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NONLOCAL CHIRAL QUARK MODEL WITH CONFINEMENT

A. E. Radzhabov [1], *M. K. Volkov* [2]

Joint Institute for Nuclear Research, Dubna

The nonlocal version of the $SU(2) \times SU(2)$ symmetric four-quark interaction of the NJL type is considered. Each of quark lines contains the form factors. These form factors remove the ultraviolet divergences in quark loops. The additional condition for constituent quark mass function $m(p)$ ensures the absence of the poles in the quark propagator (quark confinement). The model contains minimal numbers of the arbitrary parameters because the constituent quark mass $m(0)$ is equal to the cut-off parameter $\Lambda = 340$ MeV in the chiral limit. These parameters are fixed by the experimental value of the weak pion decay constant $F_\pi = 93$ MeV and allow us to describe the mass of the light scalar meson and decays $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ in the qualitative agreement with experimental data.

Рассмотрена нелокальная версия $SU(2)\times SU(2)$-симметричного четырехкваркового взаимодействия. Каждая кварковая линия содержит формфактор. Формфакторы устраняют ультрафиолетовые расходимости в кварковых петлях. Дополнительное условие на массовую функцию конституентного кварка обеспечивает отсутствие полюсов в пропагаторе кварка (кварковый конфайнмент). Модель содержит минимальное число произвольных параметров, так как конституентная кварковая масса $m(0)$ равна параметру обрезания $\Lambda = 340$ МэВ в киральном пределе. Эти параметры фиксируются по экспериментальному значению константы слабого распада пиона $F_\pi = 93$ МэВ и позволяют описать массу легкого скалярного мезона и распады $\rho \to \pi\pi$ и $a_1 \to \rho\pi$ в качественном согласии с экспериментальными данными.

INTRODUCTION

Recently it has been proposed that the nonlocal $SU(2)\times SU(2)$ chiral quark model allows us to describe the intrinsic properties and strong interaction of the scalar, pseudoscalar, vector and axial-vector mesons [1]. Unlike the local Nambu–Jona-Lasinio (NJL) model [2], in this model ultraviolet (UV) divergences are absent and the quark confinement takes place. These properties of the model are provided by the form factors which are connected with each of the quark fields. The existence of these form factors is motivated by the instanton model [3–6]. It has been shown that the main low-energy theorems are fulfilled in the framework of this model. The model [1] is the development of a number of similar models [5–8].

This work is devoted to further development of these models. Before describing our approach let us shortly recall the basic method used in [1]. There the special condition, Eq. (8), for the form of the quark mass function was proposed which provided the absence of the poles in the quark propagator (quark confinement) (see Sect. 3). This method was close to the works [9]. Eq. (8) can be motivated by the existence of the nonlocal quark condensate [8].

[1]E-mail: aradzh@thsun1.jinr.ru
[2]E-mail: volkov@thsun1.jinr.ru

As a result, several solutions to the quark mass function $m(p)$ appeared. Of these solutions only one satisfies the general requirement of the form-factor behaviour in the whole domain of the p^2 and leads to satisfactory physical predictions. It is self-consistent only at definite values of the model parameters.

In the present work we propose the new representation for the quark propagator which leads to a simpler solution to the dynamical quark mass. This representation can also be connected with the nonlocal quark condensate which appeared in the gap equation. As a result, we obtain a simple expression for the dynamical quark mass that contains only one arbitrary parameter. The value of the quark mass at $p^2 = 0$ is equal in the chiral limit to the cut-off parameter Λ, $m(0) = \Lambda = 340$ MeV. This value corresponds to the experimental value of the pion weak decay constant $F_\pi = 93$ MeV. The mass of the scalar meson and decays $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ are described in qualitative agreement with experimental data. The π–a_1 transitions in this model, like in other models of this kind, are very small and can be omitted [1, 7].

The paper is organized as follows. In Sect. 1, we consider a nonlocal four-quark interaction and after bosonization derive the gap equation for dynamical quark mass. The quark mass function $m(p)$ is defined in Sect. 2. In Sect. 3, the masses and couplings of the scalar and pseudoscalar mesons are obtained and the main parameters of the model are fixed. In Sect. 4, calculations of the four-quark coupling constant G_ρ, G_{a_1}, a_1, ρ-meson coupling constant, and the decays $\rho \to \pi\pi$, $a_1 \to \rho\pi$ are given. The π–a_1 transitions are considered. The last section is devoted to the discussion of our results.

1. $SU(2) \times SU(2)$ QUARK MODEL WITH NONLOCAL INTERACTION

The $SU(2) \times SU(2)$ symmetric action with the nonlocal four-quark interaction has the form

$$\mathcal{S}(\bar{q}, q) = \int d^4x \left\{ \bar{q}(x)(i\hat{\partial}_x - m_c)q(x) + \frac{G_\pi}{2}\left(J_\sigma(x)J_\sigma(x) + J_\pi^a(x)J_\pi^a(x)\right) - \right.$$
$$\left. - \frac{G_\rho}{2} J_\rho^{\mu a}(x) J_\rho^{\mu a}(x) - \frac{G_{a_1}}{2} J_{a_1}^{\mu a}(x) J_{a_1}^{\mu a}(x) \right\}, \quad (1)$$

where $\bar{q}(x) = (\bar{u}(x), \bar{d}(x))$ are the u and d quark fields; m_c is the diagonal matrix of the current quark masses. The nonlocal quark currents $J_I(x)$ are expressed as

$$J_I(x) = \int\int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{q}(x - x_1)\, \Gamma_I\, q(x + x_2), \quad (2)$$

where the nonlocal function $f(x)$ is normalized by $f(0) = 1$. In (2) the matrices Γ_I are defined as

$$\Gamma_\sigma = \mathbf{1}, \quad \Gamma_\pi^a = i\gamma^5\tau^a, \quad \Gamma_\rho^{\mu\, a} = \gamma^\mu\tau^a, \quad \Gamma_{a_1}^{\mu\, a} = \gamma^5\gamma^\mu\tau^a,$$

where τ^a are the Pauli matrices and γ^μ, γ^5 are the Dirac matrices.

In this article, we mainly consider the strong interactions. The electroweak fields may be introduced by gauging the quark field by the Schwinger phase factors (see, cf. [6, 7]).

After bosonization the action becomes

$$\mathcal{S}(q,\bar{q},\sigma,\pi,\rho,a) = \int d^4x \Big\{ -\frac{1}{2G_\pi}(\tilde{\sigma}(x)^2 + \pi^a(x)^2) + \\ + \frac{1}{2G_\rho}(\rho^{\mu a}(x))^2 + \frac{1}{2G_{a_1}}(a_1^{\mu a}(x))^2 + \bar{q}(x)(i\hat{\partial}_x - m_c)q(x) + \\ + \int\int d^4x_1 d^4x_2\, f(x-x_1)f(x_2-x)\bar{q}(x_1)(\tilde{\sigma}(x) + \\ + \pi^a(x)i\gamma^5\tau^a + \rho^{\mu\, a}(x)\gamma^\mu\tau^a + a_1^{\mu a}(x)\gamma^5\gamma^\mu\tau^a)q(x_2) \Big\}, \quad (3)$$

where $\tilde{\sigma},\pi,\rho,a$ are the σ, π, ρ, a_1 meson fields, respectively. The field $\tilde{\sigma}$ has a nonzero vacuum expectation value $\langle\tilde{\sigma}\rangle_0 = \sigma_0 \neq 0$. In order to obtain a physical scalar field with zero vacuum expectation value, it is necessary to shift the scalar field as $\tilde{\sigma} = \sigma + \sigma_0$. This leads to the appearance of the nonlocal quark mass $m(p)$ instead of the current quark mass m_c:

$$m(p) = m_c + m_{\rm dyn}(p), \quad (4)$$

where $m_{\rm dyn}(p) = -\sigma_0 f^2(p)$ is dynamical quark mass. From the action, Eq. (3), by using

$$\left\langle \frac{\delta S}{\delta\sigma} \right\rangle_0 = 0, \quad (5)$$

one can obtain the gap equation for dynamical quark mass

$$m_{\rm dyn}(p) = G_\pi \frac{8N_c}{(2\pi)^4} f^2(p) \int d_E^4k\, f^2(k) \frac{m(k)}{k^2 + m^2(k)}. \quad (6)$$

The right-hand side of this equation is the tadpole of the quark propagator taken in the Euclidean domain. Eqs. (4), (6) have the following solution:

$$m(p) = m_c + (m_q - m_c) f^2(p), \quad (7)$$

where $m_q \equiv m(0)$.

2. DYNAMICAL QUARK MASS

Let us recall the representation for the quark propagator in the chiral limit used in our last work [1]. We demand the absence of pole singularities in the scalar part of the quark propagator:

$$\frac{m(p^2)}{m^2(p^2) + p^2} \equiv \frac{1}{2} Q(p^2), \quad (8)$$

where $Q(p^2)$ is considered as an entire function in the complex p^2 plane decreasing in the Euclidean domain as $p^2 \to \infty$. Note that the Gaussian function was used for $Q(p^2)$:

$$Q(p^2) = \frac{1}{\mu} \exp\left(-\frac{p^2}{\Lambda^2} \right), \quad (9)$$

where μ and Λ are arbitrary parameters. Eq. (8) has the following solutions:

$$m_{\pm}(p^2) = Q^{-1}(p^2)\left(1 \pm \sqrt{1 - p^2 Q^2(p^2)}\right). \tag{10}$$

Then three different situations occur at various values of the parameters μ and Λ:

1. There is some region of real p^2 where $p^2 Q^2(p^2) > 1$. This situation leads to the appearance of complex values of the quark mass. This case was not considered in [1].

2. The relation $p^2 Q^2(p^2) < 1$ is fulfilled in the whole domain of real p^2. Then, from two possible solutions only the solution $m_-(p)$ can be used which decreases as $p^2 \to \infty$. However, this solution predicts the σ-meson mass and decays $\sigma \to \pi\pi$, $\rho \to \pi\pi$ that are in disagreement with the experiment.

3. The function $p^2 Q^2(p^2)$ equals 1 at a single real point p_0. In this case the continuous mass function is

$$m(p^2) = Q^{-1}(p^2)\left(1 - \mathrm{sgn}\,(p^2 - p_0^2)\sqrt{1 - p^2 Q^2(p^2)}\right). \tag{11}$$

The last case is defined by the conditions

$$p^2 Q^2(p^2)|_{p^2=p_0^2} = 1, \quad (p^2 Q^2(p^2))'|_{p^2=p_0^2} = 0. \tag{12}$$

As a result, we come to a complicated form of solution that exists only under a special choice of model parameters.

Here we propose a somewhat different representation for the quark propagator that leads to a simpler solution to the quark mass function

$$\frac{m^2(p^2)}{m^2(p^2) + p^2} \equiv \tilde{Q}(p^2), \tag{13}$$

where

$$\tilde{Q}(p^2) = \exp\left(-\frac{p^2}{\Lambda^2}\right), \tag{14}$$

Note that the left-hand side of Eq. (13) corresponds to the integrand in the gap equation (6) taking into account Eq. (7). In contrast to Eq. (8), Eq. (13) leads to a simpler solution to the mass function[1]

$$m(p^2) = \sqrt{\frac{p^2}{\tilde{Q}^{-1}(p^2) - 1}} = \sqrt{\frac{p^2}{2}\left(\mathrm{cth}\left(\frac{p^2}{2\Lambda^2}\right) - 1\right)}. \tag{15}$$

Note that we also have only one free parameter Λ; $m(p^2)$ does not have any singularities in the whole real axis and exponentially drops as $p^2 \to \infty$ in the Euclidean domain. From Eq. (10) it follows that the form factors that provide the absence of UV divergences in our model behave similarly. At $p^2 = 0$ the mass function is equal to the cut-off parameter Λ,

[1] We use only the solution with positive sign.

$m(0) = \Lambda$. The pole part of the quark propagator also does not contain singularities that provide quark confinement[1]

$$\frac{1}{m^2(p^2)+p^2} = \frac{1-\tilde{Q}(p^2)}{p^2}. \tag{16}$$

When taking into account current quark mass, Eq. (13) is modified as follows:

$$\frac{m^2(p^2)-m_c^2}{m^2(p)+p^2} = \tilde{Q}_c(p^2), \tag{17}$$

where

$$\tilde{Q}_c(p^2) = \exp\left(-\frac{p^2+m_c^2}{\Lambda^2}\right). \tag{18}$$

Here m_c^2 is introduced in the form that conserves the analytical properties of the mass function $m(p)$. Then the mass function takes the form

$$m(p^2) = \sqrt{\frac{m_c^2+p^2\tilde{Q}_c(p^2)}{1-\tilde{Q}_c(p^2)}}. \tag{19}$$

3. PSEUDOSCALAR AND SCALAR MESONS

Let us consider the scalar and pseudoscalar mesons. The meson propagators are given by

$$D_{\sigma,\pi}(p^2) = \frac{1}{-G_\pi^{-1}+\Pi_{\sigma,\pi}(p^2)} = \frac{g^2_{\sigma,\pi}(p^2)}{p^2-M^2_{\sigma,\pi}}, \tag{20}$$

where $M_{\sigma,\pi}$ are the meson masses; $g_{\sigma,\pi}(p^2)$ are the functions describing renormalization of the meson fields, and $\Pi_{\sigma,\pi}(p^2)$ are the polarization operators (see Fig. 1) defined by

$$\Pi_{\sigma,\pi}(p^2) = \\ = i\frac{2N_c}{(2\pi)^4}\int d^4k f^2(k_-^2)f^2(k_+^2)\,\mathrm{Sp}\left[\mathrm{S}(k_-)\Gamma_{\sigma,\pi}\mathrm{S}(k_+)\Gamma_{\sigma,\pi}\right], \tag{21}$$

where $k_\pm = k \pm p/2$. The meson masses $M_{\sigma,\pi}$ are found from the position of the pole in the meson propagator

$$\Pi_{\sigma,\pi}(M^2_{\sigma,\pi}) = G_\pi^{-1}, \tag{22}$$

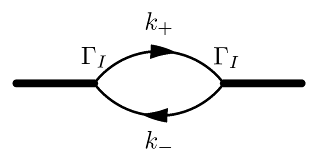

Fig. 1. Meson polarization operator. The thick lines are mesons. All loops in Figs. 1, 3–5 consist of constituent quarks (thin lines)

and the constants $g_{\sigma,\pi}(M^2_{\sigma,\pi})$ are given on meson mass shell by (see also Fig. 2)

$$g^{-2}_{\sigma,\pi}(M^2_{\sigma,\pi}) = \left.\frac{d\Pi_{\sigma,\pi}(p^2)}{dp^2}\right|_{p^2=M^2_{\sigma,\pi}}. \tag{23}$$

[1]Note that similar functions were used in [10, 11] in order to describe the quark confinement.

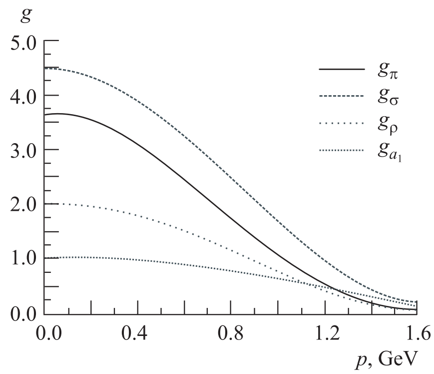

Fig. 2. Momentum dependence of the mesons strong coupling constants

In the chiral limit the pion constant $g_\pi(0)$ is given by [4]

$$g_\pi^{-2}(0) = \frac{N_c}{4\pi^2 m_q^2} \int\limits_0^\infty du\, u \frac{m^2(u) - u\, m(u) m'(u) + u^2 m'^2(u)}{(u + m^2(u))^2}. \tag{24}$$

The gap equation in the chiral limit takes the simple form

$$G_\pi \Lambda^2 = \frac{2\pi^2}{N_c}. \tag{25}$$

The quark condensate in the chiral limit is

$$\langle \bar{q} q \rangle_0 = -\frac{N_c}{4\pi^2} \int\limits_0^\infty du\, u \frac{m(u)}{u + m^2(u)}. \tag{26}$$

As is shown in [1, 6, 7], in model of this kind the Goldberger–Treiman relation holds:

$$f_\pi = \frac{m_q}{g_\pi}. \tag{27}$$

From Eqs. (24), (27) the value of the parameter $\Lambda = m_q = 340$ MeV in the chiral limit can be obtained. Then, from Eqs. (25)–(27) we obtain

$$g_\pi(0) = 3.67, \quad G_\pi = 56.6 \text{ GeV}, \quad \langle \bar{q} q \rangle_0 = -(188 \text{ MeV})^3. \tag{28}$$

In the description of pion mass it is necessary to introduce the nonzero current quark mass m_c. In our model $M_\pi^2 \ll \Lambda^2$. Therefore, we can consider only the lowest order of the expansion in small p^2. Then, one gets from Eq. (20)

$$M_\pi^2 = g_\pi^2(0) \left(\frac{1}{G_\pi} - \frac{N_c}{2\pi^2} \int\limits_0^\infty du\, u \frac{f(u)^4}{u + m^2(u)} \right). \tag{29}$$

By using the expression for G_π from the gap equation (6), the Gell-Mann–Oakes–Renner relation can be reproduced:

$$M_\pi^2 = -2\frac{m_c\langle\bar{q}q\rangle_0}{F_\pi^2} + O(m_c^2). \tag{30}$$

From Eq. (30) with $M_\pi = 140$ MeV we obtain the value of the current quark mass $m_c = 13$ MeV. The other model parameters in this case change very little:

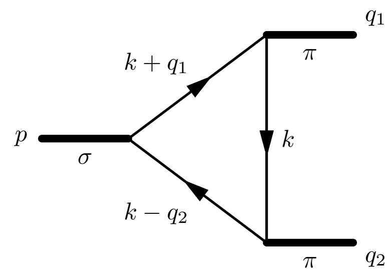

Fig. 3. Decays $\sigma \to \pi\pi$, $\rho \to \pi\pi$

$$\Lambda = 343\ \text{MeV}, \quad g_\pi(M_\pi) = 3.57, \quad G_\pi = 56.5\ \text{GeV}, \quad \langle\bar{q}q\rangle_0 = -(189\ \text{MeV})^3. \tag{31}$$

Thus, in calculations of the amplitudes of various processes we can use the values of parameters taken in the chiral limit.

With the help of the parameters (28) we get for sigma meson $M_\sigma = 420$ MeV and $g_\sigma(M_\sigma) = 3.85$. The amplitude of the decay $\sigma \to \pi\pi$, described by the diagram in Fig. 3, is equal to $A_{(\sigma\to\pi^+\pi^-)} = 1.67$ GeV. Then, the total decay width is

$$\Gamma_{(\sigma\to\pi\pi)} = \frac{3A^2_{(\sigma\to\pi^+\pi^-)}}{32\pi M_\sigma}\sqrt{1-\left(\frac{2M_\pi}{M_\sigma}\right)^2} = 150\ \text{MeV}. \tag{32}$$

Comparing these results with experimental data, one finds that M_σ is in satisfactory agreement with experiment; however, the decay width is very small.

4. VECTOR AND AXIAL-VECTOR MESONS

The propagators of the vector and axial-vector mesons have the transversal and longitudinal parts:

$$D^{\mu\nu}_{\rho,a_1} = T^{\mu\nu}D^T_{\rho,a_1} + L^{\mu\nu}D^L_{\rho,a_1}, \tag{33}$$

where $T^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu/p^2$, $L^{\mu\nu} = p^\mu p^\nu/p^2$ and

$$D^T_{\rho,a_1} = \frac{1}{G^{-1}_{\rho,a_1} + \Pi^T_{\rho,a_1}(p^2)} = \frac{g^2_{\rho,a_1}(p^2)}{M^2_{\rho,a_1} - p^2},\ D^L_{\rho,a_1} = \frac{1}{G^{-1}_{\rho,a_1} + \Pi^L_{\rho,a_1}(p^2)}. \tag{34}$$

Here, Π^T_{ρ,a_1} and Π^L_{ρ,a_1} are the transversal and longitudinal parts of the polarization operator $\Pi^{\mu\nu}_{\rho,a_1}(p^2)$:

$$\Pi^{\mu\nu}_{\rho,a_1}(p^2) = i\frac{2N_c}{(2\pi)^4}\int d^4k f^2(k_-)f^2(k_+)\,\mathrm{Sp}\left[\mathrm{S}(k_-)\Gamma_{\rho,a_1}\mathrm{S}(k_+)\Gamma_{\rho,a_1}\right].$$

The constants G_{ρ,a_1} are fixed by physical meson masses:

$$G^{-1}_{\rho,a_1} = -\Pi^T_{\rho,a_1}(M_{\rho,a_1})$$

and numerically equal $G_\rho = 6.5$ GeV^{-2}, $G_{a_1} = 0.67$ GeV^{-2}. Note that there is no pole in the longitudinal part of the vector meson propagators.

The constants $g_{\rho,a_1}(M^2_{\rho,a_1})$ are equal to

$$g^{-2}_{\rho,a_1}(M^2_{\rho,a_1}) = -\frac{d\Pi^T_{\rho,a_1}(p^2)}{dp^2}\bigg|_{p^2=M^2_{\rho,a_1}}. \tag{35}$$

From Eq. (35) we obtain $g_\rho(M_\rho) = 1.23$, $g_a(M_{a_1}) = 0.43$. At $p^2 = 0$ we have $g_\rho(0) = 2$, $g_a(0) = 1$ (see Fig. 2).

The decay $\rho \to \pi\pi$ is described by the triangle diagram similar to the diagram in Fig. 3. The amplitude for the process is

$$A^\mu_{(\rho\to\pi\pi)} = a_{(\rho\to\pi\pi)}(q_1 - q_2)^\mu, \tag{36}$$

where q_i are momenta of the pions. We obtain $a_{(\rho\to\pi\pi)} = 5.72$ and the decay width

$$\Gamma_{(\rho\to\pi\pi)} = \frac{a^2_{(\rho\to\pi\pi)}M_\rho}{48\pi}\left(1 - \left(\frac{2M_\pi}{M_\rho}\right)^2\right)^{3/2} = 135 \text{ MeV}, \tag{37}$$

which is in qualitative agreement with the experimental value (149.2 ± 0.7) MeV [12].

The decay $a_1 \to \rho\pi$ is described in a similar manner. The amplitude for the process $a_1 \to \rho\pi$ is

$$A^{\mu\nu}_{(a_1\to\rho\pi)} = a_{(a_1\to\rho\pi)}g^{\mu\nu} + b_{(a_1\to\rho\pi)}p^\nu q^\mu, \tag{38}$$

where p, q are momenta of a_1, ρ mesons, respectively. We obtain $a_{(a_1\to\rho\pi)} = -1.26$ GeV, $b_{(a_1\to\rho\pi)} = 26.8$ GeV^{-1}. As a result, the decay width is equal to $\Gamma_{(a_1\to\rho\pi)} = 170$ MeV. This value has the same order as experimental data 250–600 MeV. Note that the width of the decay $a_1 \to \rho\pi$ strongly depends on mass of the a_1 meson. Indeed, for $M_{a_1} = 1.3$ GeV we have $\Gamma_{(a_1\to\rho\pi)} = 260$ MeV.

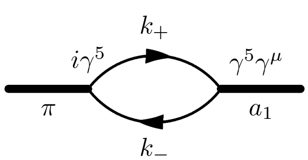

Fig. 4. Transition loop describing π–a_1 mixing

The longitudinal component of the a_1-meson field is mixed with the pion, as is illustrated in Fig. 4. The amplitude describing this mixing has the form

$$A^\mu_{(\pi\to a_1)}(p^2) = i\Lambda g_{a_1}(p^2)g_\pi(p^2)C_{(\pi\to a_1)}(p^2)p^\mu = id(p^2)p^\mu, \tag{39}$$

where $C_{(\pi\to a_1)}(p^2)$ at the point $p^2 = 0$ in the chiral limit is

$$C_{(\pi\to a_1)}(0) = \frac{N_c}{4\pi^2\Lambda^3}\int_0^\infty du\, u\frac{m^2(u)}{(u+m^2(u))^2}(2m(u) - um'(u)) = 0.061. \tag{40}$$

As a result, $d(0)$ is equal to 80 MeV.

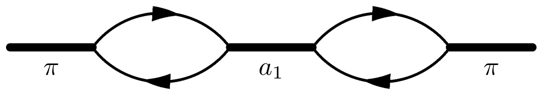

Fig. 5. Diagram describing additional renormalization of the pion field

The diagram (see Fig. 5) gives the additional pion kinetic term $\Delta L_{\text{kin}} = \Delta \cdot \dfrac{p^2}{2} \pi^a(p)^2$. Let us estimate this term in the chiral limit:

$$\Delta = \frac{(\Lambda g_{a_1}(0) g_\pi(0) C_{(\pi\to a_1)}(0))^2}{g_{a_1}^2(0)(G_{a_1}^{-1} + \Pi_{a_1}^L(0))} \approx \Lambda^2 g_\pi^2(0) C^2_{(\pi\to a_1)}(0) G_{a_1} \approx 0.004. \quad (41)$$

As one can see, Δ is very small and the effect of the π–a_1 mixing can be neglected.

DISCUSSION AND CONCLUSION

In this work we have considered one more possibility of constructing the nonlocal chiral quark model providing the absence of UV divergences and quark confinement. These features of the model are specified by the nonlocal kernel which appears in the four-quark interaction. Such a structure of the four-quark interaction can be motivated by the instanton model [4–6].

Similar models were considered in [1, 5–8]. Thus, in [7] nonlocal form factor was chosen in the Gaussian form that exponentially decreases in the Euclidean domain of momenta. In [1, 8] it was proposed to relate the functions defining the nonlocal kernel with the nonlocal quark condensate. This relation provides quark confinement. However, Eq. (8), which was used in [1], leads to complicated solutions for the mass function at different values of model parameters. Therefore, in the present work we have changed conditions for quark mass function, Eq. (13), in order to obtain a simpler solution for it. We preserve all requirements providing the absence of UV divergences and the confinement quarks in our model. It is worth noting that, though the quark mass and the cut-off parameter are connected by the condition $m(0) = \Lambda$, we can satisfactorily describe the scalar meson mass and strong decays $\rho \to \pi\pi$, $a_1 \to \rho\pi$.

Note that in our model, like in all models of this kind, π–a_1 transitions can be neglected. Actually, in local models additional renormalization of pion field with allowance made for the π–a_1 mixing is about 40 %, whereas in model of this kind it is almost one order smaller. In particular, in our model this correction does not exceed 1 %.

The worst prediction of our model is the decay $\sigma \to \pi\pi$. The failure of the model to describe the σ meson is expectable. Similar problems appeared in the QCD sum rule method. In the scalar channel with vacuum quantum numbers the corrections from different sources may be valuable. Indeed, it has recently been shown that the $1/N_c$ corrections in this channel are rather big [13], and the Hartree–Fock approximation may be inadequate in this case. Moreover, for correct description of the scalar meson it is necessary to take into account the mixing with the four-quark state [14] and the scalar glueball [15].

In conclusion, let us summarize the main results of the present work. The pseudoscalar, scalar, vector and axial-vector sectors of the model have been considered. The masses and strong coupling constants of the mesons were calculated. The strong coupling constants of the mesons were shown to noticeably decrease with increasing p^2 in the physical domain (see Fig. 2). Among satisfactory predictions of the model are the decays widths $\rho \to \pi\pi$, $a_1 \to \rho\pi$ and the mass of the sigma meson.

In the future, we plan to describe electromagnetic interactions in the framework of this model, calculate the e. m. pion radius, polarizability of the pion and consider the processes $\pi^0 \to \gamma\gamma$, $\gamma^* \to \gamma\pi$ (here γ^* is a virtual photon). We also plan to generalize this model to the $U(3) \times U(3)$ chiral group by introducing new parameters: mass of strange quark m_s and cut-off Λ_s, which allows us to describe intrinsic properties and interactions of strange mesons.

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