Письма в ЭЧАЯ. 2011. Т. 8, № 5(168). С. 800–803

SCALING INVARIANCE FOR DISSIPATIVE CHAOS

T. V. Gevorgyan [a], *S. B. Manvelyan* [a], *A. R. Shahinyan* [b,1], *G. Yu. Kryuchkyan* [a,b]

[a] Institute for Physical Research of the National Academy of Sciences, Ashtarak, Armenia

[b] Yerevan State University, Yerevan

It is known that dissipative chaos might satisfy scaling invariance. We discuss this statement from the point of view of quantum-classical transition for a model of anharmonic driven dissipative oscillator with time-modulated parameters. We apply the scaling ideology to study the ranges of chaos in quantum and semiclassical dynamics. Chaotic dynamics is analyzed numerically within the framework of both the Poincaré section in classical description and the Poincaré section of a single trajectory in quantum description. We concentrate on analysis of nontrivial regimes for which the system has chaotic behavior in quantum treatment, while the dynamics is not chaotic in classical description.

Известно, что диссипативный хаос может проявлять свойства масштабной инвариантности. Это положение исследуется с точки зрения квантово-классического перехода для модели ангармонического осциллятора с зависящими от времени параметрами. Масштабная инвариантность также используется для нахождения различных областей хаоса системы. Хаотическая динамика исследуется численно в рамках секции Пуанкаре при его классическом описании и секции Пуанкаре для одной квантовой траектории — при квантовом описании. Особо исследуются нетривиальные режимы, когда система имеет хаос при квантовом описании, однако при классическом описании хаос отсутствует.

PACS: 05.45.Mt

INTRODUCTION

The problem of quantum chaos has attracted considerable attention from many points of physics, including quantum optics and quantum computation. Particularly, it is evident that chaotic dynamics of the system should destroy entanglement between states but there are some works in this area that show even improvement of entanglement for some specific cases [1,2]. It was also shown that sub-Poissonian statistics of oscillatory numbers is improved for the model of dissipative anharmonic oscillator in chaotic regime [3]. Quantum chaos is also interesting for investigating quantum-classical correspondence [4] which is fundamental problem in quantum mechanics. Recently, it was shown in [5] that chaos is induced by quantum effects on the case of a Duffing oscillator.

In this paper, we continue the discussion of chaotic dynamics for the dissipative anharmonic oscillator with time-dependent amplitude of driving force following the papers [3,6].

[1]E-mail: anna_shahinyan@ysu.am

We concentrate on the scaling invariance properties of the system in the chaotic regime on the one hand, and investigate scenarios to develop chaotic regime from the regular one due to quantum effects on the other hand. It is known that quantum chaos is usually treated as a quantized classical chaotic system. We mention here the counterargument considering the situation when dynamics of the system is regular in the semiclassical treatment, while in quantum one it is chaotic for the certain range of the parameters. For numerical analysis we use the quantum state diffusion method [7] and as an indicator of quantum chaotic dynamics we used Poincaré section based on single quantum trajectory [8].

1. MODEL OF THE SYSTEM

We treat dissipation and decoherence microscopically using a master equation which is solved numerically in the framework of QSD approach [8]. The model is a periodically driven anharmonic oscillator for which the evolution of reduced density matrix is governed by the master equation:

$$\frac{d\rho}{dt} = \frac{-i}{\hbar}[H_0 + H_{\rm int}, \rho] + \sum_{i=1,2}\left(L_i\rho L_i^+ - \frac{1}{2}L_i^+ L_i\rho - \frac{1}{2}\rho L_i^+ L_i\right).$$

In the interaction picture the Hamiltonian has the following formulation:

$$\begin{gathered} H_0 = \hbar\Delta a^+ a, \\ H_{\rm int} = \hbar\chi(a^+a)^2 + \hbar((f_0 + f_1\, \exp{(-i\delta t)})a^+ + (f_0 + f_1\, \exp{(-i\delta t)})^* a). \end{gathered} \tag{1}$$

Here $\Delta = \omega_0 - \omega$ is the detuning; $\delta = \omega_0 - \omega_1$ is the difference between driving frequencies, which works as modulation frequency; a and a^+ are boson annihilation and creation operators; L_i are the Lindblad operators,

$$L_1 = \sqrt{(N+1)\gamma}a, \quad L_2 = \sqrt{N\gamma}a^+, \tag{2}$$

γ is the spontaneous decay rate of the dissipation process and N denotes the mean number of quanta of a heat bath. The couplings with two driving forces are given by Rabi frequencies f_0 and f_1, χ is the strength of anharmonicity. Here we focus on the pure quantum effects and assume $N = 0$. In the classical limit the corresponding equation of motion for the dimensionless amplitude is

$$\frac{d\alpha}{dt} = -\frac{\gamma}{2}\alpha - i(\Delta + \chi(1 + 2|\alpha|^2))\alpha - i(f_0 + f_1\, \exp{(-\delta t)}). \tag{3}$$

This equation is invariant for the scaling transformation of complex amplitude $\alpha = \lambda\alpha$ if the other parameters transforms like: $\Delta \to \Delta' = \Delta + \chi(1 - 1/\lambda^2)$, $\chi \to \chi' = \chi/\lambda^2$, $f \to f' = \lambda f$, $\gamma \to \gamma' = \gamma$. This scaling property leads to symmetry of strange attractor, they have the same form in the phase space and differ only by scale. Strictly speaking, scaling invariance is not valid in quantum limit.

2. RESULTS

At first, we illustrate scaling invariance of chaotic dynamics of dissipative time-modulated anharmonic oscillator (DAO). As has been shown in [9], the system has chaotic dynamics when $f_1 \simeq f_0$ and $\delta \geqslant \gamma$. The Poincaré sections obtained by recording (x, p) at time intervals of $2\pi/\delta$ for this case are shown in Fig. 1 in the form of strange attractors for two sets of scaled parameters. Here DAO is discussed in the vicinity of classically chaotic behavior if the parameters are tuned near the chaotic range, the transformation to regular dynamics might be realized. Indeed, for the considered parameters the system dynamics continues to be chaotic till $f_1/\gamma = 4.9$, while chaotic dynamics became regular when $f_1/\gamma = 4.8$.

We test quantum dynamics through Poincaré section of single QSD trajectory which is plotted in Fig. 2, *a*. It is evident that the system behavior is regular. Indeed, as we see in Fig. 2, *a*, the stochastic points in phase space are distributed regularly near two ranges. Calculations show that the Poincaré section of the system for the same parameters in semiclassical treatment also consists of two points in contrast to Fig. 1. Now, the scaling invariance of the semiclassical Eq. (3) is used to get the chaotic behavior ranges. The Poincaré section in Fig. 2, *c* displays chaotic dynamics of the system for the scaling parameter $\lambda = 2$. As is seen from unlikeness of the sections in Figs. 2, *a* and *c* the scaling invariance is violated and the quantum-classical correspondence is lost due to quantum noise effects. It is interesting that the shape of Poincaré section in Fig. 2, *c* qualitatively coincides with Poincaré sections from Figs. 1, *a* and *b*, which indicates that the quantum chaos in classically regular case is similar to classical chaos in chaotic range of parameters. We can conclude that in quantum limit system dynamics continues to be chaotic although its classical dynamics is regular. It is connected to quantum basic concept of uncertainty principle. In spite of the ideology that chaos is mainly classical concept, it exists in quantum system, while the same system in classical limit is regular. So using scaling invariance, we found the system parameters when dynamics continues to be chaotic in quantum treatment.

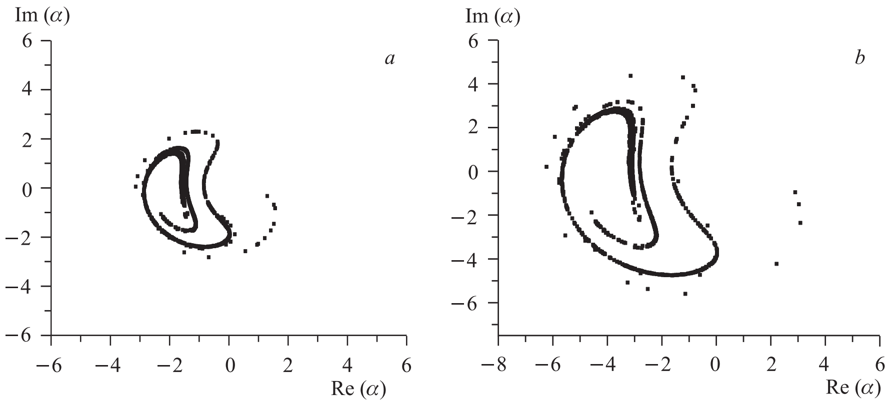

Fig. 1. The Poincaré section for the following parameters: $\Delta/\gamma = -15$, $\chi/\gamma = 2$, $f_0/\gamma = 5.8$, $f_1/\gamma = 4.9$, $\delta/\gamma = 3$ (*a*); for the scaled $\lambda = 2$ parameters (*b*). The Poincaré sections are generated for time evaluation 5000 force periods

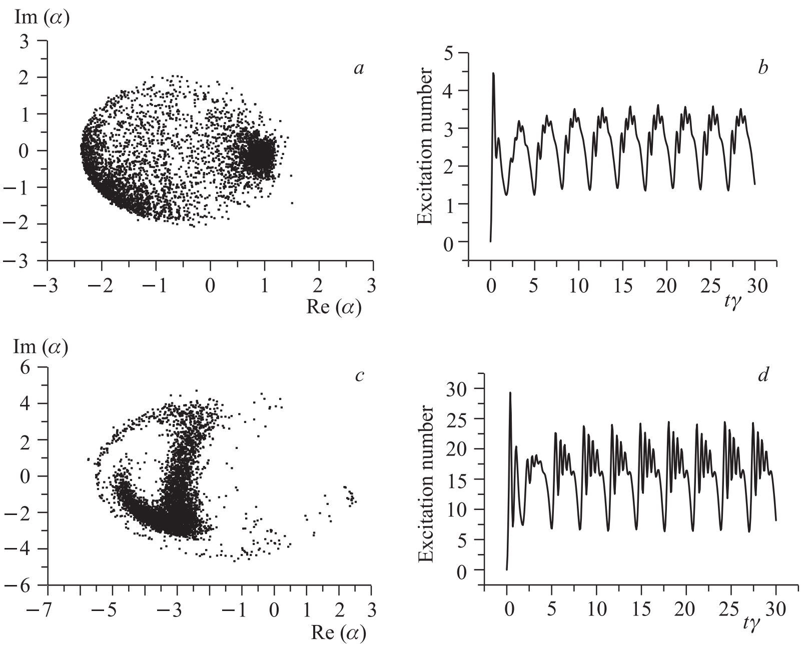

Fig. 2. The Poincaré section calculated on the basis of a single trajectory for the following parameters: $\Delta/\gamma = -15$, $\chi/\gamma = 2$, $f_0/\gamma = 5.8$, $f_1/\gamma = 4.8$, $\delta/\gamma = 3$ (*a*); the excitation number for the same parameters (*b*). Panels *c* and *d* correspond to scaled $\lambda = 2$ parameters. The Poincaré sections are generated for time evaluation 5000 force periods

Acknowledgements. This work was supported by grants NFSAT/CRDF UCEP-02/07, ISTC A-1451 and A-1606.

REFERENCES

1. *Furuya K. et al.* // Phys. Rev. Lett. 1998. V. 80. P. 5524.
2. *Miller P. A., Sarkar S.* // Phys. Rev. E. 1999. V. 60. P. 1542.
3. *Kryuchkyan G. Yu., Manvelyan S. B.* // Phys. Rev. A. 2003. V. 68. P. 013823.
4. *Ota Yu., Ohba I.* // Phys. Rev. E. 2005. V. 71. P. 015201(R).
5. *Kapulkin A., Pattanayak A. K.* // Phys. Rev. Lett. 2008. V. 101. P. 074101.
6. *Gevorgyan T. V., Shahinyan A. R., Kryuchkyan G. Yu.* // Phys. Rev. A. 2009. V. 79. P. 062905.
7. *Gisin N., Percival I. C.* // J. Phys. A. 1992. V. 25. P. 5677; 1993. V. 26. P. 2233; 2245; *Percival I. C.* Quantum State Diffusion. Cambridge: Cambridge Univ. Press, 2000.
8. *Brun T. A., Percival I. C., Schack R.* // J. Phys. A. 1996. V. 29. P. 2077.
9. *Adamyan H. H., Manvelyan S. B., Kryuchkyan G. Yu.* // Phys. Rev. E. 2001. V. 64. P. 046219.
10. *Adamyan H. H., Manvelyan S. B., Kryuchkyan G. Yu.* // Phys. Rev. A. 2003. V. 68. P. 013823.