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CONTRACTION OF ELECTROWEAK MODEL CAN EXPLAIN THE INTERACTIONS OF NEUTRINOS WITH MATTER

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The very rare interactions of neutrinos with matter and the dependence of the corresponding cross section on neutrinos energy are explained as contraction of the gauge group of the electroweak model already at the level of classical gauge fields. Small contraction parameter is connected with the universal Fermi constant of weak interactions and neutrino energy as $\epsilon^2(s) = \sqrt{G_F s}$.

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INTRODUCTION

The modern theory of electroweak interactions — standard Electroweak Model — is gauge theory based on gauge group $SU(2) \times U(1)$. In physics the operation of group contraction is well known [1], which transforms, for example, a simple or semisimple group to a nonsemisimple one. For better uderstanding of a complicated physical system it is useful to investigate its limit cases for limit values of its physical parameters. For symmetric system similar limit values are often connected with contraction parameters of its symmetry group. In this paper we discuss the modified Electroweak Model with the contracted gauge group $SU(2;\epsilon) \times U(1)$. We explain, at the level of classical fields, the vanishingly small interactions of neutrinos with matter especially for low energies and the decrease of the neutrinos-matter cross section when energy tends to zero with the help of contraction of gauge group. We connect dimensionless contraction parameter $\epsilon \to 0$ with neutrinos energy.

1. MODIFICATION OF THE STANDARD ELECTROWEAK MODEL

We shall follow the books [2–4] in description of standard Electroweak Model. From the viewpoint of electroweak interactions, all known leptons and quarks are divided into three generations. In what follows, we shall regard only first generations of leptons and quarks.

We consider a model where the contracted gauge group $SU(2;\epsilon)\times U(1)$ acts in the boson, lepton, and quark sectors. The contracted group $SU(2;\epsilon)$ is obtained [5] by the consistent rescaling of the fundamental representation of $SU(2)$ and the space C_2

$$z'(\epsilon)=\left(\begin{array}{c}\epsilon z'_1\\ z'_2\end{array}\right)=\left(\begin{array}{cc}\alpha & \epsilon\beta\\ -\epsilon\bar\beta & \bar\alpha\end{array}\right)\left(\begin{array}{c}\epsilon z_1\\ z_2\end{array}\right)=u(\epsilon)z(\epsilon),\qquad (1)$$
$$\det u(\epsilon)=|\alpha|^2+\epsilon^2|\beta|^2=1,\quad u(\epsilon)u^\dagger(\epsilon)=1$$

in such a way that the Hermitian form $z^\dagger z(\epsilon)=\epsilon^2|z_1|^2+|z_2|^2$ remains invariant, when contraction parameter tends to zero $\epsilon\rightarrow 0$ or is equal to the nilpotent unit $\epsilon=\iota$, $\iota^2=0$. The actions of $U(1)$ and the electromagnetic subgroup $U(1)_{\rm em}$ in the fibered space $C_2(\iota)$ with the base $\{z_2\}$ and the fiber $\{z_1\}$ are given by the same matrices as in the space C_2.

The space $C_2(\epsilon)$ of the fundamental representation of $SU(2;\epsilon)$ group can be obtained from C_2 by substituting z_1 by ϵz_1. Substitution $z_1\rightarrow\epsilon z_1$ induces another ones for Lie algebra generators $T_1\rightarrow\epsilon T_1$, $T_2\rightarrow\epsilon T_2$, $T_3\rightarrow T_3$. As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely: $A^1_\mu\rightarrow\epsilon A^1_\mu$, $A^2_\mu\rightarrow\epsilon A^2_\mu$, $A^3_\mu\rightarrow A^3_\mu$, $B_\mu\rightarrow B_\mu$. For the new gauge fields these substitutions are as follows:

$$W^\pm_\mu\rightarrow\epsilon W^\pm_\mu,\quad Z_\mu\rightarrow Z_\mu,\quad A_\mu\rightarrow A_\mu.\qquad (2)$$

The fields $L_l=\left(\begin{array}{c}\nu_l\\ e_l\end{array}\right)$, $Q_l=\left(\begin{array}{c}u_l\\ d_l\end{array}\right)$ are $SU(2)$-doublets, so their components are transformed in the similar way as components of z:

$$\nu_l\rightarrow\epsilon\nu_l,\quad e_l\rightarrow e_l,\quad u_l\rightarrow\epsilon u_l,\quad d_l\rightarrow d_l.\qquad (3)$$

The right lepton and quark fields are $SU(2)$-singlets and therefore are not transformed.

After these transformations and spontaneous symmetry breaking with $\phi^{\rm vac}=\left(\begin{array}{c}0\\ v/\sqrt{2}\end{array}\right)$, the standard boson Lagrangian [2] can be represented in the form

$$L_B(\epsilon)=L_B^{(2)}(\epsilon)+L_B^{\rm int}(\epsilon)=$$
$$=\frac{1}{2}\left(\partial_\mu\chi\right)^2-\frac{1}{2}m_\chi^2\chi^2-\frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu}+\frac{1}{2}m_Z^2Z_\mu Z_\mu-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}+$$
$$+\epsilon^2\left\{-\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^-_{\mu\nu}+m_W^2W^+_\mu W^-_\mu\right\}+L_B^{int}(\epsilon),\qquad (4)$$

where as usual second-order terms describe the boson particles content of the model, and higher-order terms $L_B^{\rm int}$ are regarded as their interactions. The standard lepton Lagrangian [2] in terms of electron and neutrino fields takes the form

$$L_L(\epsilon) = e_l^\dagger i\tilde{\tau}_\mu\partial_\mu e_l + e_r^\dagger i\tau_\mu\partial_\mu e_r - m_e(e_r^\dagger e_l + e_l^\dagger e_r) + \\ + \frac{g\cos 2\theta_w}{2\cos\theta_w} e_l^\dagger\tilde{\tau}_\mu Z_\mu e_l - e e_l^\dagger\tilde{\tau}_\mu A_\mu e_l - g'\cos\theta_w e_r^\dagger\tau_\mu A_\mu e_r + \\ + g'\sin\theta_w e_r^\dagger\tau_\mu Z_\mu e_r + \epsilon^2\Big\{\nu_l^\dagger i\tilde{\tau}_\mu\partial_\mu\nu_l + \frac{g}{2\cos\theta_w}\nu_l^\dagger\tilde{\tau}_\mu Z_\mu\nu_l + \\ + \frac{g}{\sqrt{2}}\left[\nu_l^\dagger\tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger\tilde{\tau}_\mu W_\mu^-\nu_l\right]\Big\} = L_{L,b} + \epsilon^2 L_{L,f}. \quad (5)$$

The quark Lagrangian [2] in terms of u- and d-quarks fields can be written as

$$L_Q(\epsilon) = d^\dagger i\tilde{\tau}_\mu\partial_\mu d + d_r^\dagger i\tau_\mu\partial_\mu d_r - m_d(d_r^\dagger d + d^\dagger d_r) - \frac{e}{3}d^\dagger\tilde{\tau}_\mu A_\mu d - \\ - \frac{g}{\cos\theta_w}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)d^\dagger\tilde{\tau}_\mu Z_\mu d - \frac{1}{3}g'\cos\theta_w d_r^\dagger\tau_\mu A_\mu d_r + \\ + \frac{1}{3}g'\sin\theta_w d_r^\dagger\tau_\mu Z_\mu d_r - \epsilon^2\Big\{u^\dagger i\tilde{\tau}_\mu\partial_\mu u + u_r^\dagger i\tau_\mu\partial_\mu u_r - \\ - m_u(u_r^\dagger u + u^\dagger u_r) + \frac{g}{\cos\theta_w}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w\right)u^\dagger\tilde{\tau}_\mu Z_\mu u + \\ + \frac{2e}{3}u^\dagger\tilde{\tau}_\mu A_\mu u + \frac{g}{\sqrt{2}}\left[u^\dagger\tilde{\tau}_\mu W_\mu^+ d + d^\dagger\tilde{\tau}_\mu W_\mu^- u\right] + \frac{2}{3}g'\cos\theta_w u_r^\dagger\tau_\mu A_\mu u_r - \\ - \frac{2}{3}g'\sin\theta_w u_r^\dagger\tau_\mu Z_\mu u_r\Big\} = L_{Q,b} + \epsilon^2 L_{Q,f}, \quad (6)$$

where $m_e = h_e v/\sqrt{2}$ and $m_u = h_u v/\sqrt{2}$, $m_d = h_d v/\sqrt{2}$ represents electron and quark masses.

The full Lagrangian of the modified model is the sum

$$L(\epsilon) = L_B(\epsilon) + L_Q(\epsilon) + L_L(\epsilon) = L_b + \epsilon^2 L_f. \quad (7)$$

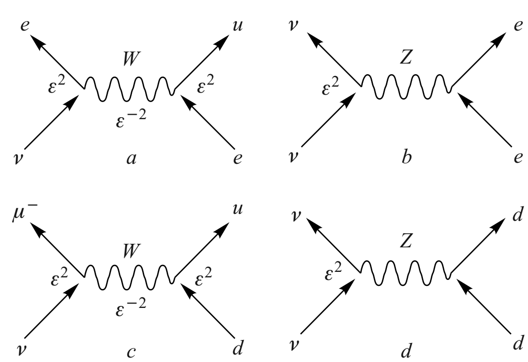
The boson Lagrangian $L_B(\epsilon)$ was discussed in [6] for standard formulation and in [7] without Higgs boson, where it was shown that masses of all particles of the Electroweak Model remain the same under contraction $\epsilon^2 \to 0$. In this limit the contribution $\epsilon^2 L_f$ of neutrino, W-boson, and u-quark fields as well as their interactions with other fields to the Lagrangian (7) will be vanishingly small in comparison with contribution L_b of electron, d-quark, and remaining boson fields. So Lagrangian (7) describes very rare interaction neutrino fields with the matter for low energies. On the other hand, contribution of the neutrino part $\epsilon^2 L_f$ to the full Lagrangian is risen when the parameter ϵ^2 is increased, that again corresponds to the experimental facts. The dependence of ϵ on neutrino energy can be obtained from the experimental data.

In the mathematical language the fields space of the standard electroweak model is fibered after the contraction in such a way that neutrino, W-boson, and u-quark fields are in the fiber, whereas all other fields are in the base. We regard locally trivial fibering, which is defined by the projection in the field space. This fibering is understood in the context of semi-Riemannian geometry [8, 9] and has nothing to do with the principal fiber bundle. The simple and best known example of such fiber space is the nonrelativistic space-time with one-dimensional base, which is interpreted as time, and a three-dimensional fiber, which is interpreted as proper space. It is well known that in nonrelativistic physics the time does not depend on the space coordinates, while the space properties can be changed in time. The space-time of the special relativity is transformed to the nonrelativistic space-time when dimensionfull contraction parameter — velocity of light c — tends to the infinity and dimensionless parameter $v/c \to 0$.

2. RARELY NEUTRINOS-MATTER INTERACTIONS FOR LOW ENERGIES AND CONTRACTION OF GAUGE GROUP

To establish the physical meaning of the contraction parameter we consider neutrino elastic scattering on electron and quarks for low energies $s \ll m_W^2$, i.e., the limit case of the Electroweak Model. The corresponding diagrams for the neutral and charged currents interactions are presented in the Figure.

Under substitutions (2), (3) both vertex of diagram in Figure, a are multiplied by ϵ^2, as it follows from lepton Lagrangian (5). The propagator of virtual fields W according to boson Lagrangian (4) is multiplied by ϵ^{-2}. Indeed, propagator is



Neutrino elastic scattering on electron and quarks inverse operator to operator of free field, but the later for W-fields is multiplied by ϵ^2. So in total, the probability amplitude for charged weak current interactions is transformed as $\mathcal{M}_W \to \epsilon^2 \mathcal{M}_W$. For diagram in Figure, b only one vertex is multiplied by ϵ^2, whereas second vertex and propagator of Z virtual field do not change, so the corresponding amplitude for neutral weak current interactions is transformed in a similar way $\mathcal{M}_Z \to \epsilon^2 \mathcal{M}_Z$. A cross section is proportional to a squared amplitude, so neutrino-electron scattering cross section is proportional to ϵ^4. For low energies $s \ll m_W^2$, this cross section is as follows [3]:

$$\sigma_{\nu e} = G_F^2 s f(\xi) = \frac{g^4}{m_W^4} \tilde{f}(\xi), \tag{8}$$

where $G_F = 10^{-5}(1/m_p^2) = 1.17 \cdot 10^{-5}$ GeV^{-2} is Fermi constant, s is squared energy in c.m. system, $\xi = \sin\theta_w$, $\tilde{f}(\xi) = f(\xi)/32$ is a function of Weinberg angle. On the other hand, taking into account that contraction parameter is dimensionless, we can write down

$$\sigma_{\nu e} = \epsilon^4 \sigma_0 = (G_F s)(G_F f(\xi)) \tag{9}$$

and obtain

$$\epsilon^2(s) = \sqrt{G_F s} \approx \frac{g\sqrt{s}}{m_W}. \tag{10}$$

Neutrino elastic scattering on quarks due to neutral and charged currents is pictured in Figures, c, d. Cross sections for neutrino–quarks scattering are obtained in a similar way as for the lepton case and are as follows [3]: $\sigma_\nu^W = G_F^2 s \hat{f}(\xi))$, $\sigma_\nu^Z = G_F^2 s\, h(\xi)$. Nucleons are some composite construction of quarks, therefore some form factors appeared in the expressions for neutrino–nucleons scattering cross sections. The final expression $\sigma_{\nu n} = G_F^2 s \hat{F}(\xi)$ coincides with (8), i.e., this cross section is transformed as (9) with the contraction parameter (10). At low energies, scattering interactions make the leading contribution to the total neutrino-matter cross section, therefore it has the same properties (9), (10) with respect to contraction of the gauge group.

CONCLUSIONS

We have suggested the modification of the standard Electroweak Model by the contraction of its gauge group. At the level of classical (nonquantum) gauge fields the very weak neutrino-matter interactions especially at low energies can be explained by this model. The zero tending contraction parameter depends on neutrino energy in accordance with the energy dependence of the neutrino matter interaction cross section.

The limit transition $c \to \infty$ in special relativity resulted in the notion of group contraction [1]. In our model, on the contrary, the notion of group contraction is used to explain the fundamental limit process of nature.

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