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INTERACTIONS FOR MASSIVE MIXED SYMMETRY FIELD

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Here we give the first two examples of nontrivial interactions for simplest massive mixed symmetry field (hook) in general $(A)dS$ space with arbitrary value of cosmological constant, including flat Minkowski space. For that purpose, using frame-like gauge-invariant description of massive higher spin particles, we extend the Fradkin–Vasiliev approach, initially developed for investigations of gravitational and other interactions for massless higher spin particles in AdS space, to the case of arbitrary combinations of massive and/or massless particles, including, e.g., electromagnetic and gravitational interactions for massive higher spin ones.

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1. MASSIVE FIELDS INTERACTIONS A LA FRADKIN–VASILIEV

As is well known, any attempt to switch on minimal gravitational interactions for higher spin fields spoils their gauge invariance:

$$e_\mu^a \Rightarrow e_\mu^a + h_\mu^a, \quad D_\mu \Rightarrow D_\mu - \omega_\mu \Longrightarrow \delta S \sim R,$$

where R stands for gravitational Riemann tensor and for massless fields in a flat Minkowski space it cannot be restored by any nonminimal corrections to the Lagrangian and/or gauge transformations. Similarly, any attempt to switch on minimal electromagnetic interactions spoils gauge invariance:

$$D_\mu \Rightarrow D_\mu + e_0 A_\mu \Longrightarrow \delta S \sim F,$$

where F stands for electromagnetic field strength and for massless fields in a flat Minkowski space it also cannot be restored by nonminimal corrections to the Lagrangian and/or gauge transformations.

But Fradkin and Vasiliev showed [1, 2] that in AdS space it is possible to restore broken gauge invariance with the introduction of nonminimal higher derivatives corrections containing Riemann tensor with the coefficients proportional to inverse powers of cosmological constant

$$\delta S \sim \sum \frac{1}{\Lambda^n} R^n$$

so that the flat limit $\Lambda \to 0$ is impossible. Similarly, for the electromagnetic interactions it is also possible to restore broken invariance with the nonminimal higher derivatives corrections containing electromagnetic field strength with the coefficients proportional to inverse powers of cosmological constant

$$\delta S \sim \sum \frac{1}{\Lambda^n} F^n.$$

Moreover, the procedure can be reversed: starting with massless field in a flat space and nonminimal interactions with the highest number of derivatives, one can reproduce minimal gravitational or electromagnetic interactions by smooth AdS deformation.

Note that the appearance of higher derivatives in the equations of motion in general increases the number of degrees of freedom, while appearance of higher derivatives of gauge parameters in gauge transformations in general increases the number of constraints, thus decreasing the number of degrees of freedom. It appears to be highly nontrivial to keep balance between these two mechanisms in such a way that interacting theory has the same number of physical degrees of freedom as the initial free one. One of the effective and convenient ways to resolve these difficulties is to use the so-called frame-like formalism [3, 4]. Such a formalism is just a higher spin generalization of the well-known frame-like formalism in gravity

$$h_{\mu\nu} \Rightarrow h_\mu^a \oplus \omega_\mu^{ab}, \quad \xi_\mu \Rightarrow \xi^a \oplus \eta^{ab},$$

where instead of symmetric tensor $h_{\mu\nu}$ one uses general tensor h_μ^a and auxiliary Lorentz connection ω_μ^{ab} with both their own gauge parameters ξ^a and η^{ab}. In this case, on the mass shell the Lorentz connection can be expressed through the first derivatives of h_μ^a $\omega \sim Dh$ and similarly $\eta \sim D\xi$.

For the description of spin-3 particles, such a formalism requires introduction of one auxiliary and one extra field:

$$\Phi_{\mu\nu\alpha} \Rightarrow \Phi_\mu^{ab} \oplus \Omega_\mu^{ab,c} \oplus \Sigma_\mu^{ab,cd},$$
$$\xi_{\mu\nu} \Rightarrow \xi^{ab} \oplus \eta^{ab,c} \oplus \zeta^{ab,cd},$$

again each with the their own gauge parameters. Thus, on the mass shell one obtains: $\Omega \sim D\Phi$, $\Sigma \sim D^2\Phi$, $\eta \sim D\xi$, $\zeta \sim D^2\xi$.

For the arbitrary spin particle, one has to introduce a whole bunch of extra fields and additional gauge parameters:

$$\Phi \Rightarrow \Phi \oplus \Omega \oplus \Sigma_1 \oplus \cdots \oplus \Sigma_{s-2},$$
$$\xi \Rightarrow \xi \oplus \eta \oplus \zeta_1 \oplus \cdots \oplus \zeta_{s-2}.$$

On the mass shell all these fields can be expressed through the higher and higher derivatives of the main field and similarly additional gauge parameters — through the higher derivatives of the main one. Thus, all these higher derivatives turn out to be hidden and do not appear explicitly.

Using such a frame-like formalism, Fradkin and Vasiliev developed an effective approach to investigations of gravitational and other interactions for massless higher spin particles in AdS space. Let us illustrate this approach using the simplest but nontrivial example — massless spin 2.

In a frame-like formalism, free Lagrangian describing massless spin-2 particle in AdS space has the form

$$\mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu^{ac}\omega_\nu^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu^{ab} D_\nu h_\alpha^c - \frac{\Lambda(d-2)}{2}\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu^a h_\nu^b. \quad (1)$$

It is invariant under the following gauge transformations:

$$\delta_0 h_\mu^a = D_\mu \xi^a + \eta_\mu^a, \quad \delta_0 \omega_\mu^{ab} = D_\mu \eta^{ab} + \Lambda e_\mu^{[a}\xi^{b]}. \quad (2)$$

It is easy to construct two gauge-invariant objects (linearized curvature and torsion):

$$\begin{aligned} R_{\mu\nu}^{ab} &= D_{[\mu}\omega_{\nu]}{}^{ab} + \Lambda e_{[\mu}{}^{[a} h_{\nu]}{}^{b]}, \\ T_{\mu\nu}^a &= D_{[\mu}h_{\nu]}^a - \omega_{[\mu,\nu]}^a. \end{aligned} \quad (3)$$

Moreover, the free Lagrangian can be rewritten through these curvatures as follows:

$$\mathcal{L}_0 = -\frac{1}{32\Lambda(d-3)}\left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd}. \quad (4)$$

Now let us consider the most general corrections for curvatures quadratic in fields, as well as the most general corrections to gauge transformations linear in fields, and require that variations of deformed curvatures be proportional to curvatures themselves. It is not hard to check that with the deformed curvatures

$$\begin{aligned} \hat{R}_{\mu\nu}{}^{ab} &= D_{[\mu}\omega_{\nu]}{}^{ab} + \Lambda e_{[\mu}{}^{[a} h_{\nu]}{}^{b]} + \omega_{[\mu}{}^{ac}\omega_{\nu]}{}^{bc} - \Lambda h_{[\mu}{}^a h_{\nu]}{}^b, \\ \hat{T}_{\mu\nu}{}^a &= D_{[\mu}h_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a - \omega_{[\mu}{}^{ab}h_{\nu]}{}^b \end{aligned} \quad (5)$$

and corresponding corrections to gauge transformations

$$\delta\omega_\mu{}^{ab} = \eta^{c[a}\omega_\mu{}^{b]c} - \Lambda h_\mu{}^{[a}\xi^{b]}, \quad \delta h_\mu^a = \eta^{ab}h_\mu^b - \omega_\mu{}^{ab}\xi^b, \quad (6)$$

we indeed obtain

$$\delta\hat{R}_{\mu\nu}{}^{ab} = \eta^{c[a}R_{\mu\nu}{}^{b]c} - \Lambda T_{\mu\nu}{}^{[a}\xi^{b]}, \quad \delta\hat{T}_{\mu\nu}{}^a = \eta^{ab}T_{\mu\nu}{}^b - R_{\mu\nu}{}^{ab}\xi^b. \quad (7)$$

Now we can easily construct an interacting Lagrangian keeping the same form as for the free one but with curvatures replaced by the deformed ones:

$$\mathcal{L}_0 = -\frac{1}{32\Lambda(d-3)} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd}. \tag{8}$$

As we have seen, two main ingredients of the Fradkin–Vasiliev approach are gauge invariance and frame-like formalism. But there exists a frame-like gauge invariant description for massive fields (both symmetric and mixed symmetry ones) [5–8] that nicely works in $(A)dS$ spaces and flat Minkowski space, including all possible massless and partially massless limits. Thus, it seems natural to extend the Fradkin–Vasiliev approach to such a frame-like gauge-invariant formalism for investigation of possible interactions for any combination of massive and/or massless particles. In what follows, we consider two examples of such an approach, namely, electromagnetic and gravitational interactions for the simplest massive mixed symmetry field (hook).

2. ELECTROMAGNETIC INTERACTIONS FOR MASSIVE HOOK

In this section, we consider electromagnetic interactions for massive hook [9]. The frame-like gauge-invariant description requires two pairs of fields: ($\Omega_\mu{}^{abc}$, $\Omega_\mu{}^{ab}$) and (C^{abc}, B^{ab}) (all of them are antisymmetric in local indices). Free Lagrangian for AdS space looks as follows:

$$\begin{aligned}\mathcal{L}_0 = -\frac{3}{4} \left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{acd} \Omega_\nu{}^{bcd} - \frac{3}{8} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{abd} D_\nu \Omega_\alpha{}^{cd} - \frac{1}{6} C_{abc}{}^2 - \\ - \frac{1}{4} e^\mu{}_a C^{abc} D_\mu B^{bc} - m_2 e^\mu{}_a \left[\frac{3}{2} \Omega_\mu{}^{abc} B^{bc} + C^{abc} \Omega_\mu{}^{bc} \right] - \\ - \frac{m_1^2}{2} \left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{ac} \Omega_\nu{}^{bc} - \frac{\tilde{m}_1^2}{4} B_{ab}{}^2,\end{aligned} \tag{9}$$

where $m_{1,2}$ — two mass-like parameters such that $3m_2^2 - m_1^2 \sim \Lambda$. This Lagrangian is invariant under the following gauge transformations:

$$\begin{aligned}&\delta\Omega_\mu{}^{abc} = D_\mu \eta^{abc} + \frac{4m_1^2}{3(d-3)} e_\mu{}^{[a} \eta^{bc]}, \quad \delta\Omega_\mu{}^{ab} = D_\mu \eta^{ab} - 2\eta_\mu{}^{ab}, \\ &\delta C^{abc} = 6m_2 \eta^{abc}, \quad \delta B^{ab} = -4m_2 \eta^{ab}.\end{aligned} \tag{10}$$

It is straightforward to construct corresponding gauge-invariant objects (we will call them curvatures) for all four fields:

$$\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{abc} &= D_{[\mu}\Omega_{\nu]}{}^{abc} + \frac{4m_2}{3(d-3)} e_{[\mu}{}^{[a} C_{\nu]}{}^{bc]} + \frac{4m_1^2}{3(d-3)} e_{[\mu}{}^{[a}\Omega_{\nu]}{}^{bc]}, \\
\mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} + 2\Omega_{[\mu,\nu]}{}^{ab} - \frac{2m_2}{(d-3)} e_{[\mu}{}^{[a} B_{\nu]}{}^{b]}, \\
\mathcal{C}_{\mu}{}^{abc} &= D_\mu C^{abc} - 6m_2\Omega_\mu{}^{abc} - \frac{2m_1^2}{(d-3)} e_\mu{}^{[a} B^{bc]}, \\
\mathcal{B}_\mu{}^{ab} &= D_\mu B^{ab} + \frac{4}{3} C_\mu{}^{ab} + 4m_2\Omega_\mu{}^{ab}.
\end{aligned} \tag{11}$$

Moreover, the free Lagrangian can be rewritten as an expression quadratic in these curvatures:

$$\begin{aligned}
\mathcal{L}_0 = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \left[a_1 \mathcal{R}_{\mu\nu}{}^{abe}\mathcal{R}_{\alpha\beta}{}^{cde} + a_2 \mathcal{F}_{\mu\nu}{}^{ab}\mathcal{F}_{\alpha\beta}{}^{cd} \right] + \\
+ \left\{ {}^{\mu\nu}_{ab} \right\} \left[a_3 \mathcal{C}_\mu{}^{acd}\mathcal{C}_\nu{}^{bcd} + a_4 \mathcal{B}_\mu{}^{ac}\mathcal{B}_\nu{}^{bc} \right],
\end{aligned} \tag{12}$$

where $a_{1,3} \sim 1/m_1^2$, $a_{2,4} \sim 1$.

Now let us consider appropriate deformations for curvatures. As for the hook's curvatures, it turns out that their deformation corresponds to standard minimal substitution $D_\mu \Rightarrow D_\mu + e_0 A_\mu$. After that they transform as follows:

$$\delta\hat{R}_{\mu\nu}{}^{abc,i} = e_0 \varepsilon^{ij} F_{\mu\nu}\eta^{abc,j}, \quad \delta\hat{\mathcal{F}}_{\mu\nu}{}^{ab,i} = e_0\varepsilon^{ij} F_{\mu\nu}\eta^{ab,j}. \tag{13}$$

In this case, deformation for electromagnetic field strength appears to be nontrivial and looks like

$$\begin{aligned}
\hat{F}_{\mu\nu} = F_{\mu\nu} + a_0\varepsilon^{ij} \Bigg[\Omega_{[\mu}{}^{abc,i}\Omega_{\nu]}{}^{abc,j} - \frac{2}{3(d-3)} C_{[\mu}{}^{ab,i} C_{\nu]}{}^{ab,j} + \\
+ \frac{2m_1^2}{(d-3)} \Omega_{[\mu}{}^{ab,i}\Omega_{\nu]}{}^{ab,j} - \frac{2m_1^2}{(d-3)^2} B_{[\mu}{}^{a,i} B_{\nu]}{}^{a,j} \Bigg],
\end{aligned} \tag{14}$$

while transformations for deformed field strength have the form

$$\delta\hat{F}_{\mu\nu} = 2a_0\varepsilon^{ij} \left[\mathcal{R}_{\mu\nu}{}^{abc,i}\eta^{abc,j} + \frac{2m_1^2}{(d-3)} \mathcal{F}_{\mu\nu}{}^{ab,i}\eta^{ab,j} \right]. \tag{15}$$

At last, we consider an interacting Lagrangian which has the same form as the free one but with curvatures replaced by the deformed ones:

$$\begin{aligned}
\mathcal{L} = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \left[a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe}\hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab}\hat{\mathcal{F}}_{\alpha\beta}{}^{cd} \right] + \\
+ \left\{ {}^{\mu\nu}_{ab} \right\} \left[a_3 \hat{\mathcal{C}}_\mu{}^{acd}\hat{\mathcal{C}}_\nu{}^{bcd} + a_4 \hat{\mathcal{B}}_\mu{}^{ac}\hat{\mathcal{B}}_\nu{}^{bc} \right] - \frac{1}{4}\hat{F}_{\mu\nu}{}^2.
\end{aligned} \tag{16}$$

In this case, the requirement that this Lagrangian be invariant in the linear approximation gives one relation on the parameters:

$$e_0 \sim a_0 m_1^2.$$

A few remarks are in order.

• It is possible to take a smooth massless limit in AdS space: $m_2 \to 0$, $m_1^2 \sim -\Lambda$ and thus obtain electromagnetic interactions for massless hook in AdS.

• Similarly, nothing prevents us from considering flat limit: $m_2^2 \to 3m_1^2$, $\Lambda \to 0$, thus giving us one more nontrivial example of electromagnetic interactions for massive particle in a flat Minkowski space.

• At the same time, it turns out to be impossible to take a massless limit in dS space $m_1 \to 0$ without switching off electromagnetic interactions.

3. GRAVITATIONAL INTERACTIONS FOR MASSIVE HOOK

In this section, we consider gravitational interactions for massive hook [10], using the same description for free hook, so we will not repeat it here. Instead, we begin directly with the search for appropriate curvatures deformations.

As for the hook's curvatures, this time they also correspond just to the standard minimal substitution rule:

$$e_\mu^a \Rightarrow e_\mu^a + h_\mu^a, \quad D_\mu \Rightarrow D_\mu - \omega_\mu.$$

In this case, deformed curvatures transform as follows:

$$\begin{gathered} \delta\hat{\mathcal{R}}_{\mu\nu}{}^{abc} = R_{\mu\nu}{}^{d[a}\eta^{bc]d} - \frac{4m_1^2}{3(d-3)} T_{\mu\nu}{}^{[a}\eta^{bc]}, \\ \delta\hat{\mathcal{F}}_{\mu\nu}{}^{ab} = 2\eta^{abc}T_{\mu\nu}{}^{c} - R_{\mu\nu}{}^{c[a}\eta^{b]c}, \\ \delta\hat{\mathcal{C}}_\mu{}^{abc} = 0, \quad \delta\hat{\mathcal{B}}_\mu{}^{ab} = 0. \end{gathered} \tag{17}$$

At the same time, deformed Riemann tensor (we will not need deformed torsion here) has the form

$$\begin{aligned} \hat{R}_{\mu\nu}{}^{ab} = R_{\mu\nu}{}^{ab} + a_0 \Bigg[\Omega_{[\mu}{}^{acd}\Omega_{\nu]}{}^{bcd} + \frac{4}{9(d-3)} C_{[\mu}{}^{ca} C_{\nu]}{}^{bc} - \\ - \frac{4m_1^2}{3(d-3)} \Omega_{[\mu}{}^{ca}\Omega_{\nu]}{}^{bc} - \frac{m_1^2}{3(d-3)^2} B_{[\mu}{}^{a} B_{\nu]}{}^{b} \Bigg], \end{aligned} \tag{18}$$

while its transformations look like

$$\delta\hat{R}_{\mu\nu}{}^{ab} = -a_0\eta^{cd[a}\mathcal{R}_{\mu\nu}{}^{b]cd} + \frac{4m_1^2 a_0}{3(d-3)} \eta^{c[a}\mathcal{F}_{\mu\nu}{}^{b]c}. \tag{19}$$

Now we consider an interacting Lagrangian which has the same form as the free one but with curvatures replaced by the deformed ones:

$$\mathcal{L}_0 = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \left[a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd} \right] + \\ + \left\{ {}^{\mu\nu}_{ab} \right\} \left[a_3 \hat{\mathcal{C}}_\mu{}^{acd} \hat{\mathcal{C}}_\nu{}^{bcd} + a_4 \hat{\mathcal{B}}_\mu{}^{ac} \hat{\mathcal{B}}_\nu{}^{bc} \right] - \\ - \frac{1}{32\Lambda(d-3)} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd} \quad (20)$$

and require that it be gauge-invariant in the linear approximation. This gives us

$$a_0 = \frac{9(d-3)\Lambda}{32 m_1^2}.$$

A few remarks are in order.

• Massless limit in AdS space is nonsingular, and we obtain gravitational interactions for massless hook in AdS that agree with previously obtained results [11].

• Flat limit is nontrivial due to massless graviton:

$$\mathcal{L}_g \sim \frac{1}{\Lambda} \hat{R}^2, \quad \delta\hat{R} \sim \frac{\Lambda}{m_1^2} \mathcal{R}(\mathcal{F}),$$

but at least for cubic interactions it is nonsingular, giving us an example of gravitational interactions for massive field in a flat Minkowski space.

• Similarly to the electromagnetic case, it is impossible to take massless limit in dS space ($m_1 \to 0$) without switching off minimal interactions.

CONCLUSION

Thus, we have seen that the Fradkin–Vasiliev approach with frame-like gauge-invariant formalism allows one to effectively investigate possible interactions for any set of massive and/or massless fields both in AdS and in flat Minkowski space. In particular, it turns out to be very well suited for investigations of electromagnetic and gravitational interactions for massive higher spin particles. One of the issues that deserve further study is flat limit for interactions of massive higher spin particles with massless gravity. Also, it would be interesting to understand the striking difference between massless limits in AdS and dS spaces as far as possibility to switch on interactions is concerned.

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