, mean field, taking into account only exchange interaction (Pauli's principle). In the plasma approximation [10] in Eq. (9) the term $[U'_{ab}(12), g'_{ab}(12)]$ which describes the direct interaction of two particles (1), (2) is not taken into account. Let us consider the homogeneous case. In the Wigner representation the kinetic equation (9) takes the form:

$$\frac{\partial}{\partial t} f_a(\mathbf{p}) =$$
$$= J_a(\mathbf{p}) = 2\hbar^2 \sum_b \int d\mathbf{p}'\, d\mathbf{k}[U_{ab}(\mathbf{k}) + \delta_{ab}\eta_a U_{ab}(\mathbf{p}' - \mathbf{p})]g'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k}). \quad (13)$$

Here the spin variables are omitted for simplicity. The solution of the equation for the pair correlation function $g'_{ab}(\mathbf{p}, \mathbf{p}', \mathbf{k})$ in the Wigner form can be expressed in the spatially homogeneous case in terms of the resolvent of equation (10) and its source (12).

$$g'_{ab}(p, p', \mathbf{k}, t) =$$
$$= \sum_{a'b'} \int d\mathbf{q}\, d\mathbf{q}' R_{ab,a'b'}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}', \mathbf{k}, z, \mu t) A'_{a'b'}(\mathbf{q}, \mathbf{q}', \mathbf{k}, \mu t)\Big|_{\omega=0}, \quad (14)$$

$$i\hbar A'_{ab}(\mathbf{p},\mathbf{p}',\mathbf{k},\mu t) = \\ = U_{ab}(\mathbf{k})\left\{ f_a(\mathbf{p})f_b(\mathbf{p}')\left[1+\eta_a f_a\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)\right]\left[1+\eta_b f_b\left(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2}\right)\right] - \right. \\ \left. - f_a\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right) f_b\left(\mathbf{p}'-\frac{\hbar\mathbf{k}}{2}\right)[1+\eta_a f_a(\mathbf{p})][1+\eta_b f_b(\mathbf{p}')]\right\} \quad (15)$$

with the resolvent $R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},z,\mu t)$ in Eq. (14) being a product of two resolvents

$$R_{ab,a'b'}(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}',\mathbf{k},t-t',\mu t') = \\ = R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},t-t',\mu t')R_{bb'}(\mathbf{p}',\mathbf{q}',\mathbf{k},t-t',\mu t'), \quad (16)$$

which satisfy the linearized Hartree–Fock equation

$$[\hbar z + \Delta_k E_a(\mathbf{p})]R_{aa'}(\mathbf{p},\mathbf{q},\mathbf{k},z,t) = \delta_{aa'}\delta(\mathbf{p}-\mathbf{q}) + \Delta_k f_a(\mathbf{p})\times \\ \times \sum_c \int d\mathbf{p}'\left[U_{ac}(\mathbf{k}) + \delta_{ac}\eta_a U_{ac}\left(\frac{\mathbf{p}-\mathbf{p}'}{\hbar}\right)\right] R_{ca'}(\mathbf{p}',\mathbf{q},\mathbf{k},z,t), \quad (17)$$

where

$$\Delta_k E_a(\mathbf{p}) = E_a\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right) - E_a\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right); \\ \Delta_k f_a(\mathbf{p}) = f_a\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right) - f_a\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right), \quad (18)$$

$$E_a(\mathbf{p}) = \frac{\mathbf{p}^2}{2m_a} + \eta_a \int d\mathbf{p}' U_{aa}\left(\frac{\mathbf{p}-\mathbf{p}'}{\hbar}\right) f_a(\mathbf{p}'). \quad (19)$$

The solution of Eq. (17) takes the form

$$R_{aa'}(\mathbf{p},\mathbf{p}',\mathbf{k},z,t) = \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\delta_{aa'}}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')} + \frac{U_{aa'}(\mathbf{k})}{\varepsilon^{\mathrm{HF}}(\omega,\mathbf{k})}\Psi_a^{(1)}(\mathbf{p})\Psi_{a'}^{(2)}(\mathbf{p}'), \quad (20)$$

where we introduced the notations

$$\Psi_a^{(1)}(\mathbf{p}) = \int d\mathbf{p}'' \frac{\Gamma_a(\mathbf{p},\mathbf{p}'')\Delta_k f_a(\mathbf{p}'')}{\hbar z - \Delta_k E_a(\mathbf{p}'')}, \\ \Psi_{a'}^{(2)}(\mathbf{p}') = \int d\mathbf{p}'' \frac{\Gamma_{a'}(\mathbf{p}'',\mathbf{p}')}{\hbar z - \Delta_k E_{a'}(\mathbf{p}')}, \quad (21)$$

and

$$\varepsilon^{\mathrm{HF}}(\omega,\mathbf{k}) = 1 - \Phi(\mathbf{k})\sum_a e_a^2 \int d\mathbf{p}\, d\mathbf{p}' \frac{\Gamma_a(\mathbf{p},\mathbf{p}')\Delta_k f_a(\mathbf{p}')}{\hbar z - \Delta_k E_a(\mathbf{p}')} \quad (22)$$

is the dielectric function with exchange interaction. The exchange scattering amplitude $\Gamma_a(\mathbf{p},\mathbf{p}')$ for Eqs. (20)–(22)) satisfies an integral equation, which contains only the exchange interaction potential:

$$\Gamma_a(\mathbf{p},\mathbf{p}') = \\ = \delta(\mathbf{p}-\mathbf{p}') + \eta_a \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \int d\mathbf{p}'' U_{aa}\left(\frac{\mathbf{p}-\mathbf{p}''}{\hbar}\right) \Gamma_a(\mathbf{p}'',\mathbf{p}'). \quad (23)$$

$\Gamma_a(\mathbf{p},\mathbf{p}')$ depends on $\mathbf{k}$ and z as on parameters and is similar to the vertex function, well known in many-particle perturbation theory. The solution of this equation in case of Coulomb interaction of the particles is difficult and requires an appropriate approximation. The simplest approximation is the replacement of the expression under the integral of (23) by the averaged over the impulse value

$$U_{aa}(\mathbf{k})G(z,\mathbf{k}) = \int d\mathbf{p}'' U_{aa}\left(\frac{\mathbf{p}-\mathbf{p}''}{\hbar}\right) \Gamma_a(\mathbf{p}'',\mathbf{p}'). \quad (24)$$

Then, the dielectric function, taking into account exchange interaction particles, takes the form:

$$\varepsilon^{\mathrm{HF}}(z,\mathbf{k}) = 1 - P(z,\mathbf{k})[1 + P(z,\mathbf{k})G(z,\mathbf{k})]^{-1}, \quad (25)$$

where

$$P(z,\mathbf{k}) = \sum_a U_{aa}(\mathbf{k}) \int d\mathbf{p} \frac{\Delta_k f_a(\mathbf{p})}{\hbar z - \Delta_k E_a(\mathbf{p})} \quad (26)$$

is the polarization. In the special case of the Hubbard approximation [11], one has $G(z,\mathbf{k}) = \frac{1}{2}\frac{k^2}{k^2 + k_f^2}$. One form of $G(z,\mathbf{k})$ was found for equilibrium state using a variation procedure [12]. Using the expression for the pair correlation function, we find the collision integral

$$J_a(\mathbf{p}) = 4\pi^2 e^4 \int \Phi^2(\mathbf{k})(1 - G(z,\mathbf{k}))\, d\mathbf{k}\, d\mathbf{q} \frac{\delta(\Delta_k E(\mathbf{q}) - \Delta_k E(\mathbf{p}))}{\left|\widetilde{\varepsilon}(\Delta_k E(\mathbf{q})/\hbar,\mathbf{k})\right|^2} \times \\ \times \left\{ f\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right) f\left(\mathbf{q}-\frac{\hbar\mathbf{k}}{2}\right) \left[1 - f\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right)\right] \left[1 - f\left(\mathbf{q}+\frac{\hbar\mathbf{k}}{2}\right)\right] - \right. \\ \left. - f\left(\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right) f\left(\mathbf{q}+\frac{\hbar\mathbf{k}}{2}\right) \left[1 - f\left(\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right)\right] \left[1 - f\left(\mathbf{q}-\frac{\hbar\mathbf{k}}{2}\right)\right] \right\}, \quad (27)$$

where

$$\widetilde{\varepsilon}(z,\mathbf{k}) = 1 - (1 - G(z,\mathbf{k}))P(z,\mathbf{k}). \quad (28)$$

From the collision integral (27) follows that $\left|\widetilde{\varepsilon}(z, \mathbf{k})\right|^2$ plays the role of the screening of the interaction potential $\Phi(\mathbf{k})$. It is interesting to note that Eq. (27) is differed from the corresponding Balescu's expressions by taking into account the exchange interaction in this screening. Moreover, the collision integral (27) contains the additional renormalization of the interaction $(1 - G(z,\mathbf{k}))$. However, $\widetilde{\varepsilon}(z, \mathbf{k})$ does not serve as linear responce function, in contrast to the Hartree–Fock dielectric function $\varepsilon^{\mathrm{HF}}(z, \mathbf{k})$ in Eq. (25).

CONCLUSION

Using the operator technique within BBGKY hierarchy we obtained a closed set of equations for the one- and two-particle density matrices, referring to the plasma approximation which considers also the exchange interaction. The equation for the pair correlation function is solved with the help of the resolvent of the Hartree–Fock equation. The expression obtained for the pair correlation function takes into account the exchange interaction. The latter is described by the scattering amplitude which is subject to the integral equation formulated above. The expression for the time-dependent nonlocal collision integral and the internal energy are obtained with the exchange interaction and polarization taken into account.

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