ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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GAUGE-INVARIANT QUARK GREEN'S FUNCTIONS WITH POLYGONAL WILSON LINES

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Properties of gauge-invariant two-point quark Green's functions, defined with polygonal Wilson lines, are studied. Green's functions can be classified according to the number of straight line segments their polygonal lines contain. Functional relations are established between Green's functions with different numbers of segments on the polygonal lines. An integrodifferential equation is obtained for Green's function with one straight line segment, in which the kernels are represented by a series of Wilson loop vacuum averages along polygonal contours with an increasing number of segments and functional derivatives on them. The equation is exactly solved in the case of two-dimensional QCD in the large-N_c limit. The spectral properties of Green's function are displayed.

Изучаются свойства калибровочно-инвариантных двухточечных кварковых функций Грина, определенных с помощью полигональных функций Вильсона. Функции Грина классифицируются согласно числу отрезков прямых линий, содержащихся в их полигональных линиях. Устанавливаются функциональные соотношения между функциями Грина с различным числом сегментов на полигональной линии. Получено интегродифференциальное уравнение для функции Грина с одним прямым сегментом, в котором ядра представлены серией вильсоновских петлевых вакуумных усреднений вдоль полигональных контуров с растущим числом сегментов и функциональных призводных на них. Уравнение точно решается в случае двумерного КХД в пределе большого N_c. Показываются специальные свойства функций Грина.

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INTRODUCTION

Path-ordered gluon field phase factors (Wilson lines) play a fundamental role in defining gauge-invariant Green's functions in QCD [1, 2]. As complementary objects to these, Wilson loops appear through the role of potentials [3–12] or kernels [13, 14] in the evaluation of physical quantities. Properties of Wilson loops were thoroughly studied in the past [15–20].

The present paper is a summary of recent investigations of the author to obtain integrodifferential equations for gauge-invariant quark Green's functions constructed with phase factors along polygonal lines [21, 22]. Polygonal lines are

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of particular interest, since they can be decomposed as a succession of straight line segments. The latter are Lorentz-invariant in the form and have an unambiguous geometric limit when their end points approach each other. Furthermore, polygonal lines can be classified according to the number of segments they contain and this, in turn, leads to a similar classification of Green's functions themselves.

A polygonal line representation appears as a natural generalization for light quarks [21, 23] of the representation of heavy quark propagators in the presence of external fields [6]. Although the explicit use of representations of propagators in external fields is not a requisite to obtain functional equations for Green's functions, since the use of the equations of motion of the latter, associated with appropriate integrations, may produce the same effect, the former may provide a more illustrative algorithm of the various mechanisms at work.

Polygonal lines for Wilson lines have also been introduced in the literature, for the study of the high-energy behavior of scattering amplitudes. At high energies, the propagators of fast moving quarks in the presence of external gluon fields are well approximated by including the contributions of the latter in Wilson lines along straight line segments collinear to the world lines of the quarks [24–26]. These naturally generate lightlike polygonal lines or lightlike Wilson loops associated with high-energy scattering amplitudes.

The aim of our approach is to investigate the nonperturbative regime of QCD, and therefore we focus on exact functional equations, at least on formal grounds, of gauge invariant quark Green's functions, in analogy with the Dyson–Schwinger equations satisfied by ordinary Green's functions. Applications in two-dimensional QCD in the large-N_c limit allow us to check the consistency and the degree of predictivity of the approach.

1. QUARK GREEN'S FUNCTIONS WITH POLYGONAL LINES

We designate by $U(y,x)$ a path-ordered phase factor along an oriented straight line segment going from x to y. A displacement of one end point of the rigid segment, while the other end point remains fixed, generates also a displacement of the interior points of the segment. This defines a *rigid path displacement*. Parametrizing the interior points of the segment with a linear parameter λ varying between 0 and 1, such that $z(\lambda) = \lambda y + (1-\lambda)x$, the rigid path derivative operations with respect to y or x yield

$$\frac{\partial U(y,x)}{\partial y^\alpha} = -igA_\alpha(y)\,U(y,x) + ig(y-x)^\beta \times \\ \times \int_0^1 d\lambda\,\lambda U(y,z(\lambda))\,F_{\beta\alpha}(z(\lambda))\,U(z(\lambda),x), \quad (1)$$

$$\frac{\partial U(y,x)}{\partial x^{\alpha}} = +igU(y,x)\,A_{\alpha}(x) + ig(y-x)^{\beta}\times \\ \times \int_0^1 d\lambda(1-\lambda)\,U(y,z(\lambda))\,F_{\beta\alpha}(z(\lambda))\,U(z(\lambda),x), \quad (2)$$

where A is the gluon potential; F, its field strength; and g, the coupling constant.

In gauge-invariant quantities, the end-point contributions of the segments are usually cancelled by other neighboring-point contributions and one remains only with the interior-point contributions of the segments, represented by the integrals above. We introduce for them a condensed notation:

$$\frac{\bar{\delta}U(y,x)}{\bar{\delta}y^{\alpha+}} = ig(y-x)^{\beta}\int_0^1 d\lambda\,\lambda U(y,z(\lambda))\,F_{\beta\alpha}(z(\lambda))\,U(z(\lambda),x), \quad (3)$$

$$\frac{\bar{\delta}U(y,x)}{\bar{\delta}x^{\alpha-}} = ig(y-x)^{\beta}\int_0^1 d\lambda(1-\lambda)\,U(y,z(\lambda))\,F_{\beta\alpha}(z(\lambda))\,U(z(\lambda),x). \quad (4)$$

The superscript «+» or «−» on the derivative variable takes account of the orientation on the segment and specifies, in the case of joined segments, the segment on which the derivative acts.

The vacuum expectation value (or vacuum average) of a Wilson loop along a contour C will be designated by $W(C)$. In the case of a polygonal contour C_n, with n segments and n junction points $x_1, x_2, \ldots, x_n$, it will be designated by W_n and represented as an exponential functional [17, 19]:

$$W_n = W(x_n, x_{n-1}, \ldots, x_1) = \exp\left[F_n(x_n, x_{n-1}, \ldots, x_1)\right] = \mathrm{e}^{F_n}. \quad (5)$$

The two-point gauge-invariant quark Green's function (2PGIQGF) with a phase factor along a polygonal line composed of n segments and $(n-1)$ junction points is designated by $S_{(n)}$:

$$S_{(n)}(x,x';t_{n-1},\ldots,t_1) = \\ = -\frac{1}{N_c}\langle\overline{\psi}(x')U(x',t_{n-1})U(t_{n-1},t_{n-2})\cdots U(t_1,x)\psi(x)\rangle, \quad (6)$$

the quark fields, with mass parameter m, belonging to the fundamental representation of the color gauge group $SU(N_c)$ and the vacuum expectation value being defined in the path integral formalism. (Spinor indices are omitted and the color indices are implicitly summed.) The simplest such function is $S_{(1)}$, having a phase factor along a straight line segment:

$$S_{(1)}(x,x') \equiv S(x,x') = -\frac{1}{N_c}\langle\overline{\psi}(x')U(x',x)\psi(x)\rangle. \quad (7)$$

(We shall generally omit the index 1 from that function.)

For the internal parts of rigid path derivatives, we have definitions of the type

$$\frac{\bar{\delta} S_{(n)}(x, x'; t_{n-1}, \ldots, t_1)}{\bar{\delta} x^{\mu -}} = \\ = -\frac{1}{N_c} \langle \overline{\psi}(x') U(x', t_{n-1}) U(t_{n-1}, t_{n-2}) \cdots \frac{\bar{\delta} U(t_1, x)}{\bar{\delta} x^{\mu -}} \psi(x) \rangle. \quad (8)$$

2. FUNCTIONAL RELATIONS

The above Green's functions satisfy the following equations of motion concerning the quark field variables:

$$(i\gamma \cdot \partial_{(x)} - m) S_{(n)}(x, x'; t_{n-1}, \ldots, t_1) = \\ = i\delta^4(x - x') e^{F_n(x, t_{n-1}, \ldots, t_1)} + i\gamma^\mu \frac{\bar{\delta} S_{(n)}(x, x'; t_{n-1}, \ldots, t_1)}{\bar{\delta} x^{\mu -}}, \quad (9)$$

which become for $n = 1$

$$(i\gamma \cdot \partial_{(x)} - m)\ S(x, x') = i\delta^4(x - x') + i\gamma^\mu \frac{\bar{\delta} S(x, x')}{\bar{\delta} x^{\mu -}}. \quad (10)$$

Similar equations, with slight changes, also hold with the variable x'.

Multiplying Eq. (9) with $S(t_1, x)$ and integrating with respect to x, one obtains functional relations between various 2PGIQGFs. A typical such relation is

$$S_{(n)}(x, x'; t_{n-1}, \ldots, t_1) = S(x, x') \exp\left[F_{n+1}(x', t_{n-1}, \ldots, t_1, x)\right] + \\ + \left(\frac{\bar{\delta} S(x, y_1)}{\bar{\delta} y_1^{\alpha_1 +}} + S(x, y_1) \frac{\bar{\delta}}{\bar{\delta} y_1^{\alpha_1 -}} \right) S_{(n+1)}(y_1, x'; t_{n-1}, \ldots, t_1, x). \quad (11)$$

(Integrations on intermediate variables are implicit and will not be written throughout this paper. Here, y_1 is an integration variable.) Equation (11) can also be obtained, within a two-step quantization method, by using a specific representation for the quark propagator in the external gluon field, based on a gauge-covariant expansion with phase factors along polygonal lines [21].

Equation (11) expresses $S_{(n)}$ in terms of two quantities: the first one, which plays the role of a driving term, contains the simplest 2PGIQGF, with one straight line segment, together with a Wilson loop average along a polygonal contour with $(n + 1)$ segments; the second term, which appears as a corrective term, is represented by the contribution of a higher-index 2PGIQGF. However, since this equation is valid for any $n \geqslant 1$, one can use it again in its right-hand side for $S_{(n+1)}$. One therefore generates an iterative procedure that eliminates successively the higher-index 2PGIQGFs in terms of the lowest-index one, $S_{(1)}$. Assuming that the terms rejected to infinity are negligible, one ends up with a series where only $S_{(1)}$ appears together with Wilson loop averages along polygonal contours with an increasing number of sides and rigid path derivatives along the segments. This result shows that among the set of the 2PGIQGFs $S_{(n)}$, $n = 1, 2, \ldots$, it is only $S_{(1)}$, having a phase factor along one straight line segment, that is a genuine dynamical independent quantity. Higher-index 2PGIQGFs could, in principle, be eliminated in terms of $S_{(1)}$, together with polygonal Wilson loops and their rigid path derivatives.

3. INTEGRODIFFERENTIAL EQUATION

The construction of S proceeds from the resolution of the equation of motion (10). It is then necessary to evaluate the action of the rigid path derivative on S as it appears in the right-hand side of the equation. This is done by using again the functional relations (11), where the driving term of the right-hand side gives the main contribution. Thus, the rigid path derivative acting along the segment xt_1 of $S_{(n)}$, acts in the right-hand side in the first place on the logarithm of the Wilson loop average F_{n+1}; it also acts on the remainder containing $S_{(n+1)}$. Using back Eq. (11), one obtains an equation for $\bar{\delta}S_{(n)}/\bar{\delta}x^-$ which expresses the latter as a product of $\bar{\delta}F_{n+1}/\bar{\delta}x^-$ with $S_{(n)}$ plus a remainder containing the derivative of $S_{(n+1)}$. Continuing the procedure, one factorizes in front of every $S_{(n')}$ $(n' > n)$ derivatives of Wilson loop averages.

Selecting in the above set of equations the case $n = 1$, one finds

$$\begin{aligned}
\frac{\bar{\delta}S(x,x')}{\bar{\delta}x^{\mu -}} &= \frac{\bar{\delta}F_2(x',x)}{\bar{\delta}x^{\mu -}}\, S(x,x') - \frac{\bar{\delta}^2 F_3(x',x,y_1)}{\bar{\delta}x^{\mu -}\bar{\delta}y_1^{\alpha_1 +}}\, S(x,y_1)\,\gamma^{\alpha_1}\, S_{(2)}(y_1,x';x) - \\
&- \left(\frac{\bar{\delta}S(x,y_1)}{\bar{\delta}y_1^{\alpha_1 +}} + S(x,y_1)\frac{\bar{\delta}}{\bar{\delta}y_1^{\alpha_1 -}}\right)\gamma^{\alpha_1}\,\frac{\bar{\delta}^2 F_4(x',x,y_1,y_2)}{\bar{\delta}x^{\mu -}\bar{\delta}y_2^{\alpha_2 +}}\, S(y_1,y_2)\,\gamma^{\alpha_2}\times \\
&\times S_{(3)}(y_2,x';x,y_1) - \sum_{n=4}^{\infty}\left(\frac{\bar{\delta}S(x,y_1)}{\bar{\delta}y_1^{\alpha_1 +}} + S(x,y_1)\frac{\bar{\delta}}{\bar{\delta}y_1^{\alpha_1 -}}\right)\gamma^{\alpha_1}\times\cdots\times \\
&\times\left(\frac{\bar{\delta}S(y_{n-3},y_{n-2})}{\bar{\delta}y_{n-2}^{\alpha_{n-2}+}} + S(y_{n-3},y_{n-2})\frac{\bar{\delta}}{\bar{\delta}y_{n-2}^{\alpha_{n-2}-}}\right)\gamma^{\alpha_{n-2}}\times \\
&\times\frac{\bar{\delta}^2 F_{n+1}(x',x,y_1,\ldots,y_{n-1})}{\bar{\delta}x^{\mu -}\bar{\delta}y_{n-1}^{\alpha_{n-1}+}}\, S(y_{n-2},y_{n-1})\,\gamma^{\alpha_{n-1}}\times \\
&\times S_{(n)}(y_{n-1},x';x,y_1,\ldots,y_{n-2}). \qquad (12)
\end{aligned}$$

The above equation displays the structure of the rigid path derivative of S in terms of Wilson loop averages and other 2PGIQGFs. It is the analogue of the self-energy Dyson–Schwinger equation for ordinary quark Green's functions. One observes in its right-hand side the appearance of the whole set of 2PGIQGFs.

Gluon propagators are replaced here by Wilson loop averages along polygonal contours and rigid path derivatives acting on them. This ensures gauge invariance of every term of the expansion. We notice that each derivative acts on a different segment from the others and therefore one does not meet singularities arising from the action of functional derivatives on the same point.

One should complete representation of Eq. (12) by bringing all derivatives to the utmost right. The resulting expression, for the first few terms, can be found in [21]. One finds that the 2PGIQGF $S_{(n)}$ is accompanied by one or several Wilson loop averages and by the 2PGIQGF S, these factors being globally submitted to n derivatives, S being individually submitted to at most one derivative. The structure of the terms allows a classification of the kernels into categories that can be described as connected, crossed, and nested, all of them being of the irreducible type.

One major difference of the integrals present in the right-hand side of Eq. (12) with those of the Dyson–Schwinger equation is the property that they are not of the convolution type. This is due to the presence of the Wilson loops, whose contours pass by all points of the accompanying terms and do not allow for a convolutive factorization in x space.

The equation of motion (9), together with the result (12), takes now the following form:

$$\begin{aligned}(i\gamma\cdot\partial_{(x)}-m)\,S(x,x') = i\delta^4(x-x') + i\gamma^\mu \bigg\{ & K_{1\mu-}(x',x)\,S(x,x') + \\ & + K_{2\mu-}(x',x,y_1)\,S_{(2)}(y_1,x';x) + \\ + \sum_{n=3}^{\infty} K_{n\mu-}(x',x,y_1,\ldots,y_{n-1})\,& S_{(n)}(y_{n-1},x';x,y_1,\ldots,y_{n-2}) \bigg\}, \quad (13)\end{aligned}$$

where the kernels K_n ($n=1,2,\ldots$) contain Wilson loop averages along polygonal contours that are at most $(n+1)$-sided and $(n-1)$ 2PGIQGF S and its derivative. The total number of derivatives contained in K_n is n. Once the Wilson loop averages and the various derivatives have been evaluated and the high-index $S_{(n)}$s have been expressed in terms of S, Eq. (13) becomes an integro-differential equation in S, which is the primary unknown quantity to be solved.

Equations (12), (13) can also be analyzed, at least superficially, from the viewpoint of a perturbative expansion. According to Eqs. (3), (4), each derivative operator results in an insertion of the gluon field strength, leading to the appearance of a valence gluon, accompanied multiplicatively by the coupling constant. In the short-distance regime, where perturbative QCD should be applicable, a naive counting of the number of derivatives would give us an indication about the size of the corresponding term, the leading terms corresponding to those having the least number of derivatives. Here, perturbation theory would be affected in the presence of the polygonal Wilson loops for each term. At large distances, it is expected that Wilson loop averages are saturated by minimal surfaces [17, 23]. Here also, increasing the number of derivatives would lead to less dominant terms. It thus seems reasonable to assume, as a starting hypothesis, that Eq. (12) does, on practical grounds, represent a perturbative expansion. The first term of the series, corresponding to a single derivative term, is null for symmetry reasons. Therefore, the leading term of the series would be represented by the two-derivative term with a Wilson loop average along a triangular contour.

4. SPECTRAL REPRESENTATION

The analyticity properties of the 2PGIQGFs could be studied by passing to momentum space. The polygonal structure of the phase factor lines facilitates this task. The 2PGIQGF $S_{(n)}$ is a translation-invariant function of its $(n+1)$ x-space arguments, having well defined Lorentz transformation properties, the geometric configuration of the straight line segments being completely defined by their end-point positions. The situation is even simpler for S which depends only on the variable $(x-x')$; in momentum space it is dependent on a single momentum p.

The singularities of the 2PGIQGFs in momentum space could be studied by inserting in them a complete set of intermediate states. Here, however, a difficulty arises: intermediate states inserted in expressions (6) or (7) are necessarily colored states, since no color singlet states can be made from gluons and single quarks belonging to the fundamental representation. This means that if the theory is confining, hadronic states, which are expected to form the complete sets of states, could not contribute to the singularities of the 2PGIQGFs. One might then conclude that the latter do not have singularities at all and are completely analytic functions. However, the equations of motion (9), (10) do have singularities resulting from the presence of the free quark propagator (the inverse of the Dirac operator). To remedy this difficulty, one has to admit that quark and gluon states, although colorful objects, continue forming complete sets of states. It is the solution of the equations that should provide their precise status. We admit that hypothesis and assume at the same time that the quark and gluon states contribute, like other states, with positive energies and that their fields satisfy the causality property.

With these assumptions, the analysis of the spectral properties of Green's function S can be done as for ordinary propagators [27–29]. The phase factor can be decomposed into a series of gluon fields and the contribution of each field along the straight line can be analyzed. One obtains at the end a generalized version of the Källen–Lehmann representation:

$$S(p) = i\int_0^\infty ds' \sum_{n=1}^{\infty} \frac{[\gamma\cdot p\rho_1^{(n)}(s') + \rho_0^{(0)}(s')]}{(p^2 - s' + i\varepsilon)^n}. \qquad (14)$$

Simplifications may occur by recombining several terms by integrations by parts or by the infinite summation.

The above representation, or a simplified version of it, might serve us as a guide for searching for solutions of the integrodifferential equation (13).

5. TWO-DIMENSIONAL QCD

The equations obtained in the previous sections remain also valid in two dimensions and could be analyzed more easily in that case. Two-dimensional QCD in the large-N_c limit [30, 31] provides a simplified framework for the study of the confinement properties which are expected to prevail also in four dimensions. Wilson loop averages can be explicitly calculated [32–34]: for simple contours they are equal to the exponential of the areas enclosed by the contours. In that case, the second-order derivative of the logarithm of the Wilson loop average reduces to a two-dimensional delta-function. Higher-order derivatives give zero, since they act on different segments of the polygonal contour. The case of overlapping self-intersecting surfaces, which give more complicated expressions, should be analyzed separately. A detailed analysis suggests that the residual terms they produce are probably of zero weight under the integrations that are involved. We assume that hypothesis.

In the series of terms of Eq. (12) it is only the second-order derivative that survives, and the integrodifferential equation (13) takes the following (exact) expression:

$$(i\gamma\cdot\partial - m)S(x) = i\delta^2(x) - \sigma\gamma^\mu(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\,x^\nu x^\beta \times \\ \times\left[\int_0^1 d\lambda\,\lambda^2\,S((1-\lambda)x)\gamma^\alpha S(\lambda x) + \int_1^\infty d\xi\,S((1-\xi)x)\gamma^\alpha S(\xi x)\right], \quad (15)$$

where σ is the string tension.

The above equation can be analyzed by first passing to momentum space. Designating by $S(p)$ the Fourier transform of $S(x)$, one can decompose it into Lorentz invariant components:

$$S(p) = \gamma\cdot pF_1(p^2) + F_0(p^2). \quad (16)$$

The solution of Eq. (15) can be searched for by using the analyticity properties of the 2PGIQGF as discussed in Sec. 4. It turns out that the equation can be solved exactly and in analytic form. The functions F_1 and F_0 are found having an infinite number of branch cuts located on the positive real axis of p^2 (timelike region), starting at thresholds $M_1^2, M_2^2, \ldots, M_n^2, \ldots$, with fractional power singularities equal to $-3/2$. Their expressions are [22], for complex p^2,

$$F_1(p^2) = -i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty} b_n \frac{1}{(M_n^2 - p^2)^{3/2}}, \tag{17}$$

$$F_0(p^2) = i\frac{\pi}{2\sigma}\sum_{n=1}^{\infty}(-1)^n b_n \frac{M_n}{(M_n^2 - p^2)^{3/2}}. \tag{18}$$

The masses M_n ($n = 1, 2, \ldots$) are positive, greater than the free quark mass m and ordered according to increasing values. For massless quarks they remain positive. The masses M_n and the coefficients b_n, the latter being also positive, satisfy, for general m, an infinite set of algebraic equations that are solved numerically. Their asymptotic values, for large values of n such that $n \gg m^2/(\pi\sigma)$, are

$$M_n^2 \simeq \pi n\sigma, \qquad b_n \simeq \frac{\sigma^2}{M_n + (-1)^n m}. \tag{19}$$

The functions $(M_n^2 - p^2)^{-3/2}$ are defined with cuts starting from their branch points and going to $+\infty$ on the real axis; they are real below their branch points on the real axis down to $-\infty$.

The expressions (17), (18) are represented by weakly converging series. The high-energy behavior of the functions F_1 and F_0 is obtained with a detailed study of the asymptotic tails of the series and the use of the asymptotic behaviors of the parameters M_n and b_n (Eqs. (19)). One finds that they behave as in free field theories, which is here a trivial manifestation of asymptotic freedom [35]:

$$F_1(p^2) \underset{p^2\to-\infty}{=} \frac{i}{p^2}, \tag{20}$$

$$F_0(p^2) \underset{p^2\to-\infty}{=} \frac{im}{p^2} \quad \text{for} \quad m \neq 0, \tag{21}$$

$$F_0(p^2) \underset{p^2\to-\infty}{=} -\frac{4i\sigma F_0(x=0)}{(p^2)^2} \quad \text{for} \quad m = 0. \tag{22}$$

In summary, the solution of Eq. (15) is nonperturbative and infrared finite. The masses M_n are dynamically generated, since they do not exist in the Lagrangian of the theory. They could be interpreted as dynamical masses of quarks with, however, the following particular features. First, they are infinite in number. Second, they do not appear as poles in Green's function, but rather with stronger singularities. In x space, the latter do not produce finite plane waves at large distances and therefore quarks could not be observed as free asymptotic states. Nevertheless, the above singularities being gauge-invariant should have physical significance and would show up in the infrared regions of physical processes involving quarks. Finally, since they appear only in the timelike region of real p^2, one concludes that the quark and gluon fields satisfy, even in the nonperturbative regime, the usual spectral properties of quantum field theory [27–29].

CONCLUSION

The consideration of gauge-invariant Green's functions with phase factors along polygonal lines allows for a systematic investigation of their properties through the functional relations they satisfy. An equation playing the same role as the Dyson–Schwinger equation for ordinary Green's functions has been obtained, in which the kernels are represented by Wilson loop averages along polygonal contours with rigid path derivatives on their segments.

The application of this equation to two-dimensional QCD in the large-N_c limit provides an exact nonperturbative analytic solution, not known from conventional approaches, which displays the spectral properties of quark fields: the latter behave like fields of physical particles, with the difference that their singularities in momentum space are stronger than simple poles.

The consistency of the results obtained in two-dimensional QCD is a positive test for the general approach presently developed and provides hints for the continuation of investigations in four dimensions.

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