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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

MULTIPLE OPTICAL INTERACTIONS FOR QUANTUM GATES

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We present results of quasi-phase matching (QPM) interactions in one-dimensional multilayered media consisting of layers with different $\chi^{(2)}$ nonlinearities which are interchanged by linear dispersive layers. We exploit the idea of manipulating overall group delay mismatches between the various fields in each layer by appropriate choosing of the dispersive parameters and consider both multiple optical QPM interactions and preparation of pure photon states in application to quantum gates.

Представлены результаты исследования квазифазового синхронизма в одномерной среде, состоящей из слоев с различными оптическими нелинейностями, которые чередуются дисперсионными слоями. Используется идея управления фазового синхронизма между взаимодействующими электромагнитными полями в каждом из слоев путем дисперсии. Рассмотрены многочастичные взаимодействия, основанные на квазифазовом синхронизме, и приготовление чистых фотонных состояний в применении к квантовым логическим элементам.

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INTRODUCTION

Quantum communication and optical quantum computation rely on the controlled preparation and manipulation of specific photonic states. Such states are usually prepared in multiple optical QPM interactions realized in $\chi^{(2)}$ nonlinear media [1, 2]. Particularly, preparation of pure multiphotonic states is an important starting point for implementation of many schemes of quantum information processing including efficient scalable quantum computing with single photons, linear optical elements, and projective measurement [3]. Two-qubit unitary gates such as controlled-NOT (CNOT) gate, or controlled phase gates have also been built in this way.

In this paper, we discuss the problem of realizing the strong QPM interactions between photons in multilayered structures on the one hand and analyze production of multiphoton pure states in type-II collinear down-conversion in these structures on the other hand. We focus on the structures consisting of layers with different susceptibilities of nonlinearity and consider detailed description of elementary interactions in each layer, including dispersions of interacting waves.

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Traditionally, the technique of QPM between interacting waves is realized in periodically poled nonlinear crystals (PPNC) or in photonic crystals. In the papers [4, 5] the spectral properties of joint states of photon pairs produced by pulsed parametric down-conversion in a multilayered structures of second-order nonlinear and linear materials have been investigated in application to production of both two-photon states with an arbitrary degree of entanglement as well as pure photonic wave packets. The idea is the manipulation of overall group delay mismatches between the various fields in structured materials. The standard method of production of pure multiphotonic states is the method of conditional measurements in a spontaneous parametric down-conversion (SPDC) generating correlated photon pairs. According to the method of conditional preparation, counting n photons in one of the correlated mode projects the other mode in an n-photon Fock state.

1. MULTIPLE INTERACTIONS IN DYSON SERIES REPRESENTATION

In this section we consider multiple three-wave interaction in $\chi^{(2)}$ medium that is the multilayered structure and write interaction Hamiltonian as

$$H(t) = H^{(-)}(t) + H^{(+)}(t), \tag{1}$$

$$H^{(-)}(t) = \int_0^L dz\chi^{(2)}(z)E_p(z,t)E_1^{(-)}(z,t)E_2^{(-)}(z,t) = \\ = \sum_n \int_{z_n}^{z_{n+1}} dz\chi^{(2)}E_{pn}E_{1n}^{(-)}(z,t)E_{2n}^{(-)}(z,t), \tag{2}$$

where L is the length of the medium and

$$E_{1n}^{(-)}(z,t) = \int \mathrm{e}^{-i(\omega t - k_{1n}(\omega)z)}a^+(\omega)\,d\omega, \quad E_{2n}^{(-)}(z,t) = \int \mathrm{e}^{-i(\omega t - k_{2n}(\omega)z)}b^+(\omega)\,d\omega \tag{3}$$

describe electromagnetic fields of two modes (1) and (2) in each of the layers, and

$$E_{pn}(z,t) = \int f_p(\omega)\,\mathrm{e}^{-i(\omega - k_{pn}(\omega)z)}\,d\omega \tag{4}$$

is the corresponding pump field, while a^+ and b^+ are creation operators for two fields.

We specify our three-photon interaction as leading to type-II parametric down-conversion and investigate the state vector in all order of the perturbation theory as

$$|\psi(t)\rangle = T\exp\left\{-\left(\frac{i}{\hbar}\right)\int_{-\infty}^{t} d\tau(H^{(-)}(\tau) + H^{(+)}(\tau))|0\rangle_1|0\rangle_2\right\}. \tag{5}$$

Particularly, in the second order of the perturbation theory we get

$$|\psi(t)\rangle = (1+\gamma)|0\rangle_1|0\rangle_2 - |\psi^{(1)}(t)\rangle - \frac{1}{2}|\psi^{(1)}(t)\rangle^2, \tag{6}$$

where

$$|\psi^{(1)}(t)\rangle = \int d\omega_1\, d\omega_2\, \Phi(\omega_1,\omega_2) a^+(\omega_1) b^+(\omega_2) |0\rangle_1 |0\rangle_2, \tag{7}$$

γ is the correction to vacuum state and $\Phi(\omega_1,\omega_2)$ is the spectral two-photon amplitude, that is the product of pump field and the effective nonlinear coefficient $G(\Delta k)$:

$$\Phi(\omega_1,\omega_2) = f_L(\omega_1+\omega_2) G(\Delta k). \tag{8}$$

2. PREPARATION OF PURE PHOTON STATES IN PPNC

In this section, we investigate the process of type-II down-conversion in periodically poled nonlinear crystals (PPNC) with dispersive elements. This structure consists of $N/2$ segments of length l_1 with positive χ and negative $-\chi$ susceptibilities and $N/2$ linear optical spacers of length l_2. This approach helps us associate our results with real length, number of slights, coefficients of dispersions, and other parameters relating to real structures. The calculations lead to the following result for the effective nonlinear coefficient [6]:

$$G(\Delta k, \Delta\kappa) = l_1 \chi\, \mathrm{e}^{-i\phi} \sin c\left(\frac{l_1}{2}\Delta k\right) \frac{\sin\left(\dfrac{N(l_1+l_2)\Delta K}{4}\right)}{\sin\left(\dfrac{(l_1+l_2)\Delta K}{4}\right)}. \tag{9}$$

Here we assume two mismatch functions Δk and $\Delta\kappa$ corresponding to nonlinear $n = 1, 3, 5, \ldots$ and $n = 2, 4, \ldots$ segments, $\Delta K = \bar{l_1}\Delta k + \bar{l_2}\Delta\kappa - q_m$, $\Delta k = k_p(\omega_0) - k_1(\omega_1) - k_2(\omega_2)$, $\Delta\kappa = \kappa_p(\omega_0) - \kappa_1(\omega_1) - \kappa_2(\omega_2)$, $|k_p| = (\omega/c)n_p$, $|k_i| = (\omega_i/c)n_i$ $(i = 1,2)$, $\phi = (1/2)l_1\Delta k + (1/2)\Delta K(l_1+l_2)(N/2+1)$. $L = N/2(l_1+l_2)$ is the total length of the medium, $\bar{l_i} = l_i/(l_1+l_2)$ $(i = 1,2)$, $\bar{l_1} + \bar{l_2} = 1$ and $q_m = 2\pi m/d$ is the harmonic grating wave vector, m is an arbitrary odd number, which is specified for a concrete process, $d = 2(l_1+l_2)$. Thus, the total effective interaction coefficient is presented as the product of two separate functions. One describes each individual nonlinear crystal segment and the other function describes the modifications of the QPM function in the superlattice.

Now we will find conditions for preparation of pure photon states, rewriting two-photon amplitude in the Gaussian form and assuming that pump field has the form $f_p(\omega) \approx \left(\frac{\tau_p^2}{2}(\omega - \omega_0)^2\right)$, choosing the phase-matching conditions as $\Delta k^{(0)} = 0$ and $\bar{l}_2\Delta\kappa^{(0)} = q_m$. We look the case of large number of segments $N \gg 1$ and expand phase-matching functions Δk and $\Delta\kappa$ into Taylor series up to the second order. In this case, it is not difficult to get the Gaussian-like form from Eqs. (8) and (9). Note that the two-photon amplitude in the Gaussian form can be factorized as

$$\Phi(\omega_1,\omega_2) = \varphi(\omega_1)\, \phi(\omega_2), \tag{10}$$

where

$$\varphi(\omega_1) = \exp\left[\left(-\frac{i}{2}(\alpha_1+\beta_1)\right)\nu_1^2\right] \times \\ \times \exp\left[\left(-\frac{1}{20}N^2(t_1+T_1)^2 - \frac{1}{2}\tau_p^2 - \frac{1}{6}N(\alpha_1+\beta_1)\right)\nu_1^2\right],$$

$$\phi(\omega_2) = \exp\left[\left(-\frac{i}{2}(\alpha_2+\beta_2)\right)\nu_2^2\right] \times \\ \times \exp\left[\left(-\frac{1}{20}N^2(t_2+T_2)^2 - \frac{1}{2}\tau_p^2 - \frac{1}{6}N(\alpha_2+\beta_2)\right)\nu_2^2\right], \quad (11)$$

under the conditions

$$-\frac{1}{10}N^2(t_1+T_1)(t_2+T_2) - \tau_p^2 = 0, \quad (\alpha_p+\beta_p) = 0. \quad (12)$$

Here

$$T_\mu = \frac{l_1}{2}\left(\left.\frac{dk_0(\omega)}{d\omega}\right|_{\omega=\omega_0} - \left.\frac{dk_\mu(\omega)}{d\omega}\right|_{\omega=\frac{\omega_0}{2}}\right),$$
$$t_\mu = \frac{l_2}{2}\left(\left.\frac{d\kappa_0(\omega)}{d\omega}\right|_{\omega=\omega_0} - \left.\frac{d\kappa_\mu(\omega)}{d\omega}\right|_{\omega=\frac{\omega_0}{2}}\right),$$
$$\beta_\mu = \frac{l_1}{2}\left(\left.\frac{d^2k_0(\omega)}{d\omega^2}\right|_{\omega=\frac{\omega_0}{2}} - \left.\frac{d^2k_\mu(\omega)}{d\omega^2}\right|_{\omega=\frac{\omega_0}{2}}\right), \quad (13)$$
$$\alpha_\mu = \frac{l_2}{2}\left(\left.\frac{d^2\kappa_0(\omega)}{d\omega^2}\right|_{\omega=\frac{\omega_0}{2}} - \left.\frac{d^2\kappa_\mu(\omega)}{d\omega^2}\right|_{\omega=\frac{\omega_0}{2}}\right),$$

$$\beta_p = \frac{l_1}{2}\left.\frac{d^2k_0(\omega)}{d\omega^2}\right|_{\omega=\omega_0}, \quad \alpha_p = \frac{l_2}{2}\left.\frac{d^2\kappa_0(\omega)}{d\omega^2}\right|_{\omega=\omega_0},$$

$\mu = 1, 2$ and $\nu_\mu = \omega_\mu - \omega_0/2$.

These conditions guarantee that all correlations between the signal and idler photons be eliminated and hence mean the preparation of pure photon wave packets. The conditions have been considered in [5], but for the case of nonlinear materials without linear dispersive elements. As we can see, the condition (12) involves dispersive coefficients of both nonlinear and linear segments. For the system we investigate we have additional parameter: group velocity matching function of the dispersive linear element, beside τ_p, which helps us to implement (realize) the condition. The first condition of Eq. (12) could be achieved easily, as the group velocity matching function of the dispersive linear element could have the opposite sign to the group velocity matching function of nonlinear segments, and we always could manage the thickness of the segments. Generally, the terms in the second condition are very small and could be neglected. But if there are materials where these terms are not vanishing, we could satisfy this condition managing the thicknesses of the segments.

In this case, the two-mode state (5) becomes

$$|\psi\rangle = \sum_{n_{\rm even}} \frac{\lambda^{n/2}(A^+B^+)^{n/2}}{(n/2)}|0\rangle_1|0\rangle_2, \tag{14}$$

where A^+ and B^+ are new discreet creation operators, in which the spectral degree of freedom no longer plays a role once integrated out

$$A^+ = \int \varphi(\omega_1)a^+(\omega_1)\,d\omega_1, \quad B^+ = \int \phi(\omega_1)b^+(\omega_2)\,d\omega_2. \tag{15}$$

They satisfy the commutator relations $[A, A^+] = [B, B^+] = 1$. Thus, the two-mode state is expressed in terms of a single Schmidt mode pair. In this case, counting n photons in one of the correlated mode projects the other mode in an n-photon Fock state

$$|\varphi\rangle = \frac{(A^+)^n}{\sqrt{n!}}|0\rangle_1, \quad \text{or} \quad |\phi\rangle = \frac{(B^+)^n}{\sqrt{n!}}|0\rangle_2. \tag{16}$$

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