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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

LINEARIZED INTERACTIONS OF SCALAR AND VECTOR FIELDS WITH THE HIGHER SPIN FIELD IN AdS_D

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The explicit form of linearized gauge and generalized «Weyl invariant» interactions of scalar and general higher even spin fields in the AdS_D space constructed in [1] is reviewed. Also a linearized interaction of vector field with general higher even spin gauge field is obtained. It is shown that the gauge-invariant action of linearized vector field interacting with the higher spin field also includes the whole tower of invariant actions for couplings of the same vector field with the gauge fields of smaller even spin.

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INTRODUCTION

After discovering the AdS_4/CFT_3 correspondence of the critical $O(N)$ sigma model [4], interest in the interacting theory of an arbitrary even high spin field drastically increased. So in the center of our attention is a theory of Fradkin–Vasiliev type [5] in Fronsdal's metric formulation [6]. This case of AdS_D/CFT_{D-1} correspondence is also of great interest because supersymmetry and BPS arguments are absent and because both conformal points of the boundary theory (i.e., unstable free field theory and critical interacting point, in the large N limit) correspond to the same higher spin theory and are connected on the boundary by a Legendre transformation which corresponds to different boundary conditions (regular dimension one or shadow dimension two) in the quantization of the bulk scalar field [7]. Existence of this scalar field in higher spin gauge theory is also an interesting and important phenomenon and supports the spontaneous symmetry breaking mechanism and mass creation for initially massless gauge fields due to corresponding possible interactions (see, for example, [8, 9]). From this point of view, any construction of a reasonable even linearized interaction is an interesting and important task in this reconstruction of the higher spin gauge theory from the holographic dual CFT and can be controlled by corresponding information about the anomalous dimensions of the dual global symmetry currents that fulfill the conservation conditions in the large N limit. Therefore, we see that construction of the conformal coupling of the scalar with a general even higher spin gauge field appears as an interesting example of an

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interaction which is applicable for many different quantum one-loop calculations such as the trace anomaly of the scalar in the external higher spin gauge field and so on [10].

In this article we construct a generalization of the well-known action for the conformally coupled scalar field in D dimensions in external gravity

$$S = \frac{1}{2}\int d^D z\sqrt{-G}\left[G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{(D-2)}{4(D-1)}R(G)\phi^2\right] \tag{1}$$

to the coupling with the linearized external higher spin ℓ gauge field. We show that the gauge and «Weyl invariant» interaction of the scalar with the spin ℓ Fronsdal gauge field can be constructed only if we add the same type of interaction with all lower even spin gauge fields. In other words, we can construct a self-consistent interaction of a gauge field with the conformally coupled scalar only with the whole finite tower of gauge fields with even spins in the range $2 \leqslant s \leqslant \ell$. We use the same notations and conventions as in [1]. In Sec. 1 we explicitly construct a linearized interaction *Lagrangian* of the conformal scalar field with the spin ℓ gauge field using Noether's procedure for higher spin *gauge* invariance. In Sec. 2 we extend our investigation including Noether's procedure for *generalized Weyl invariance* and obtain a unique interacting action after nontrivial and tedious calculations. In Sec. 3 we construct the linearized gauge-invariant interaction of electromagnetic field with the higher spin fields. Note also that some consideration of nonlinear gauge-invariant couplings of the scalar field on the level of the equation of motion can be found in [11] and on the level of the BRST formalism for higher spin fields in [12]. Finalizing introduction, we can say that this is a linearized interaction with the scalar for *conformal higher spin theory* of the type discussed in [13, 14].

1. GAUGE-INVARIANT INTERACTION FOR THE SCALAR FIELD COUPLED TO SPIN ℓ FIELD

Here we construct gauge-invariant action for coupling of the scalar to the general spin ℓ field. Following [1], we apply the following gauge transformation:

$$\delta^1_\epsilon\phi(z) = \epsilon^{\mu_1\mu_2\cdots\mu_{l-1}}_\ell(z)\nabla_{\mu_1}\nabla_{\mu_2}\cdots\nabla_{\mu_{l-1}}\phi(z), \tag{2}$$

$$\delta^0_\epsilon h^{(\ell)\mu_1\cdots\mu_l} = l\nabla^{(\mu_l}\epsilon^{\mu_1\mu_2\cdots\mu_{l-1})}_\ell, \quad \delta^0_\epsilon h^{(\ell)\alpha\mu_1\cdots\mu_{l-2}}_\alpha = 2\epsilon^{\mu_1\cdots\mu_{l-2}}_{\ell(1)}, \tag{3}$$

$$\epsilon^\alpha_{\ell\alpha\mu_3\cdots\mu_{l-1}} = 0 \tag{4}$$

to the action

$$S_0(\phi) = \frac{1}{2}\int d^D z\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi + \frac{D(D-2)}{4L^2}\phi^2\right], \tag{5}$$

and obtain the following variation for Noether's procedure[1]:

$$\delta^1_\epsilon S_0(\phi) = \int d^D z\sqrt{-g}\left\{\sum_{m=1}^{\frac{l}{2}}\binom{\ell-m-1}{m-1}\left[-\nabla^{(\mu_{2m}}\epsilon^{\mu_1\cdots\mu_{2m-1})}_{\ell(l-2m)}\Psi^{(2m)}_{\mu_1\cdots\mu_{2m}}\right]+\right.$$
$$\left.+\left[\nabla^2\phi-\frac{D(D-2)}{4L^2}\phi\right]\sum_{m=2}^{\frac{l}{2}}\binom{\ell-m-1}{m-2}\nabla_{\mu_1}\cdots\nabla_{\mu_{m-1}}(\epsilon^{\mu_1\cdots\mu_{2m-2}}_{\ell(l-2m+1)}\nabla_{\mu_m}\cdots\nabla_{\mu_{2m-2}}\phi)\right\}, \tag{7}$$

where

$$\Psi^{(2m)}_{\mu_1\cdots\mu_{2m}} = (-1)^m\left\{\nabla_{\mu_1}\cdots\nabla_{\mu_m}\phi\nabla_{\mu_{m+1}}\cdots\nabla_{\mu_{2m}}\phi-\right.$$
$$-\frac{m}{2}g_{\mu_{2m-1}\mu_{2m}}g^{\alpha\beta}\nabla_{(\mu_1}\cdots\nabla_{\mu_{m-1}}\nabla_{\alpha)}\phi\nabla_{(\mu_m}\cdots\nabla_{\mu_{2m-2}}\nabla_{\beta)}\phi-$$
$$\left.-\frac{m(D+2m-2)(D+2m-4)}{8L^2}g_{\mu_{2m-1}\mu_{2m}}\nabla_{\mu_1}\cdots\nabla_{\mu_{m-1}}\phi\nabla_{\mu_m}\cdots\nabla_{\mu_{2m-2}}\phi\right\}, \tag{8}$$

and we admitted symmetrization for the set $\mu_1,\cdots,\mu_{2m}$ of indices. So we see that miraculously the coefficients in (8) do not depend on l! All ℓ-dependence is concentrated in the second line of (7) proportional to the equation of motion for the action (5). This part like in the spin four case can be removed by appropriate field redefinition (see (13), (14), (A.6))

$$\phi\to\phi+\sum_{m=2}^{\frac{l}{2}}\frac{m-1}{2(l-2m+1)}\nabla_{\mu_1}\cdots\nabla_{\mu_{m-1}}\left(h^{(2m)\alpha\mu_1\cdots\mu_{2m-2}}_{\alpha}\nabla_{\mu_m}\cdots\nabla_{\mu_{2m-2}}\phi\right) \tag{9}$$

and we can drop these terms from our consideration. Thus, we obtain the following spin ℓ gauge-invariant action:

$$S^{\mathrm{GI}}(\phi,h^{(2)},h^{(4)},\ldots,h^{(\ell)}) = S_0(\phi)+\sum_{m=1}^{\frac{l}{2}}S_1^{\Psi^{(2m)}}\left(\phi,h^{(2m)}\right), \tag{10}$$

[1]For compactness we introduce shortened notations for divergences of the tensorial symmetry parameters

$$\epsilon^{\mu\nu\cdots}_{(1)} = \nabla_\lambda\epsilon^{\lambda\mu\nu\cdots},\quad \epsilon^{\mu\cdots}_{(2)} = \nabla_\nu\nabla_\lambda\epsilon^{\nu\lambda\mu\cdots},\cdots \tag{6}$$

where

$$S_1^{\Psi^{(2m)}}(\phi, h^{(2m)}) = \frac{1}{2m}\int d^D z\sqrt{-g}h^{(2m)\mu_1\cdots\mu_{2m}}\Psi^{(2m)}_{\mu_1\cdots\mu_{2m}} =$$
$$= \frac{(-1)^m}{2m}\int d^D z\sqrt{-g}\Big\{h^{(2m)\mu_1\cdots\mu_{2m}}\nabla_{\mu_1}\cdots\nabla_{\mu_m}\phi\nabla_{\mu_{m+1}}\cdots\nabla_{\mu_{2m}}\phi -$$
$$-\frac{m}{2}h^{(2m)\alpha\mu_1\cdots\mu_{m-1}}_{\alpha\mu_m\cdots\mu_{2m-2}}\nabla_{(\mu_1}\cdots\nabla_{\mu_{m-1}}\nabla_{\mu)}\phi\nabla^{(\mu_m}\cdots\nabla^{\mu_{2m-2}}\nabla^{\mu)}\phi -$$
$$-\frac{m(D+2m-2)(D+2m-4)}{8L^2}h_\alpha^{(2m)\alpha\mu_1\cdots\mu_{2m-2}}\nabla_{\mu_1}\cdots\nabla_{\mu_{m-1}}\phi\nabla_{\mu_m}\cdots\nabla_{\mu_{2m-2}}\phi\Big\}, \tag{11}$$

and the final form of the improved gauge transformations

$$\delta^1_\epsilon\phi(z) = \epsilon_\ell^{\mu_1\mu_2\cdots\mu_{l-1}}(z)\nabla_{\mu_1}\nabla_{\mu_2}\cdots\nabla_{\mu_{l-1}}\phi(z), \tag{12}$$

$$\delta^0_\epsilon h^{(2m)\mu_1\cdots\mu_{2m}} = 2m\nabla^{(\mu_{2m}}\varepsilon_\ell^{(2m)\mu_1\cdots\mu_{2m-1})}, \quad \delta^0_\epsilon h_\alpha^{(2m)\alpha\mu_1\cdots\mu_{2m-2}} = 2\varepsilon_{\ell(1)}^{(2m)\mu_1\cdots\mu_{2m-2}}, \tag{13}$$

$$\varepsilon_\ell^{(2m)\mu_1\cdots\mu_{2m-1}} = \binom{\ell-m-1}{m-1}\epsilon^{\mu_1\cdots\mu_{2m-1}}_{\ell(l-2m)}, \quad 2m \leqslant l. \tag{14}$$

So we found the gauge-invariant action for a general spin l gauge field coupled to a scalar and this action possesses the following property: it redefines the gauge parameters for lower spin gauge fields coupled to scalar, which means: *The gauge-invariant action* $S^{\rm GI}(\phi, h^{(2)}, h^{(4)}, \ldots, h^{(\ell)})$ *for a spin* ℓ *gauge field coupled to a scalar includes gauge-invariant actions of a tower of all smaller even spin gauge fields coupled to the same scalar in an analogous way.*

2. WEYL INVARIANT ACTION FOR A HIGHER SPIN FIELD COUPLED TO A SCALAR

In this section we introduce generalized Weyl transformations for higher spin fields and derive a Weyl invariant action for a higher spin field coupled to a scalar field. Following [1,3], we write the generalized Weyl transformation for the even spin l field in the form

$$\delta^0_\sigma h^{(\ell)\mu_1\cdots\mu_l} = l(l-1)\sigma_\ell^{(\mu_1\cdots\mu_{l-2}}g^{\mu_{l-1}\mu_l)}, \tag{15}$$

$$\delta^0_\sigma h_\alpha^{(\ell)\alpha\mu_1\cdots\mu_{l-2}} = 2(D+2l-4)\sigma_\ell^{\mu_1\cdots\mu_{l-2}}, \tag{16}$$

$$\delta^1_\sigma\phi = \Delta_\ell\sigma_\ell^{\mu_1\cdots\mu_{l-2}}\nabla_{\mu_1}\cdots\nabla_{\mu_{l-2}}\phi. \tag{17}$$

Then we assume that the Weyl invariant action for a spin l field should be accompanied with similar Weyl invariant actions for smaller spin gauge fields and therefore can be constructed from (10) by adding $l/2$ additional terms

$$S^{\rm WI}\left(\phi, h^{(2)}, h^{(4)}, \ldots, h^{(\ell)}\right) = S^{\rm GI}\left(\phi, h^{(2)}, \ldots, h^{(\ell)}\right) + \sum_{m=1}^{l/2} S_1^{r^{(2m)}}\left(\phi, h^{(2m)}\right), \tag{18}$$

where each $S_1^{r^{(2m)}}$ is gauge-invariant itself. Now we will see that the generalization of the Ricci scalar for a higher spin field, namely, the trace of Fronsdal's operator [6,9]

$$r^{(\ell)\mu_1\cdots\mu_{l-2}} = -\frac{1}{2}\mathrm{Tr}\,\mathcal{F}(h^\ell) = \nabla_\alpha\nabla_\beta h^{(\ell)\alpha\beta\mu_1\cdots\mu_{l-2}} - \Box h_\alpha^{(\ell)\alpha\mu_1\cdots\mu_{l-2}} - \\ - \frac{l-2}{2}\nabla^{(\mu_1}\nabla_\alpha h_\beta^{(\ell)\mu_2\cdots\mu_{l-2})\alpha\beta} - \frac{(l-1)(D+l-3)}{L^2}h_\alpha^{(\ell)\alpha\mu_1\cdots\mu_{l-2}}, \quad (19)$$

is the only gauge-invariant combination of two derivatives and a higher spin field which we need to construct the Weyl invariant action (18) starting from (10). We will use the following strategy for solving our problem: We apply transformation (15)–(17) to (10) and try to compensate it with the variation of

$$\sum_{m=1}^{l/2} S_1^{r^{(2m)}}(\phi, h^{(2m)}), \quad \text{where} \quad S_1^{r^{(\ell)}}(\phi, h^{(2)}, \cdots, h^{(\ell)}) = \\ = \frac{1}{2}\sum_{m=0}^{\frac{l}{2}-1}\xi_\ell^m \int d^D z\sqrt{-g}\nabla_{\mu_{2m+1}}\cdots\nabla_{\mu_{l-2}} r^{(\ell)\mu_1\cdots\mu_{l-2}}\nabla_{\mu_1}\cdots\nabla_{\mu_m}\phi\nabla_{\mu_{m+1}}\cdots\nabla_{\mu_{2m}}\phi, \quad (20)$$

introducing necessarily gauge and Weyl transformations for lower spin gauge fields

$$\delta_{\sigma_\ell} h^{(2m)\mu_1\cdots\mu_{2m}} = 2m(2m-1)C_\ell^m \sigma_{\ell(l-2m)}^{(\mu_1\cdots\mu_{2m-2}} g^{\mu_{2m-1}\mu_{2m})}, \quad m = 1,\ldots,l/2, \quad (21)$$

$$C_\ell^{\ell/2} = 1. \quad (22)$$

In other words, we solve the equation

$$\delta_{\sigma_\ell} S^{\mathrm{WI}}\left(\phi, h^{(2)},\ldots,h^{(\ell)}\right) = \delta_{\sigma_\ell}^1 S_0 + \sum_{s=1}^{l/2}\delta_{\sigma_\ell}^0 S_1^{\Psi^{(2s)}} + \sum_{s=1}^{l/2}\delta_{\sigma_\ell}^0 S_1^{r^{(2s)}} = 0, \quad (23)$$

which consists of a system of $l+1$ equations for $(l/2+1)(l/2+2)/2$ variables[1]

$$\triangle_\ell, \quad (24)$$

$$C_\ell^m, \quad m = 1,2,\ldots,l/2, \quad (25)$$

$$\xi_{2s}^n, \quad n = 0,1,\ldots,s-1;\ s = 1,\ldots,l/2, \quad (26)$$

but when we find $\xi_\ell^{\ell/2-k}$ we also find ξ_{2s}^{s-k} for any $s \geqslant k$. In other words, we find a whole

[1]This system includes also (22) as an equation for $C_\ell^{\ell/2}$.

diagonal of this triangle matrix

$$\begin{pmatrix} C_\ell^1 & C_\ell^2 & . & . & . & C_\ell^{\ell/2-1} & C_\ell^{\ell/2} & \Delta_\ell \\ \xi_\ell^0 & \xi_\ell^1 & . & . & . & \xi_\ell^{\ell/2-2} & \xi_\ell^{\ell/2-1} & \\ \xi_{\ell-2}^0 & \xi_{\ell-2}^1 & . & . & . & \xi_{\ell-2}^{\ell/2-2} & & \\ . & . & . & . & . & & & \\ . & . & . & . & & & & \\ \xi_4^0 & \xi_4^1 & & & & & & \\ \xi_2^0 & & & & & & & \end{pmatrix}, \tag{27}$$

which helps us to solve the whole system. We have two equations for any vertical line of this matrix besides the last, for which we have one equation for Δ. We start from the last vertical line and go to the left. When we take any line and two equations for that line of variables, we have only two variables to find if we have already solved all lines to the right of that one. That means that our system has a unique solution. We do not write all complicated Weyl variations of (23) and present here the resulting system of equations for the unknown variables (24)–(26):

$$\Delta_\ell = 1 - \frac{D}{2}, \tag{28}$$

$$\frac{(-1)^{l/2}}{2}\left(\Delta_\ell - \frac{l-2}{2}\right) - (D+2l-5)\xi_\ell^{\ell/2-1} = 0, \tag{29}$$

$$(-1)^m C_\ell^m + \sum_{s=m+1}^{l/2} m C_\ell^s \xi_{2s}^m = 0 \quad (m = 1, \ldots, l/2-1), \tag{30}$$

$$\frac{(-1)^{m-1}}{2}(m-1)C_\ell^m - C_\ell^m (D+4m-5)\xi_{2m}^{m-1} +$$
$$+\frac{1}{2}\sum_{s=m+1}^{l/2} C_\ell^s\left[-m(m-1)\xi_{2s}^m - (2s-2m+2)(D+2s+2m-5)\xi_{2s}^{m-1}\right] = 0$$
$$(m = 1, \ldots, l/2-1). \tag{31}$$

The solution of this system is unique $\Delta_\ell = \Delta = 1 - D/2$ and

$$\xi_\ell^m = \frac{(-1)^m}{2^{\ell-2m}(\ell/2)}\binom{\ell/2}{m}\frac{(D/2+m-1)_{\ell/2-m}}{\left(\dfrac{D+\ell-1}{2}+m-1\right)_{\ell/2-m}}, \tag{32}$$

$$C_\ell^m = \frac{(-1)^{\ell/2-m}}{2^{\ell-2m}}\binom{\ell/2-1}{m-1}\frac{(D/2+m-1)_{\ell/2-m}}{\left(\dfrac{D-1}{2}+2m\right)_{\ell/2-m}}. \tag{33}$$

These completely fix (20) and therefore the full Weyl invariant action (18) and also determine the transformation law for the whole tower of higher spin gauge fields (21).

3. SPIN ONE FIELD COUPLINGS TO THE HIGHER SPIN GAUGE FIELDS

Now we generalize the result of [2] for coupling of vector field to the spin four field to the general higher even spin case. We work in the flat space because that case is enough for our interests, although some discussion connected with AdS space is provided below. So we start from the free field Lagrangian[1]

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}(\partial A)^2, \tag{34}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \partial A = \partial_\mu A^\mu \tag{35}$$

for an electromagnetic field and use the Noether procedure with following starting variation:

$$\delta^1_\epsilon A_\mu = \epsilon_\ell^{\mu_1 \ldots \mu_{l-1}} \nabla_{\mu_1} \cdots \nabla_{\mu_{l-2}} F_{\mu_{l-1}\mu}. \tag{36}$$

From very long and tedious calculations we get

$$\begin{aligned}
\delta^1_\epsilon \mathcal{L}_0 = &\sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \left(-\nabla^{(\mu_{2m}} \epsilon_{\ell(l-2m)}^{\mu_1\cdots\mu_{2m-1})} \Psi_{\mu_1\cdots\mu_{2m}}(A_\mu)\right) + \\
&+ \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{m} \nabla_{\mu_{m+1}} \cdots \nabla_{\mu_{2m-2}} \left(\nabla_\nu \epsilon_{\ell(l-2m)\mu}^{\mu_1\cdots\mu_{2m-2}} \nabla_{\mu_1} \cdots \nabla_{\mu_{m-1}} F^\nu_{\mu_m}\right) \nabla_\alpha F^{\alpha\mu} + \\
&+ \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{2m} \nabla_{\mu_m} \cdots \nabla_{\mu_{2m-3}} \left(\epsilon_{\ell(l-2m+1)\mu}^{\mu_1\cdots\mu_{2m-3}} \nabla_{\mu_1} \cdots \nabla_{\mu_{m-2}} \nabla^\nu F_{\nu\mu_{m-1}}\right) \nabla_\alpha F^{\alpha\mu} - \\
&- \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{l-2m+1} \nabla_{\mu_m} \cdots \nabla_{\mu_{2m-2}} \left(\epsilon_{\ell(l-2m+1)}^{\mu_1\cdots\mu_{2m-2}} \nabla_{\mu_1} \cdots \nabla_{\mu_{m-2}} F_{\mu_{m-1}\mu}\right) \nabla_\alpha F^{\alpha\mu},
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
\Psi_{\mu_1\cdots\mu_{2m}}(A_\mu) = (-1)^m \Big(&-\nabla_{\mu_1} \cdots \nabla_{\mu_{m-1}} F_{\mu_m}{}^{\nu} \nabla_{\mu_{m+1}} \cdots \nabla_{\mu_{2m-1}} F_{\mu_{2m}\nu} + \\
&+ \frac{m-1}{2} g_{\mu_1\mu_2} \nabla_{\mu_3} \cdots \nabla_{\mu_m} \nabla^\alpha F_{\mu_{m+1}\beta} \nabla_{\mu_{m+2}} \cdots \nabla_{\mu_{2m-1}} \nabla^\beta F_{\mu_{2m}\alpha} + \\
&+ \frac{m}{4} g_{\mu_1\mu_2} \nabla_{\mu_3} \cdots \nabla_{\mu_{m+1}} F^{\rho\sigma} \nabla_{\mu_{m+2}} \cdots \nabla_{\mu_{2m}} F_{\rho\sigma} \Big),
\end{aligned} \tag{38}$$

and we admitted symmetrization for the set $\mu_1 \cdots \mu_{2m}$ of indices. This means that when we change our initial variation (36) to

$$\begin{aligned}
\delta^1_\epsilon A_\mu = &\epsilon_\ell^{\mu_1\cdots\mu_{l-1}} \nabla_{\mu_1} \cdots \nabla_{\mu_{l-2}} F_{\mu_{l-1}\mu} - \\
&- \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{m} \nabla_{\mu_{m+1}} \cdots \nabla_{\mu_{2m-2}} \left(\nabla_\nu \epsilon_{\ell(l-2m)\mu}^{\mu_1\cdots\mu_{2m-2}} \nabla_{\mu_1} \cdots \nabla_{\mu_{m-1}} F^\nu_{\mu_m}\right)
\end{aligned} \tag{39}$$

[1]From now on we will never make a difference between a variation of the Lagrangians or the actions discarding all total derivative terms and admitting partial integration if necessary.

and also take into account appropriate field redefinition

$$A_\mu \to A_\mu + \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{2m} \nabla_{\mu_m} \cdots \nabla_{\mu_{2m-3}} \Big(\epsilon^{\mu_1 \cdots \mu_{2m-3}}_{\ell(l-2m+1)\mu} \nabla_{\mu_1} \cdots \cdots \nabla_{\mu_{m-2}} \nabla^\nu F_{\nu\mu_{m-1}} \Big) \nabla_\alpha F^{\alpha\mu} - \\ - \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{l-2m+1} \nabla_{\mu_m} \cdots \nabla_{\mu_{2m-2}} \Big(\epsilon^{\mu_1 \mu_{2m-2}}_{\ell(l-2m+1)} \nabla_{\mu_1} \cdots \cdots \nabla_{\mu_{m-2}} F_{\mu_{m-1}\mu} \Big) \nabla_\alpha F^{\alpha\mu}, \quad (40)$$

we can see that the gauge-invariant Lagrangian for interaction of electromagnetic field with the higher even spin ℓ field is

$$\mathcal{L}_1(A_\mu, h^{(2)}, h^{(4)}, \ldots, h^{(\ell)}) = \sum_{m=1}^{\ell/2} \frac{1}{2m} h^{(2m)\mu_1 \ldots \mu_{2m}} \Psi^{(2m)}_{\mu_1 \ldots \mu_{2m}}(A_\mu). \quad (41)$$

This result is similar to the scalar case investigated in Sec. 1. The same tower of even spin gauge fields appears when we construct gauge-invariant interaction with higher spin fields. The generalization to the non-Abelian scalar or vector (Yang–Mills) fields is trivial. In scalar case we went further and constructed Weyl invariant Lagrangian. We could not generalize Weyl invariance for spin one case. That is the price for spin one manifest gauge invariance (in all interactions vector field is represented by its curvature $F_{\mu\nu}$). Here we would like to mention that AdS_D corrections to (38) have the following basic properties. As in the scalar case, there are no $1/L^4$ or higher corrections. The $1/L^2$ term is proportional to $\ell - 2$. For $1-1-2$ interaction we do not have any difference between interaction in the flat space and AdS. The $s-s-2s$ case investigated in [2] can also be written in AdS in the same form as in the flat space like $1-1-2$ case. The only difference is that curvatures of higher spin ($s > 1$) fields have analytical expansion in powers of cosmological constant [10], so the background changes interaction, but that difference is encoded in curvatures and is finite series in powers of $1/L^2$ in AdS case.

CONCLUSION

We constructed a gauge and generalized Weyl invariant interacting Lagrangian for a linearized higher even spin gauge field and a conformally coupled scalar field in AdS_D space. We also constructed gauge-invariant interaction of vector field with higher spin fields. The resulting Lagrangian for the spin ℓ field includes all lower even spin gauge fields also with the same type of interaction with the same scalar or vector field. These results can be used for construction of a more complicated interaction between different higher spin gauge fields in AdS space.

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APPENDIX

We use the following commutation relations in AdS_D:

$$\epsilon_\ell^{\mu_1\cdots\mu_{l-1}}[\nabla^\mu, \nabla_{\mu_1}\cdots\nabla_{\mu_k}]\phi = \frac{k(k-1)}{2L^2}\epsilon_\ell^{\mu\mu_2\cdots\mu_{l-1}}\nabla_{\mu_2}\cdots\nabla_{\mu_k}\phi, \tag{A.1}$$

$$[\nabla_{\mu_1}\cdots\nabla_{\mu_k}, \nabla^\mu]\epsilon_\ell^{\mu_1\cdots\mu_{l-1}} = \frac{2k(D+l-2)-k(k+1)}{2L^2}\epsilon_{\ell(k-1)}^{\mu\mu_{k+1}\cdots\mu_{l-1}}, \tag{A.2}$$

$$\epsilon_\ell^{\mu_1\cdots\mu_{l-1}}[\nabla_\mu, \nabla_{\mu_1}\cdots\nabla_{\mu_k}]\nabla^\mu\phi = \frac{k(2D+k-3)}{2L^2}\epsilon_\ell^{\mu_1\mu_2\cdots\mu_{l-1}}\nabla_{\mu_1}\cdots\nabla_{\mu_k}\phi, \tag{A.3}$$

$$\epsilon_\ell^{\mu_1\cdots\mu_{l-1}}[\nabla^2, \nabla_{\mu_1}\cdots\nabla_{\mu_k}]\phi = \frac{k(D+k-2)}{L^2}\epsilon_\ell^{\mu_1\mu_2\cdots\mu_{l-1}}\nabla_{\mu_1}\cdots\nabla_{\mu_k}\phi, \tag{A.4}$$

where $\epsilon_\ell^{\mu_1\cdots\mu_{l-1}}$ is the symmetric and traceless tensor. Finally we list all necessary binomial identities

$$\sum_{k=0}^{n-m}(-1)^k\binom{n}{k} = (-1)^{n-m}\binom{n-1}{m-1}, \quad \sum_{k=0}^{n-m}(-1)^k\binom{n}{m+k} = \binom{n-1}{m-1}, \tag{A.5}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}, \quad \binom{\ell-m-1}{m-2} = \frac{m-1}{l-2m+1}\binom{\ell-m-1}{m-1}. \tag{A.6}$$

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