Письма в ЭЧАЯ. 2011. Т. 8, № 3(166). С. 529–534

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

QUANTUM CORRECTIONS IN $N = 1$ SUPERSYMMETRIC THEORIES WITH CUBIC SUPERPOTENTIAL, REGULARIZED BY HIGHER COVARIANT DERIVATIVES

K. V. Stepanyantz

Moscow State University, Physical Faculty, Department of Theoretical Physics, Moscow

Using the higher covariant derivative regularization we calculate a two-loop β-function for the $N = 1$ supersymmetric Yang–Mills theory with the matter superfields, containing the cubic superpotential. It is found that all integrals, defining this function, are integrals of total derivatives.

PACS: 11.30.Pb

INTRODUCTION

Most calculations of quantum corrections in supersymmetric theories were made by the dimensional reduction [1]. Some of them are listed in [2]. (There are a lot of such calculations, and this list is certainly incomplete.) However, it is well known that the dimensional reduction is not self-consistent [3]. Ways, allowing to avoid this problem, are discussed in literature [4]. Nevertheless, other regularizations are also sometimes applied for calculations. For example, two-loop β-function of the $N = 1$ supersymmetric Yang–Mills theory was calculated in [5] with the differential renormalization [6].

A self-consistent regularization, which does not break the supersymmetry, is the higher covariant derivative regularization [7], which was generalized to the supersymmetric case in [8]. However, using of this regularization is rather technically complicated. Application of the higher covariant derivative regularization to calculation of quantum corrections in the $N = 1$ supersymmetric electrodynamics in two and three loops [9, 10] reveals an interesting feature of quantum corrections: all integrals, defining the Gell-Mann–Low function appear to be integrals of total derivatives and can be easily calculated. This leads the NSVZ β-function, which relates the β-function in nth loop with the β-function and the anomalous dimensions in the previous loops. Due to this, application of the higher covariant derivative regularization is very convenient in the supersymmetric case. The fact that the integrals, appearing with the higher covariant derivative regularization, in the limit of zero external momentum become integrals of total derivatives, seems to be a general feature of supersymmetric theories. That is why calculations with the higher derivative regularization seem to be interesting. In this paper we calculate a two-loop β-function for a $N = 1$ supersymmetric theory with a cubic superpotential. The presence of the cubic superpotential requires adding of the higher derivative terms not only for the gauge field, but also for the matter superfields. So, it is interesting to find out, whether the factorization of integrands into total derivatives takes place in this case.

The paper is organized as follows:

In Sec. 1 we introduce the notation and recall basic information about the higher covariant derivative regularization. The β-function for the considered theory is calculated in Sec. 2. The result is briefly discussed in Conclusion.

1. $N = 1$ SUPERSYMMETRIC YANG–MILLS THEORY AND THE HIGHER COVARIANT DERIVATIVE REGULARIZATION

We consider a general renormalizable $N = 1$ supersymmetric Yang–Mills theory, which (in the massless case) is described by the action

$$S = \frac{1}{2e^2}\,\mathrm{Re}\,\mathrm{tr}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, (\phi^*)^i (\mathrm{e}^{2V})_i{}^j \phi_j + \\ + \left(\frac{1}{6}\int d^4x\, d^2\theta\, \lambda^{ijk}\phi_i\phi_j\phi_k + \mathrm{h.c.}\right), \qquad (1)$$

where ϕ_i are chiral matter superfields and V is a real scalar gauge superfield. The superfield W_a is defined by

$$W_a = \frac{1}{8}\bar{D}^2(\mathrm{e}^{-2V} D_a\, \mathrm{e}^{2V}). \qquad (2)$$

D_a and $\bar{D}_a$ denote the right and left supersymmetric covariant derivatives, respectively, $V = \mathrm{e}\,V^A T^A$. The generators of the fundamental representation are normalized by the condition

$$\mathrm{tr}(t^A t^B) = \frac{1}{2}\delta^{AB}. \qquad (3)$$

Also we use the following notation:

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}, \quad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j, \\ f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}, \quad r \equiv \delta_{AA}. \qquad (4)$$

For calculation of quantum corrections it is convenient to use the background field method [11]: We make the substitution

$$\mathrm{e}^{2V} \to \mathrm{e}^{2V'} \equiv \mathrm{e}^{\mathbf{\Omega}^+}\, \mathrm{e}^{2V}\, \mathrm{e}^{\mathbf{\Omega}} \qquad (5)$$

in action (1), where $\mathbf{\Omega}$ is a background superfield. Then the theory is invariant under the background gauge transformations

$$\phi \to \mathrm{e}^{i\Lambda}\phi, \quad V \to \mathrm{e}^{iK}\, V\, \mathrm{e}^{-iK}, \quad \mathrm{e}^{\mathbf{\Omega}} \to \mathrm{e}^{iK}\, \mathrm{e}^{\mathbf{\Omega}}\, \mathrm{e}^{-i\Lambda}, \quad \mathrm{e}^{\mathbf{\Omega}^+} \to \mathrm{e}^{i\Lambda^+}\, \mathrm{e}^{\mathbf{\Omega}^+}\, \mathrm{e}^{-iK}, \qquad (6)$$

where K is an arbitrary real superfield, and Λ is a background-chiral superfield. This invariance allows one to set $\mathbf{\Omega} = \mathbf{\Omega}^+ = \mathbf{V}$.

It is convenient to choose a regularization and gauge fixing without breaking invariance (6). First, we fix a gauge by adding

$$S_{\rm gf} = -\frac{1}{32e^2}\,{\rm tr}\int d^4x\,d^4\theta(V\mathbf{D}^2\bar{\mathbf{D}}^2V + V\bar{\mathbf{D}}^2\mathbf{D}^2V) \tag{7}$$

to the action. The corresponding Faddeev–Popov and Nielsen–Kallosh ghost Lagrangians are constructed by the standard way. One of the possible choices of the higher derivative regularization is adding the terms

$$S_\Lambda = \frac{1}{2e^2}{\rm tr\,Re}\int d^4x\,d^4\theta V\frac{(\mathbf{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \\ +\frac{1}{8}\int d^4x\,d^4\theta\left((\phi^*)^i\left[{\rm e}^{\mathbf{\Omega}^+}{\rm e}^{2V}\frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}}{\rm e}^{\mathbf{\Omega}}\right]_i{}^j\phi_j + (\phi^*)^i\left[{\rm e}^{\mathbf{\Omega}^+}\frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}}{\rm e}^{2V}{\rm e}^{\mathbf{\Omega}}\right]_i{}^j\phi_j\right) \tag{8}$$

to the action (1). (Here $\mathbf{D}_\alpha$ denotes the background covariant derivative and we assume that $m < n$.) (Because the considered theory contains a nontrivial superpotential, it is also necessary to introduce the higher covariant derivative term for the matter superfields.)

The higher covariant derivative term does not remove divergences in the one-loop approximation [12]. In order to cancel them, it is necessary to introduce into the generating functional the Pauli–Villars determinants

$$\prod_I\left(\int D\phi_I^*D\phi_I\,{\rm e}^{iS_I}\right)^{-c_I}, \quad \sum_I c_I = 1, \quad \sum_I c_I M_I^2 = 0, \tag{9}$$

where

$$S_I = \frac{1}{8}\int d^4x\,d^4\theta\,\left((\Phi^*)^i\left[{\rm e}^{\mathbf{\Omega}^+}{\rm e}^{2V}\left(1+\frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}}\right){\rm e}^{\mathbf{\Omega}}\right]_i{}^j\Phi_j + \right. \\ \left. +(\Phi^*)^i\left[{\rm e}^{\mathbf{\Omega}^+}\left(1+\frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}}\right){\rm e}^{2V}{\rm e}^{\mathbf{\Omega}}\right]_i{}^j\Phi_j\right) + \left(\frac{1}{32}\int d^4x\,d^4\theta M_I^{ij}({\rm e}^{\mathbf{\Omega}}\Phi)_i\,\times\right. \\ \left.\times\mathbf{D}^2\frac{1}{\mathbf{D}_\alpha^2}\left(1+\frac{(\mathbf{D}_\alpha^2)^m}{\Lambda^{2m}}\right)({\rm e}^{\mathbf{\Omega}}\Phi)_j + {\rm h.c.}\right) \tag{10}$$

is the action for the Pauli–Villars fields. Their masses are proportional to the parameter Λ:

$$M_I^{ij} = a_I^{ij}\Lambda, \tag{11}$$

so that Λ is the only dimensionful parameter of the theory. Also we will choose the masses so that

$$M_I^{ij}(M_I^*)_{jk} = M_I^2\delta_k^i. \tag{12}$$

The generating functional for connected Green functions and the effective action are defined by the standard way.

In order to calculate the β-function we consider

$$\frac{d}{d\ln\Lambda}\left(d^{-1}(\alpha_0,\lambda_0,\Lambda/p) - \alpha_0^{-1}\right)\Big|_{p=0} = -\frac{d\alpha_0^{-1}}{d\ln\Lambda} = \frac{\beta(\alpha_0)}{\alpha_0^2}, \tag{13}$$

where the function d is defined by

$$\Gamma^{(2)}_V = -\frac{1}{8\pi}\,\mathrm{tr}\int\frac{d^4p}{(2\pi)^4}\,d^4\theta\,\mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\,d^{-1}(\alpha,\mu/p). \qquad (14)$$

Similarly

$$\gamma_i{}^j(\alpha_0(\alpha,\Lambda/\mu)) = -\frac{\partial}{\partial\ln\Lambda}(\ln\,Z(\alpha,\Lambda/\mu))_i^j, \qquad (15)$$

if in the massless limit:

$$\Gamma^{(2)}_\phi = \frac{1}{4}\int\frac{d^4p}{(2\pi)^4}\,d^4\theta\,(\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\mu/p). \qquad (16)$$

2. TWO-LOOP β-FUNCTION

After calculation of the supergraphs, we have obtained the following result for the two-loop β-function:

$$\beta_2(\alpha) = -\frac{3\alpha^2}{2\pi}C_2 + \alpha^2 T(R)I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r}C(R)_i{}^j C(R)_j{}^i I_2 + \\ + \alpha^3 T(R)C_2 I_3 + \alpha^2 C(R)_i{}^j\frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r}I_4, \qquad (17)$$

where

$$I_i = I_i(0) - \sum_I c_I I_i(M_I) \quad \text{for} \quad I = 0,2,3, \qquad (18)$$

and the integrals $I_0(M)$, I_1, $I_2(M)$, $I_3(M)$, and I_4 are given by

$$I_0(M) = 8\pi\int\frac{d^4q}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{q^2}\frac{d}{dq^2}\Bigg\{\frac{1}{2}\ln(q^2+M^2) - \frac{q^2}{2(q^2+M^2)} + \\ + \ln(1+q^{2m}/\Lambda^{2m}) - \frac{mq^{2m}/\Lambda^{2m}}{(1+q^{2m}/\Lambda^{2m})}\Bigg\}; \qquad (19)$$

$$I_1 = 96\pi^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{k^2}\frac{d}{dk^2}\Bigg\{\frac{1}{q^2(q+k)^2(1+q^{2n}/\Lambda^{2n})}\times \\ \times\frac{1}{(1+(q+k)^{2n}/\Lambda^{2n})}\left(\frac{n+1}{(1+k^{2n}/\Lambda^{2n})} - \frac{n}{(1+k^{2n}/\Lambda^{2n})^2}\right)\Bigg\}; \qquad (20)$$

$$I_2(M) = -16\pi^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{q^2}\frac{d}{dq^2}\Bigg\{\frac{q^2}{k^2(1+k^{2n}/\Lambda^{2n})(q^2+M^2)}\times \\ \times\frac{1}{((q+k)^2+M^2)}\Bigg[\frac{q^2(2+(q+k)^{2m}/\Lambda^{2m}+q^{2m}/\Lambda^{2m})^2}{(q^2+M^2)(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})} + \\ + \frac{mq^{2m}}{\Lambda^{2m}}\left(-\frac{1}{(1+(q+k)^{2m}/\Lambda^{2m})} + \frac{(1+(q+k)^{2m}/\Lambda^{2m})}{(1+q^{2m}/\Lambda^{2m})^2}\right)\Bigg]\Bigg\}; \qquad (21)$$

$$I_3(M) = 16\pi^2 \int \frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\Bigg\{\frac{\partial}{\partial q_\alpha}\Bigg[\frac{k_\alpha}{4k^2(k^2+M^2)(q^2+M^2)(k+q)^2}\times$$
$$\times\frac{1}{(1+(q+k)^{2n}/\Lambda^{2n})}\Bigg(-\frac{k^2(2+k^{2m}/\Lambda^{2m}+q^{2m}/\Lambda^{2m})^2}{(k^2+M^2)(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}+$$
$$+\frac{mk^{2m}/\Lambda^{2m}}{(1+q^{2m}/\Lambda^{2m})}-\frac{mk^{2m}/\Lambda^{2m}(1+q^{2m}/\Lambda^{2m})}{(1+k^{2m}/\Lambda^{2m})^2}\Bigg)\Bigg]-$$
$$-\frac{1}{k^2}\frac{d}{dk^2}\Bigg[\frac{(2+(q+k)^{2m}/\Lambda^{2m}+q^{2m}/\Lambda^{2m})^2}{2(q^2+M^2)((q+k)^2+M^2)(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}\times$$
$$\times\Bigg(\frac{1}{(1+k^{2n}/\Lambda^{2n})}+\frac{nk^{2n}/\Lambda^{2n}}{(1+k^{2n}/\Lambda^{2n})^2}\Bigg)\Bigg]\Bigg\}; \quad (22)$$

$$I_4 = 64\pi^2\int\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d}{d\ln\Lambda}\frac{1}{q^2}\frac{d}{dq^2}\Bigg[\frac{1}{k^2(q+k)^2(1+k^{2m}/\Lambda^{2m})}\times$$
$$\times\frac{1}{(1+(q+k)^{2m}/\Lambda^{2m})}\Bigg(\frac{1}{(1+q^{2m}/\Lambda^{2m})}+\frac{mq^{2m}/\Lambda^{2m}}{(1+q^{2m}/\Lambda^{2m})^2}\Bigg)\Bigg]. \quad (23)$$

It is easy to see that all these integrals are integrals of total derivatives, due to the identity

$$\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{d}{dq^2}f(q^2) = \frac{1}{16\pi^2}\Big(f(q^2=\infty)-f(q^2=0)\Big), \quad (24)$$

which can be easily proved in the four-dimensional spherical coordinates. The result is

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(-3C_2^2 + T(R)C_2 + \frac{2}{r}C(R)_i{}^jC(R)_j{}^i\Big)-$$
$$-\frac{\alpha^2 C(R)_i{}^j\lambda^*_{jkl}\lambda^{ikl}}{8\pi^3 r}+\dots \quad (25)$$

Comparing this with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda^*_{ikl}\lambda^{jkl}}{4\pi^2}+\dots, \quad (26)$$

we see agreement with the exact NSVZ β-function [13]

$$\beta(\alpha) = -\frac{\alpha^2[3C_2 - T(R) + C(R)_i^j\gamma_j^i(\alpha/r)]}{2\pi(1-C_2\alpha/2\pi)}. \quad (27)$$

Up to notation, this result is in agreement with the results of calculations made with the dimensional reduction, see, e.g., [2].

CONCLUSION

We see that the two-loop β-function in $N = 1$ supersymmetric theories can be easily calculated with the higher covariant derivative regularization. The most interesting feature of this calculation is the factorization of rather complicated integrals into integrals of total derivatives. Also it is possible to consider different forms of the regularizing terms and the Pauli–Villars action. For example, our choice is different from the one, proposed in [8]. The regularization, described here, is rather simple, but breaks the BRST-invariance of the action. That is why it is necessary to use a special subtraction scheme, which restores the Slavnov–Taylor identities in each order of the perturbation theory [14]. However, the factorization of integrals into total derivatives does not seem to depend on a particular choice. For example, another form of the regularizing terms will be considered in [15]. Possibly, this feature appears due to using of the background field method [16]. One can also try to explain this substituting solutions of Slavnov–Taylor identities into the Schwinger–Dyson equations. However, a complete proof of this fact by this method has not yet been done. Possibly, factorization of integrands into total derivatives is related with the existence of the Novikov, Shifman, Vainshtein, and Zakharov β-function, which relates n-loop contribution to the β-function with the β-function and the anomalous dimension in previous loops. In this paper we see, how this occurs at the two-loop level.

Acknowledgements. This work was partially supported by RFBR grant No. 08-01-00281a.

REFERENCES

1. *Siegel W.* // Phys. Lett. B. 1979. V. 84. P. 193.
2. *Avdeev L. V., Tarasov O. V.* // Phys. Lett. B. 1982. V. 112. P. 356;
Abbott L. F., Grisary M. T., Zanon D. // Nucl. Phys. B. 1984. V. 244. P. 454;
Parkes A., West P. // Phys. Lett. B. 1983. V. 138. P. 99;
Jack I., Jones D. R. T., North C. G. // Phys. Lett. B. 1996. V. 386. P. 138; Nucl. Phys. B. 1996. V. 473. P. 308; 1997. V. 486. P. 479;
Jack I., Jones D. R. T., Pickering A. // Phys. Lett. B. 1998. V. 435. P. 61.
3. *Siegel W.* // Phys. Lett. B. 1980. V. 94. P. 37.
4. *Stöckinger D.* // JHEP 2005. V. 0503. P. 076;
Hollik W., Stöckinger D. hep-ph/0509298;
Signer A., Stöckinger D. // Phys. Lett. B. 2005. V. 626. P. 127.
5. *Mas J., Perez-Victoria M., Seijas C.* // JHEP. 2002. V. 0203. P. 049.
6. *Freedman D. Z., Johnson K., Latorre J. I.* // Nucl. Phys. B. 1992. V. 371. P. 353.
7. *Slavnov A. A.* // Nucl. Phys. B. 1971. V. 31. P. 301.
8. *West P.* // Nucl. Phys. B. 1986. V. 268. P. 113.
9. *Soloshenko A. A., Stepanyantz K. V.* // Theor. Math. Phys. 2004. V. 140. P. 1264; hep-th/0304083.
10. *Pimenov A. B. et al.* // Rus. Phys. J. 2008. V. 51. P. 444.
11. *West P.* Introduction to Supersymmetry and Supergravity. World Sci., 1986.
12. *Faddeev L. D., Slavnov A. A.* Gauge Fields, Introduction to Quantum Theory. 2nd ed. Reading: Benjamin, 1990.
13. *Novikov V. et al.* // Nucl. Phys. B. 1983. V. 229. P. 381; Phys. Lett. B. 1985. V. 166. P. 329;
Shifman M. A., Vainshtein A. I. // Nucl. Phys. B. 1986. V. 277. P. 456.
14. *Slavnov A. A.* // Phys. Lett. B. 2001. V. 518. P. 195; Theor. Math. Phys. 2002. V. 130. P. 1;
Slavnov A. A., Stepanyantz K. V. // Theor. Math. Phys. 2003. V. 135. P. 673; 2004. V. 139. P. 599.
15. *Pimenov A. B., Shevtsova E. S., Stepanyantz K. V.* In preparation.
16. *Smilga A., Vainstein A.* // Nucl. Phys. B. 2005. V. 704. P. 445.