POLYAKOV LOOP FLUCTUATIONS AND DECONFINEMENT IN THE LIMIT OF HEAVY QUARKS

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We explore the influence of heavy quarks on color deconfinement within an effective model of gluons interacting with dynamical quarks. As the quark mass decreases, the strength of the explicit breaking of the $Z(3)$ symmetry grows, and the first-order transition in the pure $SU(3)$ gauge theory ends in a critical end point. The nature of this critical end point is examined by studying the fluctuations of the Polyakov loop quantified by the corresponding susceptibilities. The properties of the Polyakov loop fluctuations and their ratios are also studied in QCD with physical quark masses.

INTRODUCTION

In the limit of infinitely heavy quarks, deconfinement in the $SU(3)$ gauge theory is associated with the spontaneous breaking of the $Z(3)$ center symmetry [1–5]. The phase transition is of first-order and is therefore stable with respect to weak explicit symmetry breaking [6]. The presence of dynamical quarks breaks this symmetry explicitly, with a strength that increases as the quark mass decreases. It is thus expected that the transition remains discontinuous in the heavy-quark region and becomes continuous at some critical value of quark mass [7]. This defines the critical end point (CEP) for the deconfinement phase transition.

The relevant information for studying deconfinement is contained in the Polyakov loop and its susceptibilities. Recently these quantities were obtained on the lattice within $SU(3)$ pure gauge theory [8]. In particular, the ratio of the transverse and longitudinal Polyakov loop susceptibility was shown to exhibit a θ function-like behavior at the critical temperature T_d, with almost no dependence on temperature on either side of the transition. This feature makes such ratios attractive as probes of deconfinement.

The influence of the dynamical light quarks on these susceptibilities has also been studied in 2- and $(2+1)$-flavor QCD [9, 10]. The resulting susceptibility

ratios are considerably smoothed, reflecting the crossover nature of the transition. However, the theoretical understanding of these quantities is still not quite complete. It is therefore useful to explore the properties of the Polyakov loop susceptibilities as functions of quark mass in the limit of heavy quarks, thus, bridging the gap between pure gauge theory and QCD.

In this work we discuss the phase structure of the deconfinement transition for heavy quarks. We study the behavior of the Polyakov loop susceptibilities near the CEP and investigate their dependences on the quark mass and quark chemical potential. We also relate the mean-field model results with lattice studies.

1. MODELING DECONFINEMENT IN THE PRESENCE OF QUARKS

To explore the influence of heavy quarks on deconfinement, we consider the following effective model of the thermodynamic potential:

$$T^{-4}\Omega = U_G(L, L^*) + U_Q(L, L^*). \tag{1}$$

The $Z(3)$ invariant part of the potential U_G is extracted from pure gauge theory. In this study the following phenomenological Polyakov loop potential is employed [10]:

$$U_G = -\frac{1}{2}A(T)L^*L + B(T)\ln M_H + \frac{1}{2}C(T)(L^3 + L^{*3}) + D(T)(L^*L)^2, \tag{2}$$

where the $SU(3)$ Haar measure M_H is given by

$$M_H = 1 - 6L^*L + 4(L^3 + L^{*3}) - 3(L^*L)^2, \tag{3}$$

and $A, B, C,$ and D are tuned model parameters. The potential (2) was constructed to describe the lattice data on $SU(3)$ thermodynamics, including the Polyakov loop and its fluctuations. In this way, the model reproduces a first-order deconfinement phase transition at the critical temperature $T_d = 0.27$ GeV, which allows the expression of observables in physical units.

The explicit symmetry breaking term U_Q describes the coupling of quarks to the Polyakov loop. It can be obtained from the fermionic determinant under uniform background gluon field as [11]

$$\det(\hat{Q}_F) = \det((-\partial_\tau + \mu - igA_4)\gamma^0 + i\gamma \cdot \nabla - m). \tag{4}$$

In one-loop approximation, this gives the quark contribution to the effective Polyakov loop potential

$$U_Q = -2N_f\beta^4 \int \frac{d^3k}{(2\pi)^3}\left[T\ln g^+ + T\ln g^-\right], \tag{5}$$

where

$$T\ln g^+ = T\ln(1 + 3L\,e^{-\beta E^+} + 3L^*\,e^{-2\beta E^+} + e^{-3\beta E^+}) \tag{6}$$

specifies the coupling of quarks to the Polyakov loop, with $E^+ = \sqrt{k^2 + m^2} - \mu$. The function $T \ln g^-$ describes the antiquarks and is obtained from Eq. (6) by replacing $\mu \to -\mu$ and $L \leftrightarrow L^*$.

The thermal average of the Polyakov loop $\ell = \langle L \rangle$ and its conjugate $\bar{\ell} = \langle L^* \rangle$ are obtained within the mean-field approximation as solutions of the gap equations

$$\frac{\partial \Omega}{\partial L}\bigg|_{L=\ell, L^*=\bar{\ell}} = 0, \quad \frac{\partial \Omega}{\partial L^*}\bigg|_{L=\ell, L^*=\bar{\ell}} = 0. \tag{7}$$

Details of the deconfinement phase structure are revealed by examining the Polyakov loop fluctuations. In the following, we study how the position of the CEP changes with the number of quark flavors at vanishing and at finite quark density.

1.1. Deconfinement Critical End Point at $\mu = 0$. The expectation value of the transverse Polyakov loop $L_T = (L - L^*)/(2i)$ vanishes due to the symmetry of the effective potential. Consequently, only the longitudinal Polyakov loop $L_L = (L + L^*)/2$ serves as an order parameter for deconfinement. On the other hand, both the longitudinal and transverse fluctuations of the order parameter are nonvanishing and are described by the following susceptibilities [12]:

$$\chi_{L,T} = \frac{1}{2} V \left[\langle LL^* \rangle_c \pm \frac{1}{2} \langle (LL + L^* L^*) \rangle_c \right], \tag{8}$$

where $\langle \ldots \rangle_c$ denotes the connected part. Within the mean-field model, $\chi_{L,T}$ are calculated by taking derivatives of the thermodynamic potential (1) with respect to the corresponding fields [10, 12].

In Fig. 1, we show the Polyakov loop as a function of temperature for different values of quark masses. For a sufficiently large quark mass, the first-order nature of the phase transition persists, while at smaller quark masses the explicit
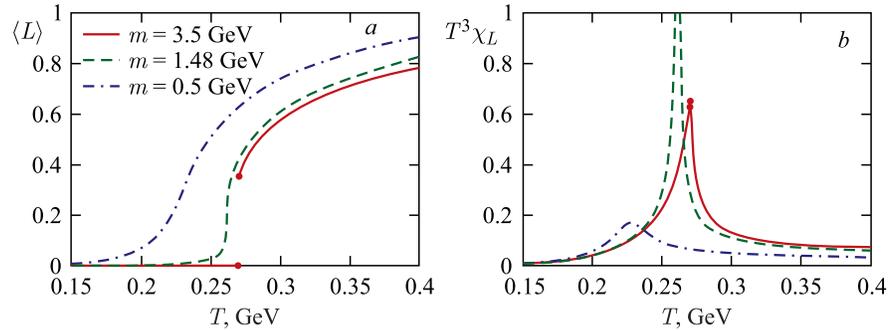

Fig. 1. Temperature dependence of the thermal averaged Polyakov loop (*a*) and its longitudinal susceptibility (*b*) for different values of quark masses: $m = 0.5$, 1.48, and 3.5 GeV at $N_f = 3$

symmetry breaking increases resulting in the transition to be a crossover. The end point of the first-order transition line defines the critical value of the quark mass m_{CEP}.

To identify the CEP, we use the longitudinal fluctuations of the Polyakov loop. In Fig. 1, we show the longitudinal susceptibility for three degenerate quark flavors. While both susceptibilities depend on the value of the quark mass, only the longitudinal one shows an enhancement near the CEP [13]. The transverse susceptibility decreases monotonically with decreasing quark mass. Thus, for a given N_f, the CEP can be located by identifying the global maximum of χ_L. For different N_f, we find the following values of the critical quark masses:

$$m_{\mathrm{CEP}} = 1.10, \ 1.35, \ 1.48 \ \mathrm{GeV} \ \ \mathrm{for} \ \ N_f = 1, 2, 3. \tag{9}$$

The resulting trend, with m_{CEP} increasing with N_f, is consistent with the recent study of [14]. The location of the deconfinement critical end point is closely related to the form of the Polyakov loop potential [14]. In the present calculation, U_G reproduces the lattice data on the equation of state as well as on the susceptibilities of the Polyakov loop. This feature is crucial for locating the CEP, which is influenced by fluctuations of the order parameter.

1.2. Deconfinement CEP at Finite Chemical Potential. At finite μ, the thermal averaged Polyakov loop ℓ and its conjugate $\bar{\ell}$ are both real but, in general, different [12, 15, 16]. This is because at nonzero μ the effective action is complex [12, 16].

The finite density results for the longitudinal susceptibility are shown in Fig. 2, a. As in the case of $\mu = 0$, only the longitudinal susceptibility is enhanced near the CEP, whereas the transverse susceptibility is insensitive to criticality.
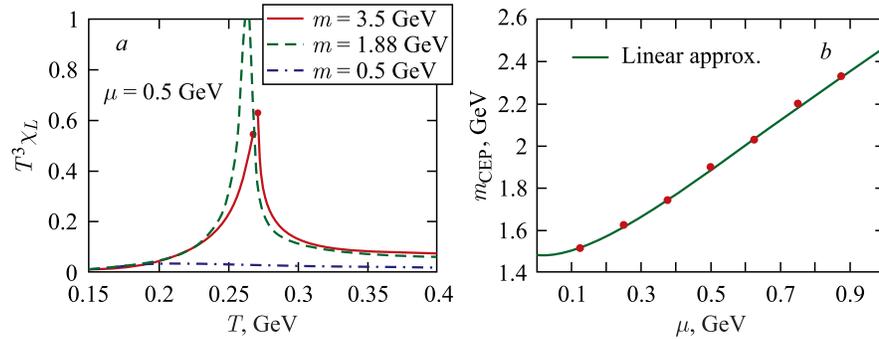

Fig. 2. a) The longitudinal χ_L susceptibility (8) as a function of T at $\mu = 0.5$ GeV for different values of quark mass. b) The critical quark mass as a function of the quark chemical potential for $N_f = 3$. Solid points represent results obtained from the global maximum of χ_L. The line represents the result obtained by keeping only the linear symmetry breaking term in the quark potential [13]

The dependence of the critical quark mass m_{CEP} on the chemical potential μ is shown in Fig. 2, b. The points in the figure are extracted from the divergence of χ_L, while the line is determined using only the leading explicit symmetry breaking term in the effective potential [13]. The increase of the critical quark mass with μ indicates that the first-order region shrinks with increasing density. This finding is consistent with the lattice results presented in [17–19].

2. INTERPLAY BETWEEN CONFINEMENT
AND CHIRAL SYMMETRY BREAKING

In QCD, the expectation value of the Polyakov loop is substantially modified by dynamical quarks. A constructive way to explore deconfinement is to study various fluctuations associated with conserved charges. In particular, the kurtosis of net-quark number fluctuations was proposed as a measure of the onset of deconfinement [20].

In Fig. 3, the ratio $R_A = \chi_A/\chi_R$ is compared with the kurtosis of the quark number fluctuations, where χ_A represents the fluctuation of the modulus of the Polyakov loop [8]. The quark deconfinement takes place evidently together with a qualitative change in R_A. Those abrupt changes in the Polyakov loop and
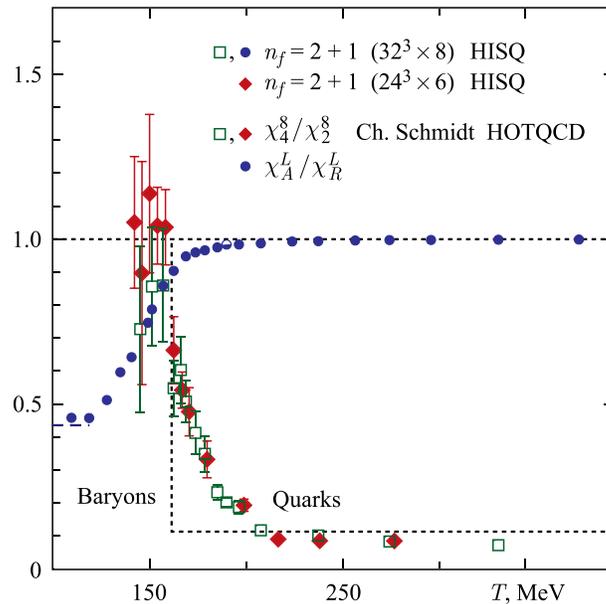

Fig. 3. The ratio of the Polyakov loop susceptibilities $R_A = \chi_A/\chi_R$ and the kurtosis of net-quark number fluctuations calculated on the lattice [10, 20]

quark number fluctuations appear in a narrow temperature range lying around the pseudocritical temperature of chiral symmetry restoration. Therefore, at vanishing chemical potential, these observables suggested that $T_{\mathrm{deconf}} \simeq T_{\mathrm{chiral}}$.

CONCLUSIONS

We have discussed the properties of the Polyakov loop susceptibilities near the deconfinement transition within the effective Polyakov loop model and in QCD on the lattice.

Within the model calculation we have concentrated on the structure of the phase diagram and discussed the properties of the deconfinement critical end point. The analysis was presented at finite and vanishing chemical potential for different number of quark flavors. It is shown that the considered effective model captures some basic properties of QCD in the limit of heavy quarks, and yields results that can be compared with lattice calculations.

We have also argued, within lattice QCD studies, that the ratio of the longitudinal and transverse Polyakov loop susceptibilities characterizes the onset of deconfinement in QCD. We have compared the properties of this ratio with the kurtosis of the net-baryon number fluctuations, which suggests that at vanishing quark chemical potential deconfinement appears in the region of the chiral crossover.

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