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$\mathcal{N} = 4$ SYM LOW-ENERGY EFFECTIVE ACTION IN $\mathcal{N} = 4$ HARMONIC SUPERSPACE

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We apply the $\mathcal{N} = 4$ harmonic superspace with $USp(4)$ harmonic variables for describing the $\mathcal{N} = 4$ SYM low-energy effective action. Scale invariance and gauge symmetry fix the leading term in the low-energy effective action uniquely, up to a constant. The value of the remaining constant can be fixed by the topological quantization condition for the Wess–Zumino term which is present in the component structure of this action.

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1. INTRODUCTION AND SUMMARY

The problem of low-energy effective action in $\mathcal{N} = 4$ SYM theory is very interesting for its applications to the D3-brane dynamics in string theory (see, e.g., [1]), but it can be considered by its own due to its wonderful properties. Indeed, the $\mathcal{N} = 4$ SYM model is a rare example of completely finite quantum field theory which respects the superconformal symmetry not only on the classical but also on the quantum level. The maximally extended supersymmetry and superconformal invariance restrict the quantum dynamics so strong that the leading terms in the effective action can be obtained without employing the perturbative calculations. In the present work we demonstrate that the $\mathcal{N} = 4$ harmonic superspace approach is very useful for studying the leading terms in the $\mathcal{N} = 4$ SYM effective action.

The $\mathcal{N} = 4$ gauge multiplet consists of $\mathcal{N} = 2$ gauge multiplet and hypermultiplet which are naturally described within the $\mathcal{N} = 2$ harmonic superspace approach [2]. In this superspace, half of supersymmetries of the $\mathcal{N} = 4$ gauge theory are realized explicitly while another half are hidden. Therefore it is quite nontrivial to respect these hidden supersymmetries on the quantum level as the symmetries of the effective action. For instance, the part of the $\mathcal{N} = 4$ SYM effective action in the sector of $\mathcal{N} = 2$ gauge multiplet was known long ago [3,4], but it took several years to find a hypermultiplet completion of this action which

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is compatible with hidden $\mathcal{N}=2$ supersymmetries [5]. Our aim is to develop such a superspace description of the low-energy effective action which makes all the supersymmetries of the $\mathcal{N}=4$ theory manifest.

In the present work we consider the $\mathcal{N}=4$ harmonic superspace with $USp(4)$ harmonic variables [6, 7]. As will be shown, the $\mathcal{N}=4$ superfield strength in this superspace is described by single chargeless analytic superfield. It is very easy to construct a scale-invariant action with this superfield. By considering the component structure of this action we show that it takes into account all terms in the $\mathcal{N}=4$ low-energy effective action up to the four-derivative order on mass shell. In particular, we derive the Wess–Zumino and F^4/X^4 terms.

2. $\mathcal{N}=4$ SUPERGAUGE MULTIPLET IN $\mathcal{N}=4$ $USp(4)$ HARMONIC SUPERSPACE

Conventional $\mathcal{N}=4$ superspace is parameterized by the coordinates $(x^m, \theta_{i\alpha}, \bar\theta^i_{\dot\alpha})$, where x^m are the Minkowski space coordinates and $\theta_{i\alpha}$, $\bar\theta^i_{\dot\alpha}$, $i=1,2,3,4$, are the Grassmann variables. We extend this superspace with the harmonic variables $u_i^{(\pm,0)}$, $u_i^{(0,\pm)}$ which obey the constraints

$$\begin{gathered} u^{(+,0)i}u_i^{(-,0)} = u^{(0,+)i}u_i^{(0,-)} = 1, \\ u_i^{(+,0)}u^{(0,+)i} = u_i^{(+,0)}u^{(0,-)i} = u_i^{(0,+)}u^{(-,0)i} = u_i^{(-,0)}u^{(0,-)i} = 0, \\ u_i^{(+,0)}u_j^{(-,0)} - u_j^{(+,0)}u_i^{(-,0)} + u_i^{(0,+)}u_j^{(0,-)} - u_j^{(0,+)}u_i^{(0,-)} = \Omega_{ij}. \end{gathered} \tag{1}$$

Here Ω_{ij} is the antisymmetric constant tensor in the $USp(4)$ group which is used for raising and lowering the indices.

The orthogonality and completeness relations (2) are used for obtaining the projections of the Grassmann variables and derivatives, e.g.,

$$\begin{gathered} \theta_\alpha^{(\pm,0)} = -u^{(\pm,0)i}\theta_{i\alpha}, \quad \theta_\alpha^{(0,\pm)} = -u^{(0,\pm)i}\theta_{i\alpha}, \\ D_\alpha^{(\pm,0)} = u_i^{(\pm,0)}D^i_\alpha, \quad D_\alpha^{(0,\pm)} = u_i^{(0,\pm)}D^i_\alpha, \end{gathered} \tag{2}$$

where D^i_α are standard $\mathcal{N}=4$ covariant spinor derivatives. There are also covariant harmonic derivatives

$$D^{(\pm\pm,0)}, \quad D^{(0,\pm\pm)}, \quad D^{(\pm,\pm)}, \quad D^{(\pm,\mp)}, \quad S_1, \qquad S_2, \tag{3}$$

which are written down explicitly in [6, 7]. The operators S_1 and S_2 in this list measure the $U(1)$ charges of the other operators and superfields.

The analytic subspace in the full superspace contains eight of sixteen Grassmann variables,

$$\{\zeta, u\} = \{(x^m_A, \theta_\alpha^{(+,0)}, \theta_\alpha^{(-,0)}, \bar\theta_{\dot\alpha}^{(0,+)}, \bar\theta_{\dot\alpha}^{(0,-)}), u\}, \tag{4}$$

where

$$x_A^m = x^m - i\theta^{(0,-)}\sigma^m\bar\theta^{(0,+)} + i\theta^{(0,+)}\sigma^m\bar\theta^{(0,-)} - \\ - i\theta^{(+,0)}\sigma^m\bar\theta^{(-,0)} + i\theta^{(-,0)}\sigma^m\bar\theta^{(+,0)}. \quad (5)$$

In this analytic subspace, the following Grassmann derivatives become short:

$$D_\alpha^{(0,\pm)} = \pm\frac{\partial}{\partial\theta^{(0,\mp)\alpha}}, \quad \bar D_{\dot\alpha}^{(\pm,0)} = \pm\frac{\partial}{\partial\bar\theta^{(\mp,0)\dot\alpha}}. \quad (6)$$

In the conventional $\mathcal{N}=4$ superspace $\{x^m, \theta_{i\alpha}, \bar\theta^i_{\dot\alpha}\}$, the $\mathcal{N}=4$ gauge superfield strength W^{ij} is constrained by [8]

$$W^{ij} = -W^{ji}, \quad \overline{W^{ij}} = \frac{1}{2}\varepsilon_{ijkl}W^{kl}, \\ D^i_\alpha W^{jk} + D^j_\alpha W^{ik} = 0, \quad \bar D_{i\dot\alpha}W^{jk} = \frac{1}{3}(\delta^j_i \bar D_{l\alpha}W^{lk} - \delta^k_i \bar D_{l\dot\alpha}W^{lj}). \quad (7)$$

Among six different harmonic projections of W^{ij}, we pick out the following one:

$$\mathcal{W} = u_i^{(0,+)}u_j^{(0,-)}W^{ij}, \quad (8)$$

which is real under special conjugation in the harmonic superspace, $\widetilde{\mathcal{W}} = \mathcal{W}$. This superfield alone is sufficient to describe the $\mathcal{N}=4$ gauge multiplet. The constraints (7) imply the following restrictions on $\mathcal{W}$:

$$D_\alpha^{(0,+)}\mathcal{W} = D_\alpha^{(0,-)}\mathcal{W} = \bar D_{\dot\alpha}^{(+,0)}\mathcal{W} = \bar D_{\dot\alpha}^{(-,0)}\mathcal{W} = 0, \\ D^{(++,0)}\mathcal{W} = D^{(--,0)}\mathcal{W} = D^{(0,++)}\mathcal{W} = D^{(0,--)}\mathcal{W} = 0, \\ (D^{(+,+)})^2\mathcal{W} = 0. \quad (9)$$

One can check that these equations kill all auxiliary field components in $\mathcal{W}$ and put the physical ones on shell. In particular, subject to (9) the component structure of $\mathcal{W}$ in the bosonic sector reads

$$\mathcal{W} = \varphi + \{^{\rangle|}(\sqcap_{[\rangle}^{(+,\prime)}\sqcap_{|]}^{(-,\prime)} - \sqcap_{[\rangle}^{(\prime,+)}\sqcap_{|]}^{(\prime,-)}) + \\ + \frac{1}{\sqrt{2}}(\theta_\alpha^{(+,0)}\theta_\beta^{(-,0)}\sigma^{m\alpha}{}_{\dot\alpha}\sigma^{n\beta\dot\alpha} - \bar\theta_{\dot\alpha}^{(0,+)}\bar\theta_{\dot\beta}^{(0,-)}\sigma^{m\dot\alpha}{}_\alpha\sigma^{n\alpha\dot\beta})F_{mn} - \\ - 4i\theta_\alpha^{(+,0)}\bar\theta_{\dot\alpha}^{(0,+)}\partial^{\alpha\dot\alpha}f^{ij}u_{[i}^{(-,0)}u_{j]}^{(0,-)} - 4i\theta_\alpha^{(-,0)}\bar\theta_{\dot\alpha}^{(0,-)}\partial^{\alpha\dot\alpha}f^{ij}u_{[i}^{(+,0)}u_{j]}^{(0,+)} + \\ + 4i\theta_\alpha^{(+,0)}\bar\theta_{\dot\alpha}^{(0,-)}\partial^{\alpha\dot\alpha}f^{ij}u_{[i}^{(-,0)}u_{j]}^{(0,+)} + 4i\theta_\alpha^{(-,0)}\bar\theta_{\dot\alpha}^{(0,+)}\partial^{\alpha\dot\alpha}f^{ij}u_{[i}^{(+,0)}u_{j]}^{(0,-)} + \\ + 4\theta_\alpha^{(+,0)}\theta_\beta^{(-,0)}\bar\theta_{\dot\alpha}^{(0,+)}\bar\theta_{\dot\beta}^{(0,-)}\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}[\varphi - f^{ij}(u_{[i}^{(+,0)}u_{j]}^{(-,0)} - u_{[i}^{(0,+)}u_{j]}^{(0,-)})]. \quad (10)$$

Here φ and $f^{ij} = -f^{ji}$ are six scalar fields, and F_{mn} is the Maxwell field strength.

3. $\mathcal{N}=4$ SYM LOW-ENERGY EFFECTIVE ACTION

The most general action with the superfield (10) which involves at most four space-time derivatives reads

$$\Gamma_4 = \int d\zeta\, dv\, \mathcal{H}(\mathcal{W}), \tag{11}$$

with some function $\mathcal{H}(\mathcal{W})$ to be determined. We point out that the analytic superspace measure $d\zeta$ is dimensionless and has eight Grassmann derivatives which are equivalent to four space-time ones. Since the mass dimension of the superfield $\mathcal{W}$ is equal to one, it is necessary to introduce a scale Λ, $[\Lambda]=1$, $\mathcal{H}(\mathcal{W})=\mathcal{H}(W/\Lambda)$. The scale invariance of the effective action fixes the form of this function $\mathcal{H}$ uniquely, up to a constant,

$$\Lambda\frac{d}{d\Lambda}\mathcal{H}(W/\Lambda)=0 \Rightarrow \mathcal{H}=c\ln\frac{\mathcal{W}}{\Lambda}. \tag{12}$$

As a result, the four-derivative part of the $\mathcal{N}=4$ SYM low-energy effective action acquires extremely simple form,

$$\Gamma_4 = c\int d\zeta\, dv\, \ln\frac{\mathcal{W}}{\Lambda}, \tag{13}$$

with c being some coefficient. For the case of gauge group $SU(2)$ spontaneously broken down to $U(1)$, this coefficient is $c=-1/(96\pi^2)$.

Substituting (10) into (13) we find that this action contains the following two terms among the others:

$$\Gamma_{F^4} = -\frac{3}{2}c\int d^4x\,\frac{F_{mn}F^{nk}F_{kl}F^{lm}-1/4(F_{pq}F^{pq})^2}{(\varphi^2+X^aX^a)^2}, \tag{14}$$

$$S_{WZ} = -\frac{8}{5}c\,\varepsilon^{mnpq}\varepsilon^{abcde}\int d^4x\,\frac{g\left(\sqrt{(X_fX_f)/\varphi^2}\right)}{\varphi^5}X_a\partial_mX_b\partial_nX_c\partial_pX_d\partial_qX_e, \tag{15}$$

where

$$g(x)=\frac{5}{8x^5}\left(3\arctan x-\frac{x\left(3+5x^2\right)}{\left(1+x^2\right)^2}\right), \tag{16}$$

and X_a are five scalar fields related with f_{ij} by means of the $SO(5)$ gamma-matrices, $X_a=\gamma_a^{ij}f_{ij}$. Here (14) is the well-known F^4/X^4 term which is explicitly $SO(6)$ invariant, while (15) is the Wess–Zumino term which is given here in the $SO(5)$ covariant form.

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