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FLUXBRANE AND S-BRANE SOLUTIONS RELATED TO LIE ALGEBRAS

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We overview composite fluxbrane and special S-brane solutions for a wide class of intersection rules related to semisimple Lie algebras. These solutions are defined on a product manifold $R_* \times M_1 \times \ldots \times M_n$ which contains n Ricci-flat spaces $M_1, \ldots, M_n$ with one-dimensional R_* and M_1. They are governed by a set of moduli functions H_s, which have polynomial structure. The powers of polynomials coincide with the components of the dual Weyl vector in the basis of simple coroots.

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INTRODUCTION

In this paper we overview fluxbrane and special S-brane solutions related to semisimple finite-dimensional (FD) Lie algebras [1, 2]. These solutions contain a subclass of (partially) supersymmetric solutions related to Lie algebras $A_1 \oplus \ldots \oplus A_1$ (at least for $M_i = \mathbf{R}^{d_i}$). The solutions are governed by functions $H_s(z) > 0$ defined on the interval $(0, +\infty)$ and obeying differential equations

$$\frac{d}{dz}\left(\frac{z}{H_s}\frac{d}{dz}H_s\right) = \frac{1}{4}B_s \prod_{s' \in S} H_{s'}^{-A_{ss'}} \tag{1}$$

with the boundary conditions imposed:

$$H_s(+0) = 1, \tag{2}$$

$s \in S$ (S is nonempty set). Here and in what follows all $B_s > 0$ are constants, and $(A_{ss'})$ is the Cartan matrix ($A_{ss} = 2$) of some semisimple FD Lie algebra $\mathcal{G}$.

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It was conjectured in [1] that Eqs. (1), (2) have polynomial solutions. For semisimple Lie algebra the powers of polynomials coincide with the components of the dual Weyl vector in the basis of simple coroots. In [1, 2] the polynomials corresponding to Lie algebras $A_1 \oplus \ldots \oplus A_1, A_2, C_2$, and G_2 were presented. The conjecture may be verified for any classical simple Lie algebra using the program from [3], where the polynomials corresponding to exceptional Lie algebras F_4 and E_6 were found as well.

1. «FLUX-S-BRANE» SOLUTIONS

We consider a model governed by the action

$$S = \int d^D x \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \triangle} \frac{\theta_a}{N_a!} \exp\,[2\lambda_a(\varphi)](F^a)^2 \right\}, \tag{3}$$

where $g = g_{MN}(x)\, dx^M \otimes dx^N$ is a metric, $\varphi = (\varphi^\alpha) \in \mathbf{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric nondegenerate $l \times l$ matrix ($l \in \mathbf{N}$), $\theta_a = \pm 1$, $F^a = dA^a$ is a N_a-form ($n_a \geqslant 1$), λ_a is a 1-form on $\mathbf{R}^l$: $\lambda_a(\varphi) = \lambda_{a\alpha}\varphi^\alpha$, $a \in \triangle$, $\alpha = 1, \ldots, l$. Here $\triangle$ is some finite set.

Let us consider a family of exact solutions to field equations corresponding to the action (3) and depending on one variable ρ. These solutions are defined on the manifold

$$M = (0, +\infty) \times M_1 \times M_2 \times \ldots \times M_n, \tag{4}$$

where M_1 is one-dimensional manifold. The solutions read [2]

$$g = \left(\prod_{s \in S} H_s^{2h_s d(I_s)/(D-2)} \right) \left\{ w d\,\rho \otimes d\rho + \left(\prod_{s \in S} H_s^{-2h_s} \right) \rho^2 g^1 + \right. \\ \left. + \sum_{i=2}^{n} \left(\prod_{s \in S} H_s^{-2h_s \delta_{iI_s}} \right) g^i \right\}, \tag{5}$$

$$\exp\,(\varphi^\alpha) = \prod_{s \in S} H_s^{h_s \chi_s \lambda_{a_s}^\alpha}, \tag{6}$$

$$F^a = \sum_{s \in S_e} (-Q_s) \left(\prod_{s' \in S} H_{s'}^{-A_{ss'}} \right) \rho d\rho \wedge \tau(I_s) + \sum_{s \in S_m} Q_s \tau(\bar{I}_s). \tag{7}$$

Functions $H_s(z) > 0$, $z = \rho^2$ obey Eq. (1) with boundary conditions (2).

In Eq. (5), $g^i = g^i_{m_i n_i}(y_i)\, dy_i^{m_i} \otimes dy_i^{n_i}$ is a Ricci-flat metric on M_i, $i = 1, \ldots, n$,

$\delta_{iI} = \sum_{j \in I} \delta_{ij}$ is the indicator of i belonging to I: $\delta_{iI} = 1$ for $i \in I$ and $\delta_{iI} = 0$ otherwise.

By definition the brane set S is the union of two sets:

$$S = S_e \cup S_m, \quad S_v = \cup_{a \in \triangle} \{a\} \times \{v\} \times \Omega_{a,v}, \tag{8}$$

$v = e, m$ and $\Omega_{a,e}, \Omega_{a,m} \subset \Omega$, where $\Omega = \Omega(n)$ is the set of all nonempty subsets of $\{1, \ldots, n\}$. Any brane index $s \in S$ has the form $s = (a_s, v_s, I_s)$, where $a_s \in \triangle$ is color index, $v_s = e, m$ is electro-magnetic index and the set $I_s \in \Omega_{a_s, v_s}$ describes the location of brane worldvolume.

The sets S_e and S_m define electric and magnetic branes, correspondingly. In Eq. (6), $\chi_s = +1, -1$ for $s \in S_e, S_m$, respectively. In Eq. (7), $\bar{I} \equiv I_0 \setminus I$, $I_0 = \{1, \ldots, n\}$.

All manifolds M_i are assumed to be oriented and connected and the volume d_i-forms $\tau_i \equiv \sqrt{|g^i(y_i)|}\ dy_i^1 \wedge \ldots \wedge dy_i^{d_i}$, and parameters $\varepsilon(i) \equiv \mathrm{sign}\,(\det \times \times (g^i_{m_i n_i})) = \pm 1$ are well-defined for all $i = 1, \ldots, n$. Here $d_i = \dim M_i$, $i = 1, \ldots, n$, $D = 1 + \sum_{i=1}^{n} d_i$. For any $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we denote $\tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}$, $d(I) \equiv \dim M(I) = \sum_{i \in I} d_i$, $\varepsilon(I) \equiv \varepsilon(i_1) \ldots \varepsilon(i_k)$.

The parameters h_s appearing in the solution satisfy the relations $h_s = K_s^{-1}$, $K_s = B_{ss}$, where

$$B_{ss'} \equiv d(I_s \cap I_{s'}) + \frac{d(I_s) d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{a_s \alpha} \lambda_{a_{s'} \beta} h^{\alpha\beta}, \tag{9}$$

$s, s' \in S$, with $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$. In Eq. (6), $\lambda^{\alpha}_{a_s} = h^{\alpha\beta} \lambda_{a_s \beta}$. Here we assume that: $(i) B_{ss} \neq 0$, for all $s \in S$, and $(ii) \det(B_{ss'}) \neq 0$, i.e., the matrix $(B_{ss'})$ is a nondegenerate one. In Eqs. (1) and (7), we put

$$(A_{ss'}) = \left(\frac{2 B_{ss'}}{B_{s's'}} \right). \tag{10}$$

In Eq. (1), $B_s = \varepsilon_s K_s Q_s^2$, $s \in S$, where $\varepsilon_s = \big(-\varepsilon[g] \big)^{(1-\chi_s)/2} \varepsilon(I_s) \theta_{a_s}$, $s \in S$, $\varepsilon[g] \equiv \mathrm{sign} \det(g_{MN})$.

The solutions presented above are valid if two restrictions on the sets of branes are satisfied, see [2].

For cylindrically symmetric case $M_1 = S^1$, $g^1 = d\phi \otimes d\phi$, $0 < \phi < 2\pi$, and $w = +1$ we get a family of composite fluxbrane solutions from [1].

2. POLYNOMIAL STRUCTURE OF H_s

In what follows we study the case $\varepsilon_s > 0$ and $K_s > 0$. In this case all $B_s > 0$.

Let us consider Eqs. (1) and (2) for the functions $H_s(z) > 0$, $s \in S$. We are interested in analytical solutions of Eq. (1) in some disc $|z| < L$:

$$H_s(z) = 1 + \sum_{k=1}^{\infty} P_s^{(k)} z^k, \tag{11}$$

where $P_s^{(k)}$ are constants, $s \in S$. The substitution of (11) into (1) gives an infinite chain of relations on parameters $P_s^{(k)}$ and B_s. The first relation in this chain

$$P_s \equiv P_s^{(1)} = \frac{1}{4} B_s = \frac{1}{4} K_s Q_s^2, \tag{12}$$

$s \in S$, corresponds to z^0-term in the decomposition of (1).

It may be shown that for analytic functions $H_s(z)$, $s \in S$ (11) ($z = \rho^2$) the metric (5) is regular at $\rho = 0$ for $w = +1$, i.e., in the fluxbrane case.

It was conjectured in [1] that there exist polynomial solutions to Eqs. (1), (2)

$$H_s = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k, \tag{13}$$

where $P_s^{(k)}$ are constants, $k = 1, \ldots, n_s$. Here $P_s^{(n_s)} \neq 0$ and

$$n_s = 2 \sum_{s' \in S} A^{ss'}, \quad s \in S. \tag{14}$$

Integers n_s are components of the so-called twice dual Weyl vector in the basis of simple coroots. For the Lie algebra A_1 we get $H_1 = 1 + Pz$.

Special solutions. Let $P_s = n_s P$, $P > 0$, $s \in S$. We get a special solution [1]

$$H_s = (1 + Pz)^{n_s}, \tag{15}$$

$s \in S$, which is valid for any semisimple FD Lie algebra.

3. EXAMPLES

$F6 \cap F3$ **fluxbrane solution related to Lie algebra** A_2**.** Let $D = 11$. $F6 \cap F3$ fluxbrane configuration with (nonstandard) A_2 intersection rules is defined on the manifold $M = (0, +\infty) \times M_1 \times M_2 \times M_3 \times M_4$, where $d_2 = 2$, $d_3 = 5$, $d_4 = 2$.

The solution reads [1]

$$g = H_e^{1/3} H_m^{2/3} \left\{ d\rho \otimes d\rho + H_e^{-1} H_m^{-1} \rho^2 \, d\phi \otimes d\phi + H_e^{-1} g^2 + H_m^{-1} g^3 + g^4 \right\}, \tag{16}$$

$$F = -Q_e H_e^{-2} H_m \rho d\,\rho \wedge d\phi \wedge \tau_2 + Q_m \tau_2 \wedge \tau_4, \tag{17}$$

where metrics g^2 and g^3 are (Ricci-flat) metrics of Euclidean signature, g^4 is the (flat) metric of the signature $(-,+)$ and

$$H_s = 1 + P_s \rho^2 + \frac{1}{4} P_1 P_2 \rho^4, \tag{18}$$

where $P_s = (1/2) Q_s^2$, $s = e, m$.

$S0$-brane solutions related to Lie algebras of rank 3. Now we consider $S0$-brane solutions defined on the manifold $M = (0, t_0) \times M_1 \times M_2$, where M_1 is a one-dimensional manifold (say S_1 or R) and M_2 is a $(D-2)$-dimensional Ricci-flat manifold, $m = 3$. Using (15) we get $H_s = X^{n_s}$, where $X = 1 + Pt$, $P < 0$. These solutions read

$$g = X^{2A} \left\{ -dt \otimes dt + X^{-2B} t^2 \, d\phi \otimes d\phi + g^2 \right\}, \tag{19}$$

$$\exp(\varphi^\alpha) = X^{B_1 \lambda_1^\alpha + B_2 \lambda_2^\alpha + B_3 \lambda_3^\alpha}, \tag{20}$$

$$F^1 = -Q_1 X^{n_2 - 2n_1} t \, dt \wedge d\phi, \quad F^2 = -Q_2 X^{n_1 - 2n_2 + k_1 n_3} t \, dt \wedge d\phi, \tag{21}$$

$$F^3 = -Q_3 X^{k_2 n_2 - 2n_3} t \, dt \wedge d\phi, \tag{22}$$

where

$$A = \frac{B}{D-2}, \quad B = \sum_{s=1}^{3} B_s, \quad B_s = n_s K_s^{-1},$$

$k_1 = (1,2,1)$, $k_2 = (1,1,2)$, for A_3, B_3 and C_3, respectively.

These solutions contain intervals with accelerated expansion of M_2-submanifold.

CONCLUSIONS

Here we have done an overview of composite fluxbrane and S-brane solutions related to semisimple FD Lie algebras. The solutions were defined on a product of Ricci-flat manifolds M_i which may have nonzero (chiral) parallel spinors. An open problem is to find the (fractional) numbers of unbroken supersymmetries for certain supergravitational solutions for various semisimple FD Lie algebras, e.g., to $A_1 \oplus \ldots \oplus A_1$ (along a line as it was done in [4] for M-branes). Another problem is to find explicit formulae for fluxbrane polynomials related to all simple FD Lie algebras.

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