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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

CHIRAL PHENOMENOLOGICAL RELATIONS BETWEEN RATES OF RARE RADIATIVE DECAYS OF KAON TO PION AND LEPTONS AND THE MESON FORM FACTORS

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In the framework of the chiral perturbation theory we obtain the phenomenological relations between decay branches of rare radiative kaon to pion and leptons $K^+ \to \pi^+ l^+ l^-$ and $K_S^0 \to \pi^0 l^+ l^-$ and meson form factors. The comparison of these results with the present-day experimental data shows us that the ChPT relations for a charge kaon can determine meson form factors from already measured decay rates at high precision level. However, in the case of the neutral kaon decays $K^0 \to \pi^0 e^+ e^- (\mu^+ \mu^-)$ the form factor data are known to a higher precision than data on the differential rates of radiative kaon decay $K^0 \to \pi^0 e^+ e^- (\mu^+ \mu^-)$.

В рамках киральной теории возмущения получаем феноменологические отношения между вероятностями редких радиационных распадов каона в пион и лептоны $K^+ \to \pi^+ l^+ l^-$ и $K_S^0 \to \pi^0 l^+ l^-$ и мезонными формфакторами. Сравнение этих результатов с современными экспериментальными данными показывает, что полученные киральные отношения могут определить мезонные формфакторы из данных по распадам на уровне современной высокой экспериментальной точности. Однако в случае распадов нейтрального каона $K^0 \to \pi^0 e^+ e^- (\mu^+ \mu^-)$ данные по формфакторам известны с более высокой точностью, чем данные по распадам.

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INTRODUCTION

New data of decay branches $\mathrm{Br}\,(K^+ \to \pi^+ l^+ l^-)$ and $\mathrm{Br}\,(K_S^0 \to \pi^0 l^+ l^-)$ were obtained a few years ago in the NA48 experiment [1–3]. In analysis of these data a number of theoretical models were used [4–8]. One of them is chiral perturbation theory with weak static interactions [7, 8] which take into account fermion loops. In this paper, we upgrade this result in order to study the relations between the decay branches and form factors including the decay $K_S^0 \to \pi^0 l^+ l^-$.

In transitions $K^+ \to \pi^+ l^+ l^-$ and $K_S^0 \to \pi^0 l^+ l^-$ the main role is played by one virtual photon exchange: $K \to \pi \gamma^* \to \pi l^+ l^-$. To describe it, we must use the theory of strong interactions (QCD) and the electroweak theory. Instead of QCD we use the chiral perturbation theory (ChPT) in bosonization form [7–10] and take into account the experimental meson electromagnetic form factors and their resonance nature.

Main difference of the present paper from other approaches (for example, [4–6, 9, 11]) is that we have only one coupling constant (g_8). Nevertheless, if we take experimentally determined charge radii of mesons and resonances, our prediction becomes more accurate. We can conclude that the chiral effective Lagrangian approach helps us to obtain the set of relations between experimental form factors and decay branches.

In this article, we ameliorate amplitudes from [7, 8], calculate the corresponding decay rates, and test them with available experiments. Really, the results of [7,8] are improved and expanded on the neutral kaon decays.

1. CHIRAL BOSONIZATION OF EW MODEL

We start with Lagrangian of weak interactions in bosonized form [7]:

$$\mathcal{L} = -\frac{e}{2\sqrt{2}\sin\theta_W}(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-),$$

$$J_\mu^\pm = [J_\mu^1 \pm i J_\mu^2]\cos\theta_C + [J_\mu^4 \pm i J_\mu^5]\sin\theta_C,$$

where Cabibbo angle $\sin\theta_C = 0.223$. Using the Gell-Mann matrices λ^k, one can define the meson current as [10]

$$i\sum \lambda^k J_\mu^k = i\lambda^k (V_\mu^k - A_\mu^k)^k = F_\pi^2\, \mathrm{e}^{i\xi}\partial_\mu\, \mathrm{e}^{-i\xi},$$

$$\xi = F_\pi^{-1}\sum_{k=1}^{8} M^k\lambda^k = F_\pi^{-1}\begin{pmatrix} \pi^0 + \dfrac{\eta}{\sqrt{3}} & \pi^+\sqrt{2} & K^+\sqrt{2} \\ \pi^-\sqrt{2} & -\pi^0 + \dfrac{\eta}{\sqrt{3}} & K^0\sqrt{2} \\ K^-\sqrt{2} & \overline{K}^0\sqrt{2} & -\dfrac{2\eta}{\sqrt{3}} \end{pmatrix},$$

here $F_\pi \simeq 92.4$ MeV. In the first orders in mesons one can write

$$V_\mu^- = \sqrt{2}(\sin\theta_C\,(K^-\partial_\mu\pi^0 - \pi^0\partial_\mu K^-) + \cos\theta_C\,(\pi^-\partial_\mu\pi^0 - \pi^0\partial_\mu\pi^-))$$

and

$$A_\mu^- = \sqrt{2}F_\pi\,(\partial_\mu K^-\sin\theta_C + \partial_\mu\pi^-\cos\theta_C).$$

This Lagrangian allows us to use the instantaneous weak interaction model [7, 8].

2. THE $K \to \pi l^+ l^-$ AMPLITUDE

In this section we briefly remind the results of paper [7], which we will use in our work. Further discussions can be found in paper [8].

According to [7, 8], for the process $K^+ \to \pi^+ l^+ l^-$ we have diagrams shown in Fig. 1, leading to the amplitude:

$$A_{K\to\pi l^+l^-} = 2g_8 e G_{\mathrm{EW}} L_\nu D_{\mu\nu}^{\gamma(\mathrm{rad})}(q)(k_\mu + p_\mu)\mathcal{T}(q^2, k^2, p^2), \tag{1}$$

where $g_8 \simeq 5.1$ is the effective parameter of enhancement [4, 7, 8],

$$G_{\mathrm{EW}} = \frac{\sin\theta_C\cos\theta_C}{8M_W^2}\frac{e^2}{\sin^2\theta_W} \equiv \sin\theta_C\cos\theta_C\frac{G_F}{\sqrt{2}},$$

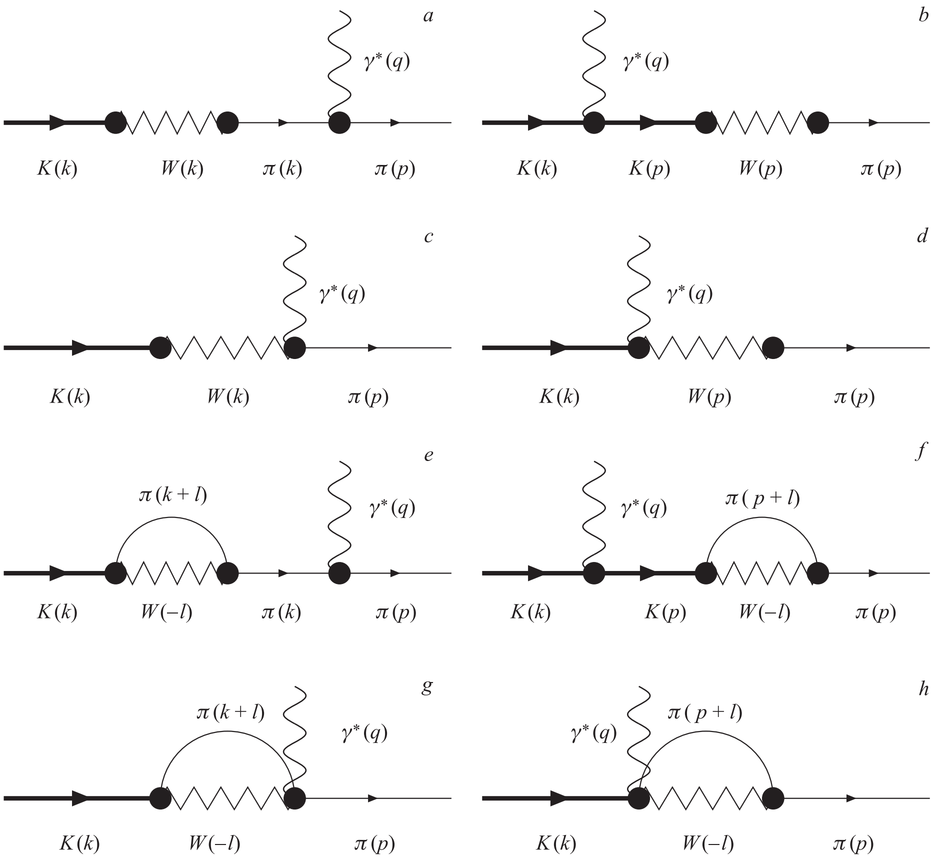

Fig. 1. Diagrams

$L_\mu = \bar{l}\gamma_\mu l$ is leptonic current and

$$\mathcal{T}(q^2, k^2, p^2) = F_\pi^2 \left(\frac{f_\pi^V(q^2)k^2}{m_\pi^2 - k^2 - i\epsilon} + \frac{f_K^V(q^2)p^2}{M_K^2 - p^2 - i\epsilon} + \frac{f_K^A(q^2) + f_\pi^A(q^2)}{2} \right). \tag{2}$$

Here $F_\pi \simeq 92.4$ MeV, $f_{\pi,K}^V(q^2)$ and $f_{\pi,K}^A(q^2)$ are phenomenological meson form factors denoted by fat dots in Fig. 1, a, b, e, f and 1, c, d, g, h, respectively.

On the mass shell the sum (2) takes the form

$$\mathcal{T}(q^2) = F_\pi^2 \left(\frac{f_K^A(q^2) + f_\pi^A(q^2)}{2} - f_\pi^V(q^2) + \left(f_K^V(q^2) - f_\pi^V(q^2)\right) \frac{m_\pi^2}{M_K^2 - m_\pi^2} \right).$$

In case of $K_S^0 \to \pi^0 l^+ l^-$ there are not diagrams Fig. 1, a–d and in the amplitude (1) instead of g_8 should be $(g_8 - 1)$.

These amplitudes lead to the decay rate [4, 8, 9]

$$\Gamma = \bar{\Gamma}_{K\to\pi l^+ l^-} \int\limits_{4m_l^2}^{(M_K - m_\pi)^2} \frac{dq^2}{M_K^2} \rho(q^2) |\hat{\phi}(q^2)|^2, \tag{3}$$

where [4]

$$\rho(q^2) = \left(1 - \frac{4m_l^2}{q^2}\right)^{1/2} \left(1 + \frac{2m_l^2}{q^2}\right) \lambda^{3/2} \left(1, \frac{q^2}{M_K^2}, \frac{m_\pi^2}{M_K^2}\right),$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca),$$

$$\bar{\Gamma}_{K^+\to\pi^+l^+l^-} = 1.37 \cdot 10^{-19}\ \text{MeV},$$

and [7]

$$\bar{\Gamma}_{K^0\to\pi^0l^+l^-} = \left(\frac{g_8 - 1}{g_8}\right)^2 \bar{\Gamma}_{K^+\to\pi^+l^+l^-}$$

$$\hat{\phi}(q^2) = \frac{(4\pi)^2 \mathcal{T}(q^2)}{q^2} = \\ = \frac{(4\pi F_\pi^2)^2}{q^2} \left(\frac{f_K^A(q^2) + f_\pi^A(q^2)}{2} - f_\pi^V(q^2) + \left(f_K^V(q^2) - f_\pi^V(q^2)\right) \frac{m_\pi^2}{M_K^2 - m_\pi^2}\right). \quad (4)$$

Thus, ChPT and instantaneous weak interaction model lead to formulas (3) and (4) as relationship between decay rates and form factors.

3. FORM FACTORS

3.1. $K^+ \to \pi^+ l^+ l^-$**.** One can make an assumption that electromagnetic form factors of the kaon and pion are saturated with resonances as in the ρ-dominance model. One of possible models of such a suturation is ChPT with both meson and baryon loops [10, 12–14], so in [7, 8] at small q^2 they were chosen in the form

$$f_\pi^V(q^2) \simeq f_K^V(q^2) \simeq f^V(q^2) = 1 + M_\rho^{-2} q^2 + \alpha_0 \Pi_\pi(q^2) + \ldots, \\ f_\pi^A(q^2) \simeq f_K^A(q^2) \simeq f^A(q^2) = 1 + M_{a_0^1}^{-2} q^2 + \ldots \quad (5)$$

We can calculate decay rates using the resonances [15]:

$$M_\rho = (775.49 \pm 0.34)\ \text{MeV}, \qquad I^G(J^{PC}) = 1^+(1^{--}), \\ M_{a_0^1} = (980 \pm 20)\ \text{MeV}, \qquad I^G(J^{PC}) = 1^-(0^{++}); \quad (6)$$

and pion loop contribution

$$\alpha_0 = \frac{4}{3} \frac{m_{\pi^+}^2}{(4\pi F_\pi)^2} = 0.01926 \pm 0.00077,$$

$$\Pi_\pi(t) = (1 - \bar{t}) \left(\frac{1}{\bar{t}} - 1\right)^{1/2} \arctan\left(\frac{\bar{t}^{1/2}}{(1 - \bar{t})^{1/2}}\right) - 1, \qquad \bar{t} = \frac{t}{(2m_{\pi^+})^2} < 1; \\ \Pi_\pi(t) = \frac{\bar{t} - 1}{2} \left(1 - \frac{1}{\bar{t}}\right)^{1/2} \left\{i\pi - \log\frac{\bar{t}^{1/2} + (\bar{t} - 1)^{1/2}}{\bar{t}^{1/2} - (\bar{t} - 1)^{1/2}}\right\} - 1, \quad \bar{t} \geqslant 1. \quad (7)$$

Let us make two remarks on this point.

First, $f_\pi^V(q^2)$ and $f_K^V(q^2)$ are nothing but electromagnetic form factors of the charged pion and kaon, but we know them much better from experiment [15]. So we can prove $f^V(q^2)$ using experimental data. At $q^2 \to 0$:

$$f^V(q^2 \to 0) \simeq 1 + \frac{\langle r^2 \rangle}{6(\hbar c)^2}\, q^2, \tag{8}$$

$$\begin{aligned} \langle r \rangle_{\pi^+} &= (0.672 \pm 0.008)\ \text{fm}, \\ \langle r \rangle_{K^+} &= (0.560 \pm 0.031)\ \text{fm}. \end{aligned} \tag{9}$$

Of course, in $\langle r \rangle$ the $\Pi_\pi(q^2)$ term is already included. To retrieve $\Pi_\pi(q^2)$ (and nontrivial q^2 dependence), expand it in series near zero:

$$\alpha_0 \Pi_\pi(q^2 \to 0) \simeq -\alpha_0 \frac{4}{3} \frac{q^2}{(2m_{\pi^+})^2}, \tag{10}$$

subtract (10) from (8) and add (7):

$$f^V(q^2 \to 0) \simeq 1 + \left(\frac{\langle r \rangle^2}{6(\hbar c)^2} + \alpha_0 \frac{4}{3} \frac{1}{(2m_{\pi^+})^2} \right) q^2 + \alpha_0 \Pi_\pi(q^2). \tag{11}$$

At large q^2, $f_\pi^V(q^2)$ and $f_K^V(q^2)$ have maximum at $q^2 = M_\rho^2$.

Second, beside a_0^1 there is

$$M_{a_0^2} = (1474 \pm 19)\ \text{MeV}, \quad I^G(J^{PC}) = 1^-(0^{++}). \tag{12}$$

If a_0^2 is not taken into account, a huge discrepancy with experiment results will be got.

Finally, using (6), (7), (9), (11), (12) we have the following improved hypothesis of (5) in Padé-type approximations:

$$\begin{aligned} f_{\pi^+}^V(q^2) &= \frac{\gamma_\pi}{1 - \dfrac{1}{\gamma_\pi}\left(\left(\dfrac{\langle r \rangle_{\pi^+}^2}{6(\hbar c)^2} + \alpha_0 \dfrac{4}{3} \dfrac{1}{(2m_{\pi^+})^2} \right) q^2 + \alpha_0 \Pi_\pi(q^2) \right)} + (1 - \gamma_\pi), \\ f_{K^+}^V(q^2) &= \frac{\gamma_K}{1 - \dfrac{1}{\gamma_K}\left(\left(\dfrac{\langle r \rangle_{K^+}^2}{6(\hbar c)^2} + \alpha_0 \dfrac{4}{3} \dfrac{1}{(2m_{\pi^+})^2} \right) q^2 + \alpha_0 \Pi_\pi(q^2) \right)} + (1 - \gamma_K), \\ f_{\pi^+}^A(q^2) &\simeq f_{K^+}^A(q^2) \simeq f^A(q^2) = \frac{1}{1 - \dfrac{q^2}{M_{a_0^1}^2}} + \frac{1}{1 - \dfrac{q^2}{M_{a_0^2}^2}} - 1 \simeq 1 + \frac{q^2}{M_{a_0^1}^2} + \frac{q^2}{M_{a_0^2}^2} + \ldots, \end{aligned} \tag{13}$$

$\gamma_\pi = 1.176677$ and $\gamma_K = 0.855628$ have been chosen to put the position of maximum of $f_{\pi^+}^V(q^2)$ and $f_{K^+}^V(q^2)$ to $q^2 = M_\rho^2$. A plot of (4) with (13) is shown in Fig. 2, $z = q^2/M_{K^+}^2$.

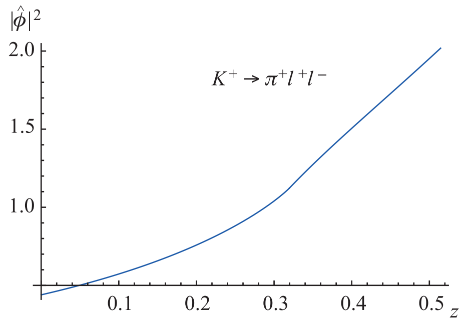

Fig. 2. The $|\hat{\phi}(q^2)|^2$ defined by (4) and (13)

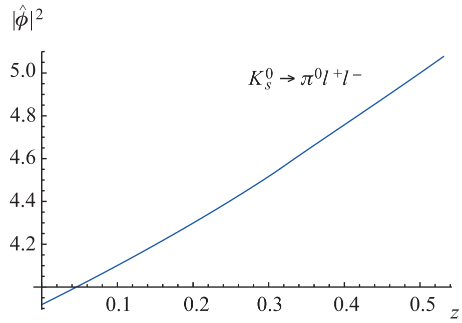

Fig. 3. The $|\hat{\phi}(q^2)|^2$ defined by (4) and (16)

3.2. $K_S^0 \to \pi^0 l^+ l^-$**.** In this case we have [15]

$$\langle r^2 \rangle_{K^0} = (-0.077 \pm 0.010)\ \mathrm{fm}^2, \tag{14}$$

and we can neglect the neutral pion electromagnetic radius [16]:

$$\langle r^2 \rangle_{\pi^0} = 0. \tag{15}$$

Notice that

$$\frac{\langle r^2 \rangle_{K^0}}{6(\hbar c)^2} \simeq -0.33 \cdot 10^{-6}\ \mathrm{MeV},$$

$$\frac{d\alpha_0 \Pi_\pi}{dq^2}(0) \simeq -0.33 \cdot 10^{-6}\ \mathrm{MeV},$$

which means that $\langle r^2 \rangle_{K^0}$ is determined almost only by $\Pi_\pi(q^2)$, that is why we will not use resonance behavior of $f_\pi^V(q^2)$ and $f_K^V(q^2)$:

$$f_{\pi^0}^V(q^2) = 0,$$

$$f_{K^0}^V(q^2) = \left(\frac{\langle r \rangle_{K^0}^2}{6(\hbar c)^2} + \alpha_0 \frac{4}{3} \frac{1}{(2m_{\pi^+})^2} \right) q^2 + \alpha_0 \Pi_\pi(q^2), \tag{16}$$

$$f_{\pi^0}^A(q^2) \simeq f_{K^0}^A(q^2) \simeq f^A(q^2) = \frac{1}{1-(q^2/M_{a_0^1}^2)} + \frac{1}{1-(q^2/M_{a_0^2}^2)} - 2 \simeq \frac{q^2}{M_{a_0^1}^2} + \frac{q^2}{M_{a_0^2}^2} + \ldots$$

A plot of (4) with (16) is shown in Fig. 3, $z = q^2/M_{K^0}^2$.

4. DECAY RATES

If we substitute formulae (13) and (16) for Eqs. (4) and (3), we get decay rates summarized in the table. We can see good agreement with experiments in all cases. Large inaccuracy in K^+ decays arises from subtraction approximately equal to f^A and $f_{\pi^+}^V$ (13) in formula (4). This table shows us that at present-day precision level, it is better to extract $f_{\pi^+}^V$ and f^A from decay rates $K^+ \to \pi^+ l^+ l^-$. Differential decay rates are presented in Fig. 4.

Decay rates compared with experiments

Decay rates	Γ, 10^{-20} MeV	$\Gamma_{\rm exp}$, 10^{-20} MeV
$K^+ \to \pi^+ e^+ e^-$	1.29 ± 0.40	1.654 ± 0.064 [1]
$K^+ \to \pi^+ e^+ e^-,\ z > 0.08$	0.94 ± 0.28	1.212 ± 0.043 [1]
$K^+ \to \pi^+ \mu^+ \mu^-$	0.39 ± 0.11	0.431 ± 0.075 [15]
$K_S^0 \to \pi^0 e^+ e^-$	5.41 ± 0.68	$4.3^{+2.2}_{-1.9}$ [2]
$K_S^0 \to \pi^0 e^+ e^-,\ q > 165$	2.90 ± 0.37	$2.2^{+1.1}_{-0.9}$ [2]
$K_S^0 \to \pi^0 \mu^+ \mu^-$	1.23 ± 0.16	$2.1^{+1.1}_{-0.9}$ [3]

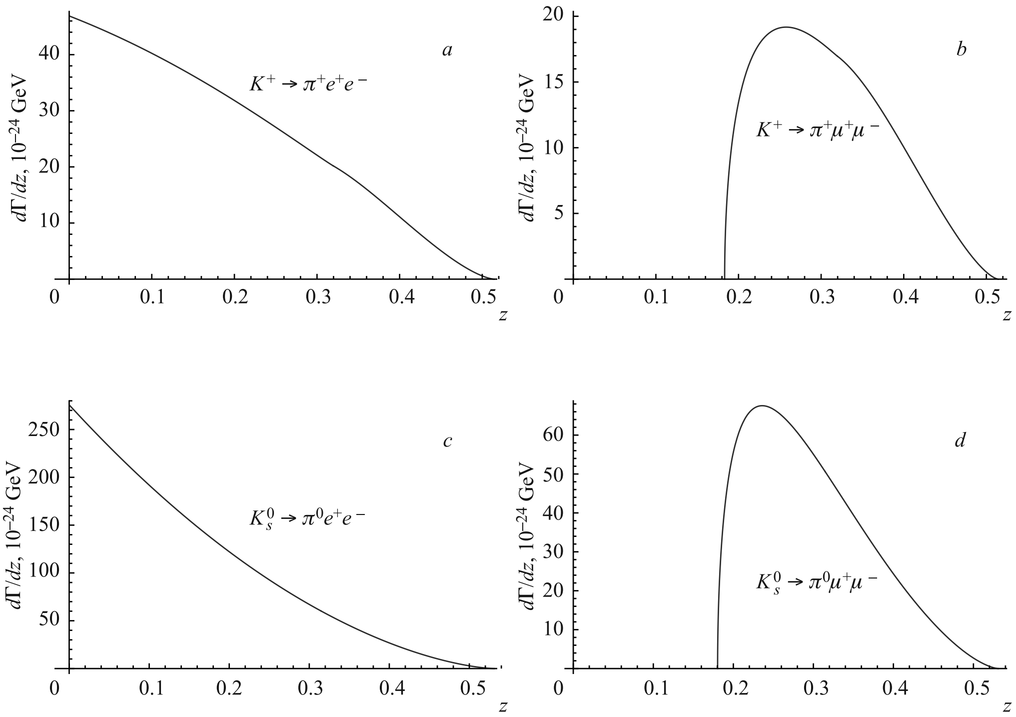

Fig. 4. The $d\Gamma/dz$ determined by relations (13), (16), (4), and (3)

CONCLUSION

In the framework of ChPT we calculated decay rates of $K^+ \to \pi^+ l^+ l^-$ and $K_S^0 \to \pi^0 l^+ l^-$ using measured electromagnetic meson radii [15] and inserting resonances with quantum numbers of a_0 meson into the instantaneous weak interaction. Taking into account the instantaneous weak interaction is the difference of our approach from other ones.

The results we obtained are in good agreement with experiments, for instance, one can determine the neutral kaon decay branch data using the meson form factor data. However, there is a large amount of inaccuracy. On the other hand, the high sensitivity of obtained decay rates allows us for a charge kaon to determine the form factors and masses of a_0 mesons from already measured $\Gamma(K^+ \to \pi^+ l^+ l^-)$ at high precision level.

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