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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

FINE STRUCTURE OF THE MUONIC ^{4}He ION

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On the basis of quasi-potential approach to the bound state problem in QED we calculate the vacuum polarization, recoil and structure corrections of orders α^5 and α^6 to the fine splitting interval $\Delta E^{\rm fs} = E(2P_{3/2}) - E(2P_{1/2})$ in muonic ${}^4_2{\rm He}$ ion. The resulting value $\Delta E^{\rm fs} = 146180.68$ μeV provides reliable guideline in performing a comparison with the relevant experimental data.

В рамках квазипотенциального подхода к проблеме связанных состояний в квантовой электродинамике вычислены поправки поляризации вакуума, отдачи и структуры ядра порядка α^5 и α^6 в интервале тонкой структуры $\Delta E^{\rm fs} = E(2P_{3/2}) - E(2P_{1/2})$ в ионе мюонного гелия $(\mu{}^4_2{\rm He})^+$. Полученная величина $\Delta E^{\rm fs} = 146180{,}68$ мкэВ представляет собой надежный ориентир для сравнения с экспериментальными данными.

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INTRODUCTION

Simple atoms play an important role in the check of quantum electrodynamics (QED), the bound state theory and precise determination of fundamental physical constants (the fine structure constant, the lepton and proton masses, the Rydberg constant, the proton charge radius, etc.) [1–3]. Light muonic atoms (muonic hydrogen (μp), muonic deuterium, ions of muonic helium, etc.) are distinguished among simple atoms by the strong influence of the vacuum polarization (VP) effects, recoil effects, nuclear structure and polarizability effects on the structure of the energy levels. The comparison of the theoretical value of the fine and hyperfine splittings in muonic helium ions with the future experimental data will lead to a more precise values of the helion and α-particle charge radii. The energy levels of muonic helium ions were theoretically studied many years ago in [4–6] both on the basis of the relativistic Dirac equation and nonrelativistic approach, accounting different corrections by the perturbation theory (PT). In these papers the basic contributions to the energies for the $(2P-2S)$ transitions in muonic helium $(\mu{}^4_2{\rm He})^+$ were evaluated with the accuracy 0.1 meV. For more than forty years, a measurement of the muonic hydrogen Lamb shift has been considered one of the fundamental experiments in atomic spectroscopy. Recently, the progress in muon beams and laser technology made such an experiment feasible. The first successful measurement of the μp Lamb shift 49881.88 (76) GHz in [7] leads to new value of the proton charge radius $r_p = 0.84184(36)(56)$ fm, where the first and second uncertainties originate respectively from the experimental uncertainty 0.76 GHz and the uncertainty 0.0049 meV in the Lamb shift value which is dominated by the proton polarizability term. The new value of proton radius r_p improves the CODATA value [3] by an order of the magnitude.

Another important project which exists now at PSI (Paul Scherrer Institute) in the CREMA (Charge Radius Experiment with Muonic Atoms) collaboration proposes to measure several transition frequencies between $2S$ and $2P$ states in muonic helium ions $(\mu_2^4\mathrm{He})^+$, $(\mu_2^3\mathrm{He})^+$ with 50 ppm precision. As a result, new values of the charge radii of the helion and α particle with the accuracy 0.0005 fm will be determined. This program suggests that the theoretical calculations of the $(2S{-}2P)$ transition frequencies will be performed with high accuracy.

In this work we continue the investigation [8] of the energy spectrum of $(\mu_2^4\mathrm{He})^+$ in the P-wave part. The aim of the present study is to calculate such contributions of orders α^5 and α^6 to the fine structure of the $2P$-state, which are connected with the electron vacuum polarization, the recoil and structure effects, the muon anomalous magnetic moment and the relativistic corrections. The role of all these effects is crucial in obtaining high theoretical accuracy. Our purpose also consists in the refinement of the earlier performed calculations in [4, 6] and in the derivation of the reliable numerical estimate for the energy intervals $(2P_{3/2}-2S_{1/2})$, $(2P_{1/2}-2S_{1/2})$ in the ion $(\mu_2^4\mathrm{He})^+$, which can be used for the comparison with experimental data. Modern numerical values of fundamental physical constants are taken from [3]: the electron mass $m_e = 0.510998910(13)\cdot 10^{-3}$ GeV, the muon mass $m_\mu = 0.1056583668(38)$ GeV, the fine structure constant $\alpha^{-1} = 137.035999679(94)$, the proton mass $m_p = 0.938272013(23)$ GeV, the mass of α particle $m_\alpha = 3.727379109(93)$ GeV, the muon anomalous magnetic moment $a_\mu = 1.16592069(60)\cdot 10^{-3}$.

1. FINE STRUCTURE OF P-WAVE ENERGY LEVELS

Our approach to the investigation of the energy spectrum of muonic helium ion $(\mu_2^4\mathrm{He})^+$ is based on the use of quasi-potential method in quantum electrodynamics [9–11], where the two-particle bound state is described by the Schrödinger equation. The basic contribution to the muon and α-particle interaction operator is determined by the Breit Hamiltonian [12, 13]:

$$H = \frac{\mathbf{p}^2}{2\mu} - \frac{Z\alpha}{r} - \frac{\mathbf{p}^4}{8m_1^3} - \frac{\mathbf{p}^4}{8m_2^3} + \frac{\pi Z\alpha}{2}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\delta(\mathbf{r}) - \\ - \frac{Z\alpha}{2m_1m_2r}\left(\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right) + \Delta V^{\mathrm{fs}}(r), \quad (1)$$

where m_1, m_2 are the masses of the muon and α particle; $\mu = m_1m_2/(m_1+m_2)$ is the reduced mass; ΔV^{fs} is the muon spin-orbit interaction:

$$\Delta V^{\mathrm{fs}}(r) = \frac{Z\alpha}{4m_1^2r^3}\left[1 + \frac{2m_1}{m_2} + 2a_\mu\left(1+\frac{m_1}{m_2}\right)\right](\mathbf{L}\boldsymbol{\sigma}_1). \quad (2)$$

The leading order $(Z\alpha)^4$ contribution to the fine structure is determined by the operator ΔV^{fs}. As it follows from Eq. (2), the potential ΔV^{fs} includes also the recoil effects (the Barker–Glover correction [14]) and the muon anomalous magnetic moment a_μ correction. The fine structure interval $(2P_{3/2}-2P_{1/2})$ for the ion $(\mu_2^4\mathrm{He})^+$ can be written in the form

$$\Delta E^{\mathrm{fs}} = E(2P_{3/2}) - E(2P_{1/2}) = \frac{\mu^3(Z\alpha)^4}{32m_1^2}\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]+ \\ +\frac{5m_1(Z\alpha)^6}{256}-\frac{m_1^2(Z\alpha)^6}{64m_2}+\frac{\alpha(Z\alpha)^6\mu^3}{32\pi m_1^2}\left[\ln\frac{\mu(Z\alpha)^2}{m_1}+\frac{1}{5}\right]+ \\ +\alpha(Z\alpha)^4A_{\mathrm{VP}}+\alpha^2(Z\alpha)^4B_{\mathrm{VP}}+A_{\mathrm{str}}(Z\alpha)^6\mu^2r_\alpha^2. \quad (3)$$

This expression includes a relativistic correction of order $(Z\alpha)^6$, which can be calculated with the aid of the Dirac equation [1, 15], the correction of order $\alpha(Z\alpha)^6$ enhanced by the factor $\ln(Z\alpha)$ [16, 17], a number of terms of fifth and sixth order in α which are determined by the effects of the vacuum polarization and the nuclear structure. The relativistic recoil effects of order $m_1(Z\alpha)^6/m_2$ in the energy spectra of hydrogenic atoms were investigated in [1,15,18–20]. In the fine splitting (3) they were calculated in [15,20]. Additional corrections of the same order were obtained in [21]. They do not depend on the muon total momentum j and give the contribution only to the Lamb shift. The contributions to the coefficients A_{VP} and B_{VP} arise in the first and second orders of perturbation theory. Numerical values of the terms in expression (3), which are presented in the analytical form, are quoted in the table for the definiteness with the accuracy 0.01 μeV. The fine structure interval (3) in the energy spectrum of electronic hydrogen is considered for a long time as a basic test of quantum electrodynamics [15, 22, 23].

The leading-order vacuum polarization potential which gives the contribution to the coefficient A_{VP}, is presented by the Feynman diagrams in Fig. 1. The one-loop vacuum polarization effects lead to the modification of both the Coulomb interaction and the muon spin-orbit interaction in expressions (1), (2) [12, 13]:

$$\Delta V_{\mathrm{VP}}^{C}(r) = \frac{\alpha}{3\pi}\int_1^\infty \rho(s)\,ds\left(-\frac{Z\alpha}{r}\right)\mathrm{e}^{-2m_e sr}, \quad (4)$$

$$\Delta V_{\mathrm{VP}}^{\mathrm{fs}}(r) = \frac{\alpha(Z\alpha)}{12\pi m_1^2 r^3}\int_1^\infty \rho(s)\,ds\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]\mathrm{e}^{-2m_e sr}(1+2m_e sr)(\mathbf{L}\boldsymbol{\sigma}_1), \quad (5)$$

where the spectral function $\rho(s)=\sqrt{s^2-1}(2s^2+1)/s^4$, m_e is the electron mass. Averaging the potential (2) over the wave functions of the $2P$-state

$$\psi_{2P}(\mathbf{r}) = \frac{1}{2\sqrt{6}}W^{5/2}r\,\exp\left(-\frac{Wr}{2}\right)Y_{1m}(\theta,\phi), \quad W=\mu Z\alpha, \quad (6)$$

we obtain the following contribution to the interval (3) (see Fig. 1, a):

$$\Delta E_1^{\mathrm{fs}} = \frac{\mu^3(Z\alpha)^4}{32m_1^2}\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]\times \\ \times\frac{\alpha}{3\pi}\int_1^\infty \rho(s)\,ds\int_0^\infty x\,dx\,\exp\left[-x\left(1+\frac{2m_e s}{W}\right)\right]\left(1+\frac{2m_e s}{W}x\right) = 129.25\ \mu\mathrm{eV}. \quad (7)$$

Fine structure of P-wave energy levels in muonic $^4_2$He ion

Contribution to the fine splitting $\Delta E^{\rm fs}$	Numerical value, μeV	Reference, equation
Contribution of order $(Z\alpha)^4$ $\frac{\mu^3(Z\alpha)^4}{32m_1^2}\left(1+\frac{2m_1}{m_2}\right)$	145563.82	[4, 13], (3)
Muon AMM contribution $\frac{\mu^3(Z\alpha)^4}{16m_1^2}a_\mu\left(1+\frac{m_1}{m_2}\right)$	330.32	[4, 13], (3)
Contribution of order $(Z\alpha)^6$	19.94	[15, 20], (3)
Contribution of order $(Z\alpha)^6 m_1/m_2$	–0.45	[15, 20], (3)
Contribution of order $\alpha(Z\alpha)^4$ in the first-order PT $\langle\Delta V^{\rm fs}_{\rm VP}\rangle$	131.67	[4, 13], (7)
Contribution of one-loop muon VP in the first-order PT $\langle\Delta V^{\rm fs}_{\rm MVP}\rangle$	0.01	[4, 13], (7)
Contribution of order $\alpha(Z\alpha)^4$ in the second-order PT $\langle\Delta V^C_{\rm VP}\cdot\tilde G\cdot\Delta V^{\rm fs}\rangle$	143.96	(10)
Contribution of order $\alpha(Z\alpha)^6$ $\frac{\alpha(Z\alpha)^6\mu^3}{32\pi m_1^2}\left[\ln\frac{\mu(Z\alpha)^2}{m_1}+\frac{1}{5}\right]$	–0.56	[1, 16, 17]
VP contribution in the second-order PT of order $\alpha^2(Z\alpha)^4$ $\langle\Delta V^C_{\rm VP}\cdot\tilde G\cdot\Delta V^{\rm fs}_{\rm VP}\rangle$	0.21	(20)
VP contribution from 1γ interaction of order $\alpha^2(Z\alpha)^4$ $\langle\Delta V^{\rm fs}_{\rm VP-VP}\rangle$	0.18	(13)
VP contribution from 1γ interaction of order $\alpha^2(Z\alpha)^4$ $\langle\Delta V^{\rm fs}_{\text{2-loop,VP}}\rangle$	0.79	(17)
VP contribution in the second-order PT of order $\alpha^2(Z\alpha)^4$ $\langle\Delta V^C_{\rm VP-VP}\cdot\tilde G\cdot\Delta V^{\rm fs}\rangle$	0.02	(18)
VP Contribution in the second-order PT of order $\alpha^2(Z\alpha)^4$ $\langle\Delta V^C_{\text{2-loop,VP}}\cdot\tilde G\cdot\Delta V^{\rm fs}\rangle$	2.08	(19)
Nuclear structure correction in 1γ interaction	–11.76	(22)
Nuclear structure correction in the second-order PT	0.45	(24), (25)
Summary contribution	146180.68	

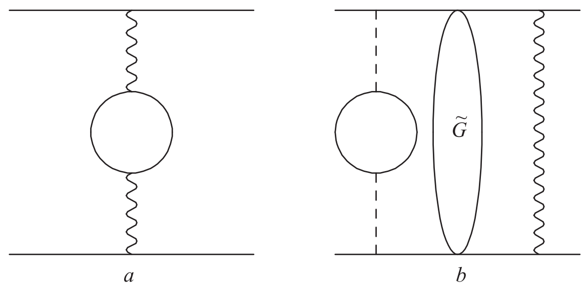

Fig. 1. One-loop vacuum polarization contributions to the fine and hyperfine structure. The dashed line corresponds to the Coulomb interaction. The wave line corresponds to the fine or hyperfine interaction. $\tilde{G}$ is the reduced Coulomb Green's function

Although the integral in Eq. (7) can be calculated analytically, we present here for simplicity only its numerical value.

Higher order corrections $\alpha^2(Z\alpha)^4$ entering in the a_μ are taken into account in this expression as well as the recoil effects. The same order contribution $\alpha(Z\alpha)^4$ can be obtained in the second-order perturbation theory (see Fig. 1, b). In this case the energy spectrum is determined by the reduced Coulomb Green's function [13, 24]:

$$G_{2P}(\mathbf{r},\mathbf{r}')=-\frac{\mu^2(Z\alpha)}{36z^2z'^2}\left(\frac{3}{4\pi}\mathbf{n}\mathbf{n}'\right)\exp\left(-\frac{z+z'}{2}\right)g(z,z'), \quad (8)$$

$$g(z,z')=24z_<^3+36z_<^3z_>+36z_<^3z_>^2+24z_>^3+36z_<z_>^3+36z_<^2z_>^3+49z_<^3z_>^3-\\-3z_<^4z_>^3-12e^{z_<}(2+z_<+z_<^2)z_>^3-3z_<^3z_>^4+12z_<^3z_>^3\left[-2C+Ei(z_<-\ln z_<-\ln z_>\right], \quad (9)$$

where $z_<=\min(z,z')$, $z_>=\max(z,z')$, $C=0.577216\ldots$ is the Euler constant, $z=Wr$. Using Eqs. (8) and (9), we transform the correction of order $\alpha(Z\alpha)^4$ to the fine structure in the second-order perturbation theory as follows:

$$\Delta E_2^{\rm fs}=-\frac{\alpha(Z\alpha)^4\mu^3}{3456\pi m_1m_2}\left[1+2a_\mu+(1+a_\mu)\frac{2m_1}{m_2}\right]\times\\ \times\int_1^\infty\rho(s)ds\int_0^\infty dx\,\exp\left[-x\left(1+\frac{2m_es}{W}\right)\right]\int_0^\infty\frac{dx'}{x'^2}\,{\rm e}^{-x'}g(x,x')=143.96\ \ \mu{\rm eV}. \quad (10)$$

Note that the coordinate integration in (10) can be done analytically. Let us consider the two-loop vacuum polarization contributions in the one-photon interaction shown in Fig. 2. They give the corrections to the fine splitting of P-levels of order $\alpha^2(Z\alpha)^4$.

In order to obtain the particle-interaction operator for the amplitude, corresponding to the diagram in Fig. 2, a, it is necessary to make the substitution

$$\frac{1}{k^2}\to\frac{\alpha}{3\pi}\int_1^\infty ds\frac{\sqrt{s^2-1}(2s^2+1)}{s^4(k^2+4m_e^2s^2)} \quad (11)$$

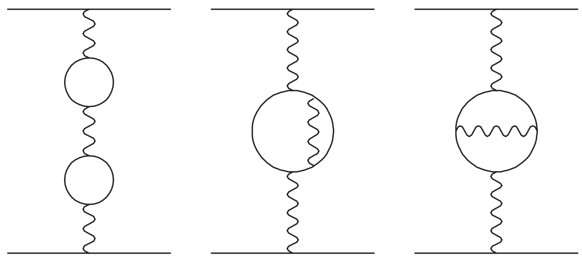

Fig. 2. Effects of two-loop electron vacuum polarization in the one-photon interaction

two times in the photon propagator. In the coordinate representation, the interaction operator has the form [25–27]

$$\Delta V^{\rm fs}_{\rm VP-VP}(r) = \frac{Z\alpha}{r^3}\left[\frac{1+2a_\mu}{4m_1^2}+\frac{1+a_\mu}{2m_1m_2}\right](\mathbf{L}\boldsymbol{\sigma}_1)\times$$
$$\times\left(\frac{\alpha}{3\pi}\right)^2\int_1^\infty \rho(\xi)d\xi\int_1^\infty \frac{\rho(\eta)d\eta}{(\xi^2-\eta^2)}[\xi^2(1+2m_e\xi r)\,\mathrm{e}^{-2m_e\xi r}-\eta^2(1+2m_e\eta r)\,\mathrm{e}^{-2m_e\eta r}]. \quad (12)$$

Averaging (12) over the wave functions (6), we obtain the following correction to the interval (3):

$$\Delta E_3^{\rm fs} = \frac{\mu^3\alpha^2(Z\alpha)^4}{72\pi^2}\left[\frac{1+2a_\mu}{4m_1^2}+\frac{1+a_\mu}{2m_1m_2}\right]\int_1^\infty\rho(\xi)d\xi\int_1^\infty\rho(\eta)d\eta\frac{1}{(\xi^2-\eta^2)}\times$$
$$\times\int_0^\infty x\,dx\left\{\xi^2\left(1+\frac{2m_e\xi}{W}x\right)\exp\left[-x\left(1+\frac{2m_e\xi}{W}\right)\right]-\right.$$
$$\left.-\eta^2\left(1+\frac{2m_e\eta}{W}x\right)\exp\left[-x\left(1+\frac{2m_e\eta}{W}\right)\right]\right\} = 0.20\ \mu\mathrm{eV}. \quad (13)$$

The two-loop vacuum polarization operator is needed to find the 1γ-interaction operator shown in Fig. 2, b, c. The modification of the photon propagator in this case has the form [1]

$$\frac{1}{k^2}\to\frac{2}{3}\left(\frac{\alpha}{\pi}\right)^2\int_0^1\frac{f(v)dv}{4m_e^2+k^2(1-v^2)}, \quad (14)$$

$$f(v) = v\left\{(3-v^2)(1+v^2)\left[\mathrm{Li}_2\left(-\frac{1-v}{1+v}\right)+2\mathrm{Li}_2\left(\frac{1-v}{1+v}\right)+\frac{3}{2}\ln\frac{1+v}{1-v}\ln\frac{1+v}{2}-\right.\right.$$
$$\left.-\ln\frac{1+v}{1-v}\ln v\right]+\left[\frac{11}{16}(3-v^2)(1+v^2)+\frac{v^4}{4}\right]\ln\frac{1+v}{1-v}+\left[\frac{3}{2}v(3-v^2)\ln\frac{1-v^2}{4}-\right.$$
$$\left.\left.-2v(3-v^2)\ln v\right]+\frac{3}{8}v(5-3v^2)\right\}. \quad (15)$$

The two-loop vacuum polarization potential and the correction to the fine structure $(2P_{3/2}-2P_{1/2})$ are the following:

$$\Delta V^{\rm fs}_{\text{2-loop,VP}}(r)=\frac{2Z\alpha^3}{3\pi^2 r^3}\left[\frac{1+2a_\mu}{4m_1^2}+\frac{1+a_\mu}{2m_1m_2}\right]\times \\ \times\int_0^1\frac{f(v)dv}{1-v^2}\,\exp\left(-\frac{2m_er}{\sqrt{1-v^2}}\right)\left(1+\frac{2m_er}{\sqrt{1-v^2}}\right)(\mathbf{L}\boldsymbol{\sigma}_1), \quad (16)$$

$$\Delta E^{\rm fs}_4=\frac{\mu^3\alpha^2(Z\alpha)^4}{12\pi^2}\left[\frac{1+2a_\mu}{4m_1^2}+\frac{1+a_\mu}{2m_1m_2}\right]\times \\ \times\int_0^\infty x\,dx\int_0^1\frac{f(v)\,dv}{1-v^2}\,\exp\left[-x\left(1+\frac{2m_e}{W\sqrt{1-v^2}}\right)\right]\left(1+\frac{2m_e}{W\sqrt{1-v^2}}x\right)=0.78\ \mu\text{eV}. \quad (17)$$

Two-loop vacuum polarization contributions in the second-order perturbation theory shown in Fig. 3 have the same order $\alpha^2(Z\alpha)^4$. For their calculation it is necessary to employ relations (2), (4), (5), and (8), and the modified Coulomb potential by the two-loop vacuum polarization [9, 11]:

$$\Delta V^C_{\rm VP-VP}(r)=\left(\frac{\alpha}{\pi}\right)^2\int_1^\infty\rho(\xi)\,d\xi\int_1^\infty\frac{\rho(\eta)\,d\eta}{\xi^2-\eta^2}\left(-\frac{Z\alpha}{r}\right)(\xi^2\,{\rm e}^{-2m_e\xi r}-\eta^2\,{\rm e}^{-2m_e\eta r}), \quad (18)$$

$$\Delta V^C_{\text{2-loop,VP}}=-\frac{2Z\alpha}{3r}\left(\frac{\alpha}{\pi}\right)^2\int_0^1\frac{f(v)\,dv}{1-v^2}\,\exp\left(-\frac{2m_er}{\sqrt{1-v^2}}\right). \quad (19)$$

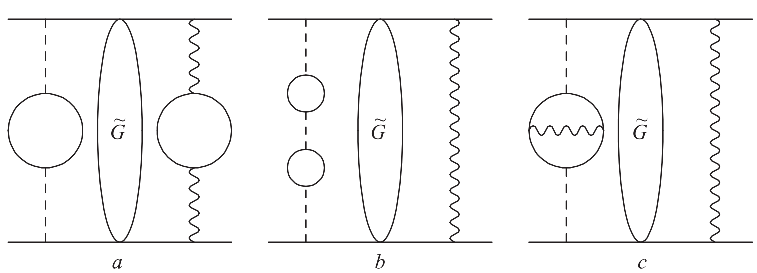

Fig. 3. Effects of two-loop electron vacuum polarization in the second-order perturbation theory. The dashed line corresponds to the Coulomb interaction. The wave line corresponds to the fine or hyperfine interaction. $\tilde G$ is the reduced Coulomb Green's function

The amplitude in Fig. 3, a gives the following correction of order $\alpha^2(Z\alpha)^4$ to the fine splitting:

$$\Delta E_5^{\rm fs} = \frac{\mu^3\alpha^2(Z\alpha)^4}{1296\pi^2}\left[\frac{1+a_\mu}{2m_1m_2}+\frac{1+2a_\mu}{4m_1^2}\right]\int_1^\infty \rho(\xi)\,d\xi\int_1^\infty \rho(\eta)\,d\eta\times$$
$$\times\int_0^\infty dx\,\exp\left[-x\left(1+\frac{2m_e\xi}{W}\right)\right]\int_0^\infty\frac{dx'}{x'^2}\left(1+\frac{2m_e\eta x'}{W}\right)\times$$
$$\times\exp\left[-x'\left(1+\frac{2m_e\eta}{W}\right)\right]g(x,x') = 0.28\ \mu\text{eV}. \quad (20)$$

We integrate (20) analytically over coordinates x, x' and numerically over parameters ξ, η. Two other contributions from the amplitudes in Fig. 3, b, c have a similar integral structure. Their numerical values are included in the table.

There exists the correction to the fine splitting due to the nuclear structure. In 1γ interaction it is related with the charge form factor of the α particle. The fine structure potential (2) is obtained in the point nuclear approximation. To generalize (2) to the case of the nucleus of the finite size, we can use the following potential in momentum representation:

$$\Delta V^{\rm fs}_{\rm str}(\mathbf{k}) = -\frac{\pi Z\alpha}{m_1^2}\frac{i[\mathbf{k}\times\mathbf{p}]\boldsymbol{\sigma}_1}{\mathbf{k}^2}F_1(k^2)\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]. \quad (21)$$

Using in (21) the dipole parameterization for the Dirac form factor $F_1(k^2)$ with the parameter $\Lambda_\alpha = \sqrt{12}/r_\alpha$, we can express the contribution of the nuclear structure to the fine splitting in the form

$$\Delta E^{\rm fs}_{\rm str} = \frac{\mu^5(Z\alpha)^6}{32m_1^2\Lambda_\alpha^2}\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]\left(-6+20\frac{W}{\Lambda_\alpha}\right) = -11.76\ \mu\text{eV}. \quad (22)$$

Numerical value of (22) is obtained using the charge radius of α particle $r_\alpha = 1.681(4)$ fm [28]. One part of the nuclear structure correction in the second-order PT $2\langle\Delta V^{\rm fs}\cdot\tilde G\cdot\Delta V^C_{\rm str}\rangle$ is determined by the potential (2) and

$$\Delta V^C_{\rm str}(r) = \frac{Z\alpha}{2r}(\Lambda_\alpha r+2)\,e^{-\Lambda_\alpha r}, \quad (23)$$

which also is obtained by means of the dipole parameterization for the charge form factor of the α particle in the Coulomb part of the potential. As a result, we find the fine structure contribution in the form

$$\Delta E^{\rm fs}_{1,\rm str} = \frac{5\mu^6(Z\alpha)^7}{16m_1^2\Lambda_\alpha^3}\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]\left(\frac{12}{5}\ln\frac{W}{\Lambda_\alpha}+\frac{29}{10}\frac{W}{\Lambda_\alpha}-1\right) = -0.07\ \mu\text{eV}. \quad (24)$$

Second part of the nuclear structure correction in second-order PT $2\langle\Delta V^{\rm fs}\cdot\tilde G\cdot\Delta V^{\rm fs}_{\rm str}\rangle$ is related with the potentials (2) and (21). Performing analytical integration over particle coordinates, we arrive at the following result:

$$\Delta E^{\rm fs}_{2,\rm str} = \frac{9\mu^6(Z\alpha)^7}{64m_1^4\Lambda_\alpha}\left[1+\frac{2m_1}{m_2}+2a_\mu\left(1+\frac{m_1}{m_2}\right)\right]^2\left(1-\frac{2W}{3\Lambda_\alpha}\ln\frac{W}{\Lambda_\alpha}-\frac{4W}{3\Lambda_\alpha}\right) = 0.52\ \mu\text{eV}. \quad (25)$$

The sum of the results (24) and (25) is presented in the table. The total result for the fine splitting $\Delta E^{\rm fs}$ in $(\mu_2^4{\rm He})^+$ is presented here as well. It takes into consideration the numerous earlier performed calculations discussed in the review article [1] and new corrections obtained in this work.

2. SUMMARY AND CONCLUSION

In the present study we have calculated QED effects in the fine structure of the P-wave energy levels in muonic helium ion $(\mu_2^4{\rm He})^+$. We have considered the electron vacuum polarization contributions of orders α^5, α^6, recoil corrections, relativistic effects of order α^6 and the nuclear structure corrections. The total numerical value of the fine splitting is presented in the table. In this table we give the references to other papers also devoted to the investigation of the fine structure of P-wave levels in the hydrogenic atoms.

Let us summarize the basic points of the calculation performed above.

1. Special attention in our investigation has been concentrated on the vacuum polarization effects. For this purpose we obtain the terms of the interaction operator in muonic helium ion which contain the one-loop and two-loop vacuum polarization corrections.

2. In each order in α we have taken into account recoil effects in the terms proportional to the ratio m_1/m_2.

3. The calculation of the nuclear structure corrections to the fine structure interval is performed on the basis of the dipole parameterization for the α-particle charge form factor.

The theoretical error of the obtained results is determined by the contributions of higher order and amounts up to 10^{-6}. Previously, the fine splitting in the ion of muonic helium was studied in [4]. Considering the numerical results obtained in [4] for different transitions $(2S{-}2P)$, we find that the fine splitting interval is equal to $\Delta E^{\rm fs} = (146.2 \pm 0.6)$ meV. Extracting the leading-order nuclear structure correction proportional to r_α^2, we can present the result of our work $\Delta E^{\rm fs} = 146.193 - 4.2124 \cdot r_\alpha^2 = (146.181 \pm 0.001)$ meV which agrees with and refines the previous calculation performed in [4] via taking into account higher order effects. It can be considered as a reliable estimate of the fine structure interval for the P-wave levels in muonic helium ion $(\mu_2^4{\rm He})^+$. In order to present here the transition frequencies between $2P$- and $2S$-states, we need the value of the Lamb shift obtained in [25]. Our calculation of the Wichmann–Kroll correction in [25] contains the error as noted in [29]. New value of the Wichmann–Kroll correction is equal to $\Delta E^{\rm WK}(2P{-}2S) = -0.0199$ meV. Then corresponding energy intervals in the fine structure of muonic helium ion $(\mu_2^4{\rm He})^+$ are $\Delta E(2P_{1/2}{-}2S_{1/2}) = 1381.561$ meV and $\Delta E(2P_{3/2}{-}2S_{1/2}) = 1527.742$ meV. A measurement of $(2P{-}2S)$ transition frequencies with the 50 ppm precision combined with the present theoretical prediction and the result of [25] will lead to the determination of the $_2^4{\rm He}$ radius to a relative accuracy $3 \cdot 10^{-4}$.

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