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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

HIGGSLESS ELECTROWEAK MODEL AND CONTRACTION OF GAUGE GROUP

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A modified formulation of the Electroweak Model with 3-dimensional spherical geometry in the target space is suggested. The *free* Lagrangian in the spherical field space along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model, whose second order terms reproduce the same experimentally verified fields with the same masses as the Standard Electroweak Model. The vector bosons masses are automatically generated, so there is no need in special mechanism of spontaneous symmetry breaking.

The limiting case of the modified Higgsless Electroweak Model, which corresponds to the contracted gauge group $SU(2;j)\times U(1)$ is discussed. In the framework of the limit model Z-boson, electromagnetic and electron fields are interpreted as external ones with respect to W-bosons and neutrino fields. The W-bosons and neutrino fields do not affect these external fields. The masses of all the particles remain the same, but the field interactions in the contracted model are more simple as compared with the standard Electroweak Model.

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INTRODUCTION

The Standard Electroweak Model based on gauge group $SU(2) \times U(1)$ gives a good description of electroweak processes. One of the unsolved problems is the origin of electroweak symmetry breaking. In the standard formulation, the scalar field (Higgs boson) performs this task via Higgs mechanism, which generates mass terms for vector bosons. Sufficiently artificial Higgs mechanism with its imaginary bare mass is a naive relativistic analog of the phenomenological description of superconductivity [1]. However, it is not yet experimentally verified whether electroweak symmetry is broken by such a Higgs mechanism, or by something else. The emergence of large number Higgsless models [2–7] was stimulated by difficulties with Higgs boson. These models are mainly based on extra dimensions of different types or larger gauge groups. A finite electroweak model without a Higgs particle which is used as regularized quantum field theory [8, 9] was developed in [7].

One of the important ingredients of the Standard Model is the simple group $SU(2)$. More than fifty years in physics there is well known the notion of group contraction [10], i.e., limit operation, which transforms, for example, a simple or semisimple group to a nonsemisimple one. From the general point of view, for better understanding of a physical system it is useful

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to investigate its properties for limiting values of their physical parameters. In partucular, for a gauge model one of the similar limiting cases corresponds to a model with contracted gauge group. The gauge theories for nonsemisimple groups which admit Lie algebras invariant nondegenerate metrics were considered in [11, 12].

In the present paper, a modified formulation of the Higgsless Electroweak Model and its limiting case for contracted gauge group $SU(2;j)\times U(1)$ is regarded. Firstly, we observe that the quadratic form $\phi^\dagger\phi=\phi_1^*\phi_1+\phi_2^*\phi_2=R^2$ of the complex matter field $\phi\in \mathbf{C}_2$ is invariant with respect to gauge group transformations $SU(2)\times U(1)$, and we can restrict fields on the quadratic form without the loss of gauge invariance of the model. This quadratic form defines three-dimensional sphere in four-dimensional Euclidean space of the real components of ϕ, where the non-Euclidean spherical geometry is realized.

Secondly, we introduce the *free* matter field Lagrangian in this spherical field space, which along with the standard gauge field Lagrangian forms the full Higgsless Lagrangian of the model. Its second order terms reproduce the same fields as the Standard Electroweak Model but without the remaining real dynamical Higgs field. The vector bosons masses are automatically generated and are given by the same formulas as in the Standard Electroweak Model, so there is no need in special mechanism of spontaneous symmetry breaking. The fermion Lagrangian of the Standard Electroweak Model is modified by replacing of the fields ϕ with the restricted on the quadratic form fields in such a way that its second order terms provide the electron mass and neutrino remains massless.

We recall the definition and properties of the contracted group $SU(2;j)$ in Sec. 1. In Sec. 2, we modify step by step the main points of the Electroweak Model for the gauge group $SU(2;j)\times U(1)$. We find transformation properties of gauge and matter fields under contractions. After that, we obtain the Lagrangian of the contracted model from the noncontracted one by the substitution of the transformed fields. The limiting case of the modified Higgsless Electroweak Model is regarded in Sec. 3. When contraction parameter tends to zero $j\to 0$ or takes nilpotent value $j=\iota$, the field space is fibered [16] in such a way that electromagnetic, Z-boson and electron fields are in the base whereas charged W-bosons and neutrino fields are in the fiber. In the framework of the limit model the base fields can be interpreted as external ones with respect to the fiber fields in the sence that the fiber fields do not affect the base fields. The field interactions are simplified under contraction.

1. CONTRACTED SPECIAL UNITARY GROUP $SU(2;j)$

Let us regard two-dimensional complex fibered vector space $\Phi_2(j)$ with one-dimensional base $\{\phi_1\}$ and one-dimensional fiber $\{\phi_2\}$ [15]. This space has two Hermitian forms: first in the base $\bar{\phi_1}\phi_1=|\phi_1|^2$ and second in the fiber $\bar{\phi_2}\phi_2=|\phi_2|^2$, where bar denotes complex conjugation. Both forms can be written by one formula

$$\phi^\dagger(j)\phi(j)=|\phi_1|^2+j^2|\phi_2|^2, \tag{1}$$

where $\phi^\dagger(j)=(\bar{\phi_1}, j\bar{\phi_2})$, parameter $j=1,\iota$ and ι is nilpotent unit $\iota^2=0$. For nilpotent unit the following heuristic rules be fulfiled: 1) division of a real or complex numbers by ι is not defined, i.e., for a real or complex a the expression a/ι is defined only for $a=0$, 2) however identical nilpotent units can be cancelled $\iota/\iota=1$.

The special unitary group $SU(2;j)$ is defined as a transformation group of $\Phi_2(j)$ which keeps invariant the Hermitian form (1), i.e.,

$$\phi'(j) = \begin{pmatrix} \phi_1' \\ j\phi_2' \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} \phi_1 \\ j\phi_2 \end{pmatrix} = u(j)\phi(j),$$
$$\det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1. \tag{2}$$

The fundamental representations of the one-parameter subgroups of $SU(2;j)$ are easily obtained

$$u_1(\alpha_1;j) = \mathrm{e}^{\alpha_1 T_1(j)} = \begin{pmatrix} \cos\frac{j\alpha_1}{2} & i\sin\frac{j\alpha_1}{2} \\ i\sin\frac{j\alpha_1}{2} & \cos\frac{j\alpha_1}{2} \end{pmatrix}, \tag{3}$$

$$u_2(\alpha_2;j) = \mathrm{e}^{\alpha_2 T_2(j)} = \begin{pmatrix} \cos\frac{j\alpha_2}{2} & \sin\frac{j\alpha_2}{2} \\ -\sin\frac{j\alpha_2}{2} & \cos\frac{j\alpha_2}{2} \end{pmatrix}, \tag{4}$$

$$u_3(\alpha_3;j) = \mathrm{e}^{\alpha_3 T_3(j)} = \begin{pmatrix} \mathrm{e}^{i\frac{\alpha_3}{2}} & 0 \\ 0 & \mathrm{e}^{-i\frac{\alpha_3}{2}} \end{pmatrix}. \tag{5}$$

The corresponding generators

$$T_1(j) = j\frac{i}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = j\frac{i}{2}\tau_1,$$
$$T_2(j) = j\frac{i}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = j\frac{i}{2}\tau_2, \tag{6}$$
$$T_3(j) = \frac{i}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{i}{2}\tau_3,$$

with τ_k being Pauli matrices, are the subject of commutation relations

$$[T_1(j), T_2(j)] = -j^2 T_3(j), \quad [T_3(j), T_1(j)] = -T_2(j), \quad [T_2(j), T_3(j)] = -T_1(j), \tag{7}$$

and form the Lie algebra $su(2;j)$ with the general element

$$T(j) = \sum_{k=1}^{3} a_k T_k(j) = \frac{i}{2}\begin{pmatrix} a_3 & j(a_1 - ia_2) \\ j(a_1 + ia_2) & -a_3 \end{pmatrix} = -T^\dagger(j). \tag{8}$$

There are two more or less equivalent ways of group contraction. We can put the contraction parameter equal to the nilpotent unit $j = \iota$ or tend it to zero $j \to 0$. Sometimes it is convenient to use the first (mathematical) approach, sometimes the second (physical) one. For example, the matrix $u(j)$ (2) has nonzero nilpotent nondiagonal elements for $j = \iota$, whereas for $j \to 0$ they are formally equal to zero. Nevertheless, both approaches lead to the same final results.

Let us describe the contracted group $SU(2;\iota)$ in detail. For $j = \iota$ it follows from (2) that $\det u(\iota) = |\alpha|^2 = 1$, i.e., $\alpha = \mathrm{e}^{i\varphi}$, therefore

$$u(\iota) = \begin{pmatrix} \mathrm{e}^{i\varphi} & \iota\beta \\ -\iota\bar{\beta} & \mathrm{e}^{-i\varphi} \end{pmatrix}, \quad \beta = \beta_1 + i\beta_2 \in \mathbf{C}. \tag{9}$$

Functions of nilpotent arguments are defined by their Taylor expansion, in particular, $\cos\iota x = 1,\ \sin\iota x = \iota x$. Then one-parameter subgroups of $SU(2;\iota)$ take the form

$$u_1(\alpha_1;\iota) = \begin{pmatrix} 1 & \iota i\dfrac{\alpha_1}{2} \\ \iota i\dfrac{\alpha_1}{2} & 1 \end{pmatrix}, \quad u_2(\alpha_2;\iota) = \begin{pmatrix} 1 & \iota\dfrac{\alpha_2}{2} \\ -\iota\dfrac{\alpha_2}{2} & 1 \end{pmatrix}. \tag{10}$$

The third subgroup does not changed and is given by (5). The simple group $SU(2)$ is contracted to the nonsemisimple group $SU(2;\iota)$, which is isomorphic to the real Euclid group $E(2)$. First two generators of the Lie algebra $su(2;\iota)$ commute $[T_1(\iota),T_2(\iota)] = 0$ and the rest commutators are given by (7). For the general element (8) of $su(2;\iota)$ the corresponding group element of $SU(2;\iota)$ is as follows:

$$u(\iota) = \mathrm{e}^{T(\iota)} = \begin{pmatrix} \mathrm{e}^{i\dfrac{a_3}{2}} & \iota i\dfrac{\bar{a}}{a_3}\sin\dfrac{a_3}{2} \\ \iota i\dfrac{a}{a_3}\sin\dfrac{a_3}{2} & \mathrm{e}^{-i\frac{a_3}{2}} \end{pmatrix}, \quad a = a_1 + ia_2 \in C. \tag{11}$$

The actions of the unitary group $U(1)$ and the electromagnetic subgroup $U(1)_{\mathrm{em}}$ in the fibered space $\Phi_2(\iota)$ are given by the same matrices as on the space Φ_2, namely,

$$u(\beta) = \mathrm{e}^{\beta Y} = \begin{pmatrix} \mathrm{e}^{i\frac{\beta}{2}} & 0 \\ 0 & \mathrm{e}^{i\frac{\beta}{2}} \end{pmatrix}, \quad u_{\mathrm{em}}(\gamma) = \mathrm{e}^{\gamma Q} = \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & 1 \end{pmatrix}, \tag{12}$$

where $Y = (i/2)\mathbf{1},\ Q = Y + T_3$.

Representations of groups $SU(2;\iota), U(1), U(1)_{\mathrm{em}}$ are linear ones, that is, they are realized by linear operators in the fibered space $\Phi_2(\iota)$.

2. ELECTROWEAK MODEL FOR $SU(2;j)\times U(1)$ GAUGE GROUP

The fibered space $\Phi_2(j)$ can be obtained from Φ_2 by substitution $\phi_2 \to j\phi_2$ in (1), which induces another ones for Lie algebra $su(2)$ generators $T_1 \to jT_1,\ T_2 \to jT_2,\ T_3 \to T_3$. As far as the gauge fields take their values in Lie algebra, we can substitute gauge fields instead of transformation of generators, namely,

$$A_\mu^1 \to jA_\mu^1, \quad A_\mu^2 \to jA_\mu^2, \quad A_\mu^3 \to A_\mu^3, \quad B_\mu \to B_\mu. \tag{13}$$

These substitutions in the Lagrangian L of the Higgsless Electroweak Model [13] give rise to the Lagrangian $L(j)$ of the contracted model with $U(2;j) = SU(2;j)\times U(1)$ gauge group

$$L(j) = L_A(j) + L_\phi(j), \tag{14}$$

where

$$L_A(j) = \frac{1}{2g^2}\mathrm{tr}(F_{\mu\nu}(j))^2 + \frac{1}{2g'^2}\mathrm{tr}(\hat{B}_{\mu\nu})^2 = -\frac{1}{4}[j^2(F_{\mu\nu}^1)^2 + j^2(F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2] - \frac{1}{4}(B_{\mu\nu})^2 \tag{15}$$

is the gauge fields Lagrangian and

$$L_\phi(j) = \frac{1}{2}(D_\mu\phi(j))^\dagger D_\mu\phi(j) \tag{16}$$

is the *free* (without any potential term) matter field Lagrangian (summation on the repeating Greek indexes is always understood). Here D_μ are the covariant derivatives

$$D_\mu\phi(j) = \partial_\mu\phi(j) + g\left(\sum_{k=1}^{3} T_k(j)A_\mu^k\right)\phi(j) + g'YB_\mu\phi(j), \tag{17}$$

where $T_k(j)$ are given by (6) and $Y = (i/2)\mathbf{1}$ is the generator of $U(1)$. Their actions on components of $\phi(j)$ are given by

$$\begin{aligned} D_\mu\phi_1 &= \partial_\mu\phi_1 + \frac{i}{2}(gA_\mu^3 + g'B_\mu)\phi_1 + j^2\frac{ig}{2}(A_\mu^1 - iA_\mu^2)\phi_2, \\ D_\mu\phi_2 &= \partial_\mu\phi_2 - \frac{i}{2}(gA_\mu^3 - g'B_\mu)\phi_2 + \frac{ig}{2}(A_\mu^1 + iA_\mu^2)\phi_1. \end{aligned} \tag{18}$$

The gauge fields

$$\begin{aligned} A_\mu(x;j) &= g\sum_{k=1}^{3} T_k(j)A_\mu^k(x) = g\frac{i}{2}\begin{pmatrix} A_\mu^3 & j(A_\mu^1 - iA_\mu^2) \\ j(A_\mu^1 + iA_\mu^2) & -A_\mu^3 \end{pmatrix}, \\ \hat{B}_\mu(x) &= g'YB_\mu(x) = g'\frac{i}{2}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} \end{aligned} \tag{19}$$

take their values in Lie algebras $su(2;j)$, $u(1)$, respectively, and the stress tensors are

$$\begin{aligned} F_{\mu\nu}(x;j) &= \mathcal{F}_{\mu\nu}(x;j) + [A_\mu(x;j), A_\nu(x;j)] = g\frac{i}{2}\begin{pmatrix} F_\mu^3 & j(F_\mu^1 - iF_\mu^2) \\ j(F_\mu^1 + iF_\mu^2) & -F_\mu^3 \end{pmatrix}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{20}$$

or in components

$$\begin{aligned} F_{\mu\nu}^1 &= \mathcal{F}_{\mu\nu}^1 + g(A_\mu^2A_\nu^3 - A_\mu^3A_\nu^2), \\ F_{\mu\nu}^2 &= \mathcal{F}_{\mu\nu}^2 + g(A_\mu^3A_\nu^1 - A_\mu^1A_\nu^3), \\ F_{\mu\nu}^3 &= \mathcal{F}_{\mu\nu}^3 + j^2g(A_\mu^1A_\nu^2 - A_\mu^2A_\nu^1), \end{aligned} \tag{21}$$

where $\mathcal{F}_{\mu\nu}^k = \partial_\mu A_\nu^k - \partial_\nu A_\mu^k$.

The Lagrangian $L(j)$ (14) describes massless fields. In a standard approach, to generate mass terms for the vector bosons the «sombrero» potential is added to the matter field Lagrangian $L_\phi(j = 1)$ (16) and after that the Higgs mechanism is used. The different way [13] is based on the fact that the quadratic form $\phi^\dagger\phi = \rho^2$ is invariant with respect to gauge transformations. This quadratic form defines the 3-dimensional sphere S_3 of the radius $\rho > 0$ in the target space Φ_2 which is $\mathbf{C}_2$ or $\mathbf{R}_4$ if real components are counted. In other words, the radial coordinates $R_+ \times S_3$ are introduced in $\mathbf{R}_4$. The vector boson masses are generated by the transformation of Lagrangian $L(j=1)$ (14) to the coordinates on the sphere S_3 and are the same as in the standard model. Higgs boson field does not appear if the sphere radius does not depend on the space-time coordinates $\rho = R = \mathrm{const}$ [13]. For $\rho \neq \mathrm{const}$ the real positive massless scalar field — analogy of dilaton or kind of Goldstone mode — is presented in the model [14].

The complex space $\Phi_2(j)$ can be regarded as 4-dimensional real space $\mathbf{R}_4(j)$. Let us introduce the real fields

$$\phi_1 = r(1 + i\psi_3), \quad \phi_2 = r(\psi_2 + i\psi_1). \tag{22}$$

The substitution $\phi_2 \to j\phi_2$ induces the following substitutions:

$$\psi_1 \to j\psi_1, \quad \psi_2 \to j\psi_2, \quad \psi_3 \to \psi_3, \quad r \to r \tag{23}$$

for the real fields.

For the real fields, the form (1) is written as $r^2(1 + \bar{\psi}^2(j)) = R^2$, where $\bar{\psi}^2(j) = j^2(\psi_1^2 + \psi_2^2) + \psi_3^2$, therefore

$$r = \frac{R}{\sqrt{1 + \bar{\psi}^2(j)}}. \tag{24}$$

Hence there are three independent real fields $\bar{\psi}(j) = (j\psi_1, j\psi_2, \psi_3)$. These fields belong to the space $\Psi_3(j)$ with non-Euclidean geometry which is realized on the 3-dimensional «sphere» of the radius R in the 4-dimensional space $\mathbf{R}_4(j)$. The fields $\bar{\psi}(j)$ are intrinsic Beltrami coordinates on $\Psi_3(j)$. The space $\Psi_3(j = 1) \equiv S_3$ has nondegenerate spherical geometry, but $\Psi_3(j = \iota)$ is fibered space of constant curvature with 1-dimensional base $\{\psi_3\}$ and 2-dimensional fiber $\{\psi_1, \psi_2\}$ [16], the so-called semispherical space [17], which can be interpreted as nonrelativistic $(1 + 2)$ kinematics with curvature or Newton kinematics [18].

The *free* Lagrangian (16) transforms to the *free* gauge invariant matter field Lagrangian $L_\psi(j)$ on $\Psi_3(j)$, which is defined with the help of the metric tensor $g_{kl}(j)$ [13] of the space $\Psi_3(j)$

$$g_{11} = \frac{1 + \psi_3^2 + j^2\psi_2^2}{(1 + \bar{\psi}^2(j))^2}, \qquad g_{22} = \frac{1 + \psi_3^2 + j^2\psi_1^2}{(1 + \bar{\psi}^2(j))^2}, \qquad g_{33} = \frac{1 + j^2(\psi_1^2 + \psi_2^2)}{(1 + \bar{\psi}^2(j))^2},$$

$$g_{12} = g_{21} = \frac{-j^2\psi_1\psi_2}{(1 + \bar{\psi}^2(j))^2}, \quad g_{13} = g_{31} = \frac{-j\psi_1\psi_3}{(1 + \bar{\psi}^2(j))^2}, \quad g_{23} = g_{32} = \frac{-j^2\psi_2\psi_3}{(1 + \bar{\psi}^2(j))^2}$$

in the form

$$L_\psi(j) = \frac{R^2}{2} \sum_{k,l=1}^{3} g_{kl}(j) D_\mu\psi_k(j) D_\mu\psi_l(j) = \\ = \frac{R^2\left[(1 + \bar{\psi}^2(j))(D_\mu\bar{\psi}(j))^2 - (\bar{\psi}(j), D_\mu\bar{\psi}(j))^2\right]}{2(1 + \bar{\psi}^2(j))^2}. \tag{25}$$

The covariant derivatives (17) are obtained from the representations of generators for the algebras $su(2)$, $u(1)$ in the space Ψ_3 [13] with the help of the substitutions (23)

$$T_1\bar{\psi}(j) = \frac{i}{2}\begin{pmatrix} -j(1 + j^2\psi_1^2) \\ j(\psi_3 - j^2\psi_1\psi_2) \\ -j^2(\psi_2 + \psi_1\psi_3) \end{pmatrix}, \qquad T_2\bar{\psi}(j) = \frac{i}{2}\begin{pmatrix} -j(\psi_3 + j^2\psi_1\psi_2) \\ -j(1 + j^2\psi_2^2) \\ j^2(\psi_1 - \psi_2\psi_3) \end{pmatrix},$$

$$T_3\bar{\psi}(j) = \frac{i}{2}\begin{pmatrix} j(-\psi_2+\psi_1\psi_3) \\ j(\psi_1+\psi_2\psi_3) \\ 1+\psi_3^2 \end{pmatrix}, \qquad Y\bar{\psi}(j) = \frac{i}{2}\begin{pmatrix} -j(\psi_2+\psi_1\psi_3) \\ j(\psi_1-\psi_2\psi_3) \\ -(1+\psi_3^2) \end{pmatrix}$$

and are as follows:

$$\begin{aligned} D_\mu\psi_1 &= \partial_\mu\psi_1 - \frac{g'}{2}(\psi_2+\psi_1\psi_3)B_\mu + \\ &\qquad + \frac{g}{2}\left[-(1+j^2\psi_1^2)A_\mu^1 - (\psi_3+j^2\psi_1\psi_2)A_\mu^2 - (\psi_2-\psi_1\psi_3)A_\mu^3\right], \\ D_\mu\psi_2 &= \partial_\mu\psi_2 + \frac{g'}{2}(\psi_1-\psi_2\psi_3)B_\mu + \\ &\qquad + \frac{g}{2}\left[(\psi_3-j^2\psi_1\psi_2)A_\mu^1 - (1+j^2\psi_2^2)A_\mu^2 + (\psi_1+\psi_2\psi_3)A_\mu^3\right], \\ D_\mu\psi_3 &= \partial_\mu\psi_3 - \frac{g'}{2}(1+\psi_3^2)B_\mu + \\ &\qquad + \frac{g}{2}\left[-j^2(\psi_2+\psi_1\psi_3)A_\mu^1 + j^2(\psi_1-\psi_2\psi_3)A_\mu^2 + (1+\psi_3^2)A_\mu^3\right]. \end{aligned} \tag{26}$$

The gauge fields Lagrangian (15) does not depend on the fields ϕ and therefore remains unchanged. So the full Lagrangian (14) is given by the sum of (15) and (25).

For small fields, the second order part of the Lagrangian (25) is written as

$$L_\psi^{(2)}(j) = \frac{R^2}{2}\left[(D_\mu\bar{\psi}(j))^{(1)}\right]^2 = \frac{R^2}{2}\sum_{k=1}^{3}\left[(D_\mu\psi_k(j))^{(1)}\right]^2, \tag{27}$$

where linear terms in covariant derivates (26) have the form

$$\begin{aligned} (D_\mu\psi_1)^{(1)} &= \partial_\mu\psi_1 - \frac{g}{2}A_\mu^1 = -\frac{g}{2}\left(A_\mu^1 - \frac{2}{g}\partial_\mu\psi_1\right) = -\frac{g}{2}\hat{A}_\mu^1, \\ (D_\mu\psi_2)^{(1)} &= \partial_\mu\psi_2 - \frac{g}{2}A_\mu^2 = -\frac{g}{2}\left(A_\mu^2 - \frac{2}{g}\partial_\mu\psi_2\right) = -\frac{g}{2}\hat{A}_\mu^2, \\ (D_\mu\psi_3)^{(1)} &= \partial_\mu\psi_3 + \frac{g}{2}A_\mu^3 - \frac{g'}{2}B_\mu = \frac{1}{2}\sqrt{g^2+g'^2}Z_\mu. \end{aligned}$$

The new fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(\hat{A}_\mu^1 \mp i\hat{A}_\mu^2\right), \quad Z_\mu = \frac{gA_\mu^3 - g'B_\mu + 2\partial_\mu\psi_3}{\sqrt{g^2+g'^2}}, \quad A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$

are transformed as

$$W_\mu^\pm \to jW_\mu^\pm, \quad Z_\mu \to Z_\mu, \quad A_\mu \to A_\mu,$$

and Lagrangian (27) is rewritten as follows:

$$L_\psi^{(2)} = j^2\frac{R^2g^2}{4}W_\mu^+W_\mu^- + \frac{R^2(g^2+g'^2)}{8}\left(Z_\mu\right)^2.$$

The quadratic part of the full Lagrangian

$$L_0(j) = L_A^{(2)}(j) + L_\psi^{(2)}(j) = -\frac{1}{4}(\mathcal{F}_{\mu\nu})^2 - \frac{1}{4}(\mathcal{Z}_{\mu\nu})^2 + \frac{m_Z^2}{2}(Z_\mu)^2 + \\ + j^2\left\{-\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^-_{\mu\nu} + m_W^2 W^+_\mu W^-_\mu\right\} \equiv L_b + j^2 L_f, \quad (28)$$

where

$$m_W = \frac{Rg}{2}, \quad m_Z = \frac{R}{2}\sqrt{g^2+g'^2}, \quad (29)$$

and $\mathcal{Z}_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\mathcal{W}^{\pm}{}_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu$ are Abelian stress tensors, describes all the experimentally verified parts of the standard Electroweek Model but does not include the scalar Higgs field.

The interaction part of the full Lagrangian in the first degree of approximation is given by

$$L^{(1)}_{\rm int}(j) = j^2\left[L_A^{(3)} + L_\psi^{(3)}\right],$$

where the third-order terms of the gauge field Lagrangian (15) are

$$L_A^{(3)} = -\frac{g}{\sqrt{g^2+g'^2}}\Biggl\{i\left(\mathcal{W}^-_{\mu\nu}W^+_\mu - \mathcal{W}^+_{\mu\nu}W^-_\mu\right)(g'A_\nu + gZ_\nu) - \\ - \frac{\sqrt{2}}{g}(g'A_\mu + gZ_\mu)\left[\mathcal{W}^+_{\mu\nu}(\partial_\nu\psi_2 - i\partial_\nu\psi_1) + \mathcal{W}^-_{\mu\nu}(\partial_\nu\psi_2 + i\partial_\nu\psi_1)\right] - \\ - \frac{2ig}{\sqrt{g^2+g'^2}}\left(\mathcal{W}^-_{\mu\nu}W^+_\mu - \mathcal{W}^+_{\mu\nu}W^-_\mu\right)\partial_\nu\psi_3 - \\ - \frac{2\sqrt{2}}{\sqrt{g^2+g'^2}}\left[\mathcal{W}^+_{\mu\nu}(\partial_\nu\psi_2 - i\partial_\nu\psi_1) + \mathcal{W}^-_{\mu\nu}(\partial_\nu\psi_2 + i\partial_\nu\psi_1)\right]\partial_\nu\psi_3 + \\ + (g'\mathcal{F}_{\mu\nu} + g\mathcal{Z}_{\mu\nu})\Biggl\{\frac{i}{4}\left[\left(W^+_\mu\right)^2 - \left(W^-_\mu\right)^2\right] + \frac{4}{g^2}\partial_\mu\psi_1\partial_\nu\psi_2 + \\ + \frac{\sqrt{2}}{g}\left[W^+_\mu(\partial_\nu\psi_2 - i\partial_\nu\psi_1) + W^-_\mu(\partial_\nu\psi_2 + i\partial_\nu\psi_1)\right]\Biggr\}\Biggr\},$$

and those of the matter field Lagrangian (25) are

$$L_\psi^{(3)} = \frac{R^2 g}{2\sqrt{2}}\Biggl\{W^+_\mu\Biggl[\psi_3(\partial_\mu\psi_2 - i\partial_\mu\psi_1) - \frac{g^2-g'^2}{g^2+g'^2}(\psi_2 - i\psi_1)\partial_\mu\psi_3 + \\ + \frac{g'(gA_\mu - g'Z_\mu)}{\sqrt{g^2+g'^2}}(\psi_2 - i\psi_1)\Biggr] + W^-_\mu\Biggl[\psi_3(\partial_\mu\psi_2 + i\partial_\mu\psi_1) - \\ - \frac{g^2-g'^2}{g^2+g'^2}(\psi_2 + i\psi_1)\partial_\mu\psi_3 + \frac{g'(gA_\mu - g'Z_\mu)}{\sqrt{g^2+g'^2}}(\psi_2 + i\psi_1)\Biggr] + \\ + \frac{1}{g}\sqrt{g^2+g'^2}Z_\mu(\psi_1\partial_\mu\psi_2 - \psi_2\partial_\mu\psi_1)\Biggr\}.$$

The fermion Lagrangian of the standard Electroweek Model is taken in the form [19]

$$L_F = L_l^\dagger i\tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i\tau_\mu D_\mu e_r - h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger\phi)e_r], \tag{30}$$

where $L_l = \left(\begin{array}{c} e_l \\ \nu_{e,l} \end{array}\right)$ is the $SU(2)$-doublet, e_r is the $SU(2)$-singlet, h_e is constant and e_r, e_l, ν_e are two-component Lorentzian spinors. Here τ_μ are Pauli matricies, $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$, $\tilde{\tau}_k = -\tau_k$. The covariant derivatives $D_\mu L_l$ are given by (17) with L_l instead of ϕ and $D_\mu e_r = (\partial_\mu + ig'B_\mu)e_r$. The convolution on the inner indices of $SU(2)$-doublet is denoted by $(\phi^\dagger L_l)$.

The matter field ϕ appears in Lagrangian (30) only in mass terms. When the gauge group $SU(2)$ is contracted to $SU(2;j)$ and the matter field is fibered to $\phi(j)$, the same takes place with doublet L_l, namely, the first component e_l does not change, but the second component is multiplied by contraction parameter: $\nu_{e,l} \to j\nu_{e,l}$. With the use of (22), (24) and these substitution, the mass terms are rewritten in the form

$$h_e[e_r^\dagger(\phi^\dagger(j)L_l(j)) + (L_l^\dagger(j)\phi(j))e_r] = \frac{h_e R}{\sqrt{1+\bar{\psi}^2(j)}}\left\{e_r^\dagger e_l + e_l^\dagger e_r + \right.$$
$$\left. + i\psi_3\left(e_l^\dagger e_r - e_r^\dagger e_l\right) + ij^2\left[\psi_1\left(\nu_{e,l}^\dagger e_r - e_r^\dagger \nu_{e,l}\right) + i\psi_2\left(\nu_{e,l}^\dagger e_r + e_r^\dagger \nu_{e,l}\right)\right]\right\}, \tag{31}$$

where the $SU(2)$-singlet e_r does not transform under contraction.

3. LIMITING CASE OF HIGGSLESS ELECTROWEAK MODEL

As was mentioned, the vector boson masses are automatically (without any Higgs mechanism) generated by the transformation of the free Lagrangian of the standard Electroweak Model to the Lagrangian (14), (15), (25) expressed in some coordinates on the sphere $\Psi_3(j)$. And this statement is true for both values of contraction parameter $j = 1, \iota$. When contraction parameter tends to zero $j^2 \to 0$, then the contribution of W-bosons fields to the quadratic part of the Lagrangian (28) will be small in comparison with the contribution of Z-boson and electromagnetic fields. In other words, the limit Lagrangian includes only Z-boson and electromagnetic fields. Therefore charged W-bosons fields do not affect these fields. The part L_f forms a new Lagrangian for W-bosons fields and their interactions with other fields. The appearance of two Lagrangians L_b and L_f for the limit model is in correspondence with two Hermitian forms of fibered space $\Phi_2(\iota)$, which are invariant under the action of contracted gauge group $SU(2;\iota)$. Electromagnetic and Z-boson fields can be regarded as external fields with respect to the W-bosons fields.

In mathematical language the field space $\left\{A_\mu, Z_\mu, W_\mu^\pm\right\}$ is fibered after contraction $j = \iota$ to the base $\{A_\mu, Z_\mu\}$ and the fiber $\left\{W_\mu^\pm\right\}$. (In order to avoid terminological misunderstanding let us stress that we have in view locally trivial fibering, which is defined by the projection pr: $\left\{A_\mu, Z_\mu, W_\mu^\pm\right\} \to \{A_\mu, Z_\mu\}$ in the field space. This fibering is understood in the context of semi-Riemannian geometry [16] and has nothing to do with the principal fiber bundle.) Then L_b in (28) presents Lagrangian in the base and L_f is Lagrangian in the fiber. In general, properties of the fiber depend on the points of a base and not on the contrary. In this sense, fields in the base are external ones with respect to fields in the fiber.

The fermion Lagrangian (30) for nilpotent value of the contraction parameter $j = \iota$ is also splited on electron part in the base and neutrino part in the fiber. This means that in the limit model, electron field is external one relative to neutrino field. The mass terms (31) for $j = \iota$ are

$$h_e[e_r^\dagger(\phi^\dagger(\iota)L_l(\iota)) + (L_l^\dagger(\iota)\phi(\iota))e_r] = \frac{h_e R}{\sqrt{1+\psi_3^2}}[e_r^\dagger e_l^- + e_l^{-\dagger} e_r + i\psi_3(e_l^{-\dagger} e_r - e_r^\dagger e_l^-)]. \quad (32)$$

Its second-order terms $h_e R(e_r^\dagger e_l^- + e_l^{-\dagger} e_r)$ provide the electron mass $m_e = h_e R$ and neutrino remains massless.

Let us note that field interactions in contracted model are more simple as compared with the standard Electroweak Model due to nullification of some terms.

CONCLUSIONS

The modified formulation of the Electroweak Model with the gauge group $SU(2) \times U(1)$ based on the 3-dimensional spherical geometry in the target space is suggested. This model describes all experimentally observed fields and does not include the (up to now unobserved) scalar Higgs field. The *free* Lagrangian in the spherical matter field space is used instead of Lagrangian with the potential of the special «sombrero» form. The gauge field Lagrangian is the standard one. There is no need in Higgs mechanism since the vector field masses are generated automatically.

We have discussed the limiting case of the modified Higgsless Electroweak Model, which corresponds to the contracted gauge group $SU(2; j) \times U(1)$, where $j = \iota$ or $j \to 0$. The masses of all the experimentally verified particles involved in the Electroweak Model remain the same under contraction, but interactions of the fields are changed in two aspects. Firstly, all field interactions become more simpler due to nullification of some terms in Lagrangian. Secondly, interrelation of the fields becomes more complicated. All fields are divided into two classes: fields in the base (Z-boson, electromagnetic and electron) and fields in the fiber (W-bosons and neutrino). The base fields can be interpreted as external ones with respect to the fiber fields, i.e., Z-boson, electromagnetic and electron fields can interact with W-bosons and neutrino fields, but W-bosons and neutrino fields do not affect these fields in the framework of the limit model.

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