

## SIMPLE ESTIMATES OF NON-FEMTOSCOPIC PARTICLE CORRELATIONS IN $p + p$ COLLISIONS

*S. V. Akkelin, Yu. M. Sinyukov*

Bogolyubov Institute for Theoretical Physics, Kiev

A simple model of non-femtoscopic particle correlations in proton–proton collisions is proposed. The model takes into account correlations induced by the conservation laws as well as correlations induced by minijets. It gives reasonable description of the non-femtoscopic correlation functions of identical pions that have been obtained from the PHOJET event generator simulations of proton–proton collision events at  $\sqrt{s} = 900$  GeV and utilized as correlation baseline by the ALICE collaboration.

PACS: 13.75.Cs; 13.85.-t

### INTRODUCTION

The two-particle femtoscopy of identical particles allows one to analyze the space-time structure of the particle emission from the systems created in heavy-ion, hadron and lepton collisions (for reviews, see, e.g., [1]). It is established that specific transverse momentum dependence of the femtoscopy scales — interferometry, or HBT radii — in heavy-ion collisions is caused by the collective expansion of the systems [2]. As for elementary particle collisions, like  $p + p$ , where the collective (hydrodynamic) behavior of the matter is open to quest, there is no unambiguous interpretation of  $p_T$ -dependence of the HBT radii. Moreover, the transverse momentum behavior of the HBT radii extracted from hadron and lepton collisions depend on correlation baseline assumption [3–5] about strength and momentum dependence of the non-femtoscopic correlations. The latter appears also between unlike particles and are, typically, long-range in momentum space. As opposite to the femtoscopic (HBT) correlations, they are not conditioned by the quantum statistics and are not directly related to the spatiotemporal scales of the emitter or to the well-studied Coulomb and strong final-state interactions. These correlations can appear because of various reasons, e.g., the well-known example of the non-femtoscopic correlations is the correlations induced by the energy- and momentum-conservation laws (see, e.g., [6]). The correlations do not affect essentially the HBT radii extracted from heavy-ion collisions, but are rather noticeable for elementary particle collisions.

Our aim here is to develop simple analytical model that can be relevant for  $p + p$  collisions, and then demonstrate that this model can fit the non-femtoscopic correlations of identical pions obtained from the PHOJET event generator [7] simulations of proton–proton collision events at  $\sqrt{s} = 900$  GeV, the latter has been utilized as the correlation baseline by the ALICE collaboration [3,4].

## 1. DEFINITIONS AND PARAMETERIZATIONS OF TWO-PARTICLE CORRELATIONS

The two-particle correlation function is defined as

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}, \quad (1)$$

where  $P(p_1, p_2)$  is the probability to observe two particles with momenta  $p_1$  and  $p_2$ , while  $P(p_1)$  and  $P(p_2)$  designate the single-particle probabilities. Experimentally, the two-particle correlation function is defined as the ratio of the distributions of the pairs from the same event and pairs of particles from the different events. In heavy-ion collisions, almost all the correlations between identical pions with low relative momentum are due to quantum statistics (QS) and final-state (FS) interactions. Then

$$C(p_1, p_2) = C_F(\mathbf{p}, \mathbf{q}), \quad (2)$$

where  $\mathbf{p} = (\mathbf{p}_1 + \mathbf{p}_2)/2$ ,  $\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1$ , and  $C_F$  denotes the femtoscopic correlation function. In the case of identical bosons  $C_F$  is often parameterized neglecting FS correlations by the Gaussian form, which for one-dimensional parametrization looks like

$$C_F(|\mathbf{p}|, q_{\text{inv}}) = 1 + \lambda \exp(-R_{\text{inv}}^2 q_{\text{inv}}^2). \quad (3)$$

Here  $\lambda$  describes the correlation strength,  $R_{\text{inv}}$  is the Gaussian «invariant» HBT radius, and  $q_{\text{inv}} = \sqrt{(\mathbf{p}_2 - \mathbf{p}_1)^2 - (E_2 - E_1)^2}$  is equal to the modulus of the momentum difference in the pair rest frame.

In elementary particle collisions, additional (non-femtoscopic) correlations, like those arising from jet/string fragmentation and from energy and momentum conservation (see, e.g., [3–5]), can also give essential contribution. Then, assuming the factorization property,

$$C(p_1, p_2) = C_F(\mathbf{p}, \mathbf{q})C_{\text{NF}}(\mathbf{p}, \mathbf{q}). \quad (4)$$

Here  $C_{\text{NF}}$  denotes the non-femtoscopic correlation function, and in the simplest case non-femtoscopic effects can be parameterized in terms of function  $C_{\text{NF}}(|\mathbf{p}|, q_{\text{inv}})$  that can be fitted as, e.g., 2nd order polynomial:

$$C_{\text{NF}}(|\mathbf{p}|, q_{\text{inv}}) = a + bq_{\text{inv}} + cq_{\text{inv}}^2. \quad (5)$$

This form can be used together with some parameterization of  $C_F$  (e.g., with (3)) in order to fit correlation function  $C(p_1, p_2)$  for small systems, as have been done, for example, by the STAR collaboration for two-pion correlation functions in  $p + p$  collisions at  $\sqrt{s} = 200$  GeV [5]. At  $c > 0$  the phenomenological parameterization (5) explicitly reproduces the well-known effect of positive correlations between particles with large relative momenta  $|\mathbf{q}|$  caused by the energy–momentum conservation laws, see the EMCIC model for  $C_{\text{NF}}$  [6]. Note that  $a$ ,  $b$ , and  $c$  in Eq.(5) depend, in general, on  $|\mathbf{p}|$ , and this dependence is the subject of the fit procedure in complicated models with relatively high number of fit parameters.

Recently, the ALICE collaboration utilized PHOJET event generator<sup>1</sup> [7] for an estimate of the correlation baseline (i.e., non-femtoscopic correlation function for identical pions)

---

<sup>1</sup>PHOJET event generator accounts for soft and hard processes, takes into account energy and momentum conservation, and does not include the effects of quantum statistics.

under the Bose–Einstein peak [3]. It was motivated by the reasonable agreement of the PHOJET event generator simulations with the experimental data for correlation functions of oppositely charged pions in proton–proton collisions at  $\sqrt{s} = 900$  GeV LHC energy<sup>2</sup>. Similar to the results for nonidentical pion correlation functions, the correlation baseline simulated by the PHOJET at relatively low  $q_{inv}$  decreases with  $q_{inv}$  for relatively high  $p_T$  and demonstrates approximately flat in  $q_{inv}$  behavior for low  $p_T$ . It was conjectured in [3,4] that such behavior is conditioned by correlations induced by minijets created in the PHOJET event generator simulations. In what follows, we propose simple analytical model with the minimal number of parameters for the non-femtoscopic correlations induced by minijets and conservation laws that can reproduce the above-mentioned results and allows one to see clearly the physical mechanisms responding for peculiarities of the  $|\mathbf{p}|$ - and  $q_{inv}$ -behavior of the correlation baseline [3].

## 2. ANALYTICAL MODEL FOR MINIJETS AND MOMENTUM-CONSERVATION INDUCED CORRELATIONS

Let us assume that  $N$  particles of the same species (say, pions) are produced with momenta  $p_1, \dots, p_N$  in multiparticle production events. Then the single-particle probability,  $P(p_1)$ , in such events can be written as

$$P(p_1) = \frac{1}{K} \int \frac{d^3 p_2}{E_2} \dots \frac{d^3 p_N}{E_N} \delta(p_1, \dots, p_N) |M_N(p_1, p_2, \dots, p_N)|^2, \quad (6)$$

where  $\delta(p_1, \dots, p_N)$  denotes constraints on the  $N$ -pion states that appear due to energy and momentum conservations in multiparticle production events,  $M_N(p_1, p_2, \dots, p_N)$  is  $N$ -pion production amplitude, and  $K$  is normalization factor,

$$K = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \dots \frac{d^3 p_N}{E_N} \delta(p_1, \dots, p_N) |M_N(p_1, p_2, \dots, p_N)|^2. \quad (7)$$

Aimed to calculate the non-femtoscopic correlations of identical particles, one can define the two-particle probability,  $P(p_1, p_2)$ , in a similar way as the single-particle one, see Eq. (6). Then the two-particle non-femtoscopic correlation function,  $C_{NF}$ , where quantum statistics is ignored, is defined by Eq. (1). Note that at the multiparticle production the final  $N$ -pion states are accompanied by the additionally produced other particle species. Therefore, one can assume that the total transverse momentum of  $N$  registered particles is equal to zero in the system's center of mass, and neglect at the same time the exact conservation of energy and longitudinal momentum, supposing that the system under consideration is barely  $N$ -pion subsystem in a small midrapidity region of the total system. Then

$$\delta(p_1, \dots, p_N) = \delta^{(2)}(\mathbf{p}_{T1} + \mathbf{p}_{T2} + \dots + \mathbf{p}_{TN}), \quad (8)$$

---

<sup>2</sup>Note that because of the different resonance contributions, the unlike-sign pion pairs cannot be directly used for calculation of the correlation baseline for identical pion correlations.

where  $\mathbf{p}_{T1}, \mathbf{p}_{T2}, \dots, \mathbf{p}_{TN}$  are transverse components of the momenta of the  $N$  particles. Then, utilizing the integral representation of the delta function by means of the Fourier transformation,  $\delta^{(2)}(\mathbf{p}_T) = (2\pi)^{-2} \int d^2 r_T \exp(i\mathbf{r}_T \mathbf{p}_T)$ , one can get

$$P(p_1) = \frac{1}{(2\pi)^2 K} \int d^2 r_T e^{i\mathbf{r}_T \mathbf{p}_{T1}} G(\mathbf{p}_1, \mathbf{r}_T), \quad (9)$$

$$P(p_1, p_2) = \frac{1}{(2\pi)^2 K} \int d^2 r_T e^{i\mathbf{r}_T (\mathbf{p}_{T1} + \mathbf{p}_{T2})} G(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T), \quad (10)$$

where

$$G(\mathbf{p}_1, \mathbf{r}_T) = \int \frac{d^3 p_2}{E_2} \dots \frac{d^3 p_N}{E_N} e^{i\mathbf{r}_T (\mathbf{p}_{T2} + \dots + \mathbf{p}_{TN})} |M_N(p_1, p_2, \dots, p_N)|^2, \quad (11)$$

$$G(\mathbf{p}_1, \mathbf{p}_2, \mathbf{r}_T) = \int \frac{d^3 p_3}{E_3} \dots \frac{d^3 p_N}{E_N} e^{i\mathbf{r}_T (\mathbf{p}_{T3} + \dots + \mathbf{p}_{TN})} |M_N(p_1, p_2, \dots, p_N)|^2. \quad (12)$$

A natural assumption based on the physical background of PHOJET [7] is that the non-femtoscopic correlations generated by the PHOJET can be caused by the jet-like and energy-momentum conservation induced correlations. Note that if the only correlations are the correlations associated with transverse momentum conservation, we have

$$|M_N(p_1, p_2, \dots, p_N)|^2 = f(p_1) f(p_2) \dots f(p_{N-1}) f(p_N), \quad (13)$$

and the calculation in large  $N$  limit results in special case of the EMCIC parameterization [6] of the correlations induced by the energy- and momentum-conservation laws. To check our assumption as for non-femtoscopic correlations generated by the PHOJET, let us assume that there are no other correlations in the production of  $N$ -pion states except correlations induced by transverse momentum conservation and cluster (minijet) structures in momentum space. For the sake of simplicity, we assume here that the only two-particle clusters appear, and that  $N \gg 1$  is high enough, allowing us not to distinguish  $N$  and  $N \pm 1$ .

A possibility of different cluster configurations of particles means, in particular, that registered particles with momenta  $p_1$  and  $p_2$  can belong either to different minijets or to the same minijet. To illustrate the corresponding combinatorics, let us consider minijet as basket and pion as ball. Then there are  $(N/2)^2$  possibilities for two balls (pions) to be distributed between  $N/2$  baskets (minijets), and among these possibilities there are  $N/2$  possibilities for two balls to fall into the same basket, and  $((N/2)^2 - N/2)$  possibilities for the ones to fall into different baskets. Then, accounting for such a combinatorics,  $|M_N(p_1, \dots, p_N)|^2$  in Eqs. (11) and (12) can be written as

$$|M_N(p_1, \dots, p_N)|^2 = \left(1 - \frac{2}{N}\right) |M_N^{1\text{jet}}(p_1, \dots, p_N)|^2 + \frac{2}{N} |M_N^{1\text{jet}}(p_1, \dots, p_N)|^2, \quad (14)$$

where

$$\begin{aligned} |M_N^{1\text{jet}}(p_1, \dots, p_N)|^2 &= \\ &= f(p_1) Q(p_1, p_2) f(p_2) f(p_3) Q(p_3, p_4) f(p_4) \dots f(p_{N-1}) Q(p_{N-1}, p_N) f(p_N), \end{aligned} \quad (15)$$

$$\begin{aligned} |M_N^{2\text{jet}}(p_1, \dots, p_N)|^2 &= \\ &= f(p_1) Q(p_1, p_3) f(p_3) f(p_2) Q(p_2, p_4) f(p_4) \dots f(p_{N-1}) Q(p_{N-1}, p_N) f(p_N), \end{aligned} \quad (16)$$

and  $Q(p_i, p_j)$  denotes the correlations between  $p_i$  and  $p_j$ , existence of such correlations means that squared modulo of  $N$ -particle amplitude cannot be expressed as a product of one-particle distributions. Then the two-particle probability function can be decomposed into the two parts:

$$P(p_1, p_2) = \left(1 - \frac{2}{N}\right) P_{2\text{jet}}(p_1, p_2) + \frac{2}{N} P_{1\text{jet}}(p_1, p_2), \quad (17)$$

the first term in the right-hand side of Eq.(17) is associated with events where the two registered particles appear from the different minijets, and the second term corresponds to events when the particles belong to the same minijet. Evidently, the latter happens relatively rarely; however, notice that the second term can be significant for small systems with not very large  $N$ .

### 3. RESULTS AND DISCUSSION

Now let us check whether this model can reproduce with reasonable parameters the non-femtoscopic correlation functions of identical pions that are generated in PHOJET simulations and utilized as the correlation baseline by the ALICE collaboration [3]. Calculations within the model will be deliberately oversimplified just to demonstrate its viability. We do not use here the approximate methods like the saddle point approach, instead we utilize appropriate analytical parameterizations of the functions of interest, namely,

$$f(p_i) = E_i \exp\left(-\frac{\mathbf{p}_{i,T}^2}{T_T^2}\right) \exp\left(-\frac{\mathbf{p}_{i,L}^2}{T_L^2}\right) \quad (18)$$

and

$$Q(p_i, p_j) = \exp\left(-\frac{(\mathbf{p}_i - \mathbf{p}_j)^2}{\alpha^2}\right), \quad (19)$$

where  $T_T$ ,  $T_L$  and  $\alpha$  are some parameters, and in what follows we assume that  $T_L \gg T_T$ . In accordance with ALICE baseline obtained from the PHOJET event generator simulations, we assume that only  $q_{\text{inv}}$  is measured for each  $\mathbf{p}_T$  bin. Assuming that longitudinal components of the registered particles are equal to zero,  $p_{1L} = p_{2L} = 0$ , we approximate  $q_{\text{inv}}^2$  as

$$q_{\text{inv}}^2 \approx \mathbf{q}_T^2 \left( \frac{m^2 + \mathbf{p}_T^2 \sin^2 \phi}{m^2 + \mathbf{p}_T^2} \right), \quad (20)$$

where  $\phi$  denotes unregistered angle between  $\mathbf{p}_T$  and  $\mathbf{q}_T$ ,  $\mathbf{p}_T \mathbf{q}_T = |\mathbf{p}_T| |\mathbf{q}_T| \cos \phi$ . Then

$$C_{\text{NF}}(|\mathbf{p}_T|, q_{\text{inv}}) = \frac{\int_0^{2\pi} d\phi P(p_1, p_2)}{\int_0^{2\pi} d\phi P(p_1) P(p_2)} \quad (21)$$

and, taking into account Eq.(17), we get

$$C_{\text{NF}}(|\mathbf{p}_T|, q_{\text{inv}}) = C_{\text{NF}}^{2\text{jet}}(|\mathbf{p}_T|, q_{\text{inv}}) + C_{\text{NF}}^{1\text{jet}}(|\mathbf{p}_T|, q_{\text{inv}}), \quad (22)$$

where

$$C_{\text{NF}}^{2\text{jet}}(|\mathbf{p}_T|, q_{\text{inv}}) = \left(1 - \frac{2}{N}\right) \frac{\int_0^{2\pi} d\phi P_{2\text{jet}}(p_1, p_2)}{\int_0^{2\pi} d\phi P(p_1)P(p_2)}, \quad (23)$$

$$C_{\text{NF}}^{1\text{jet}}(|\mathbf{p}_T|, q_{\text{inv}}) = \frac{2}{N} \frac{\int_0^{2\pi} d\phi P_{1\text{jet}}(p_1, p_2)}{\int_0^{2\pi} d\phi P(p_1)P(p_2)}. \quad (24)$$

The results of our calculations for non-femtoscopic correlations based on (9)–(19) and (22)–(24) are shown in Figs. 1–5 in comparison with the PHOJET correlation functions for different transverse momentum of pion pairs bins (actually, we performed calculations for the mean value in each bin). The data for PHOJET simulations of identical two-pion

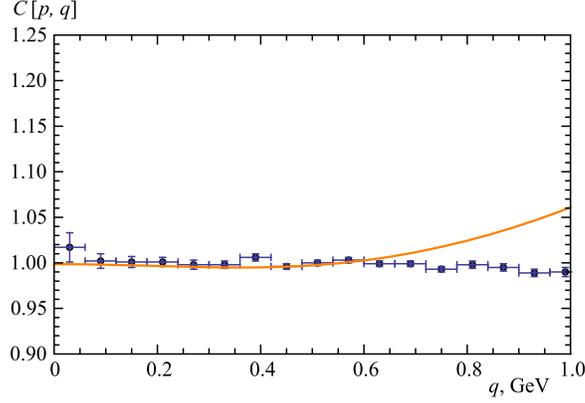


Fig. 1. The non-femtoscopic two-pion correlation functions in  $0.1 < p_T < 0.25$  GeV bin from a simulation using PHOJET [3] (solid dots) and that calculated from the analytical model: minijets + momentum conservation (solid line), see the text for details

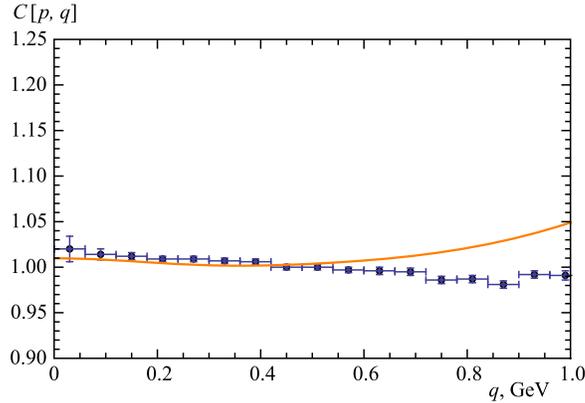


Fig. 2. The same as Fig. 1 but in  $0.25 < p_T < 0.4$  GeV bin

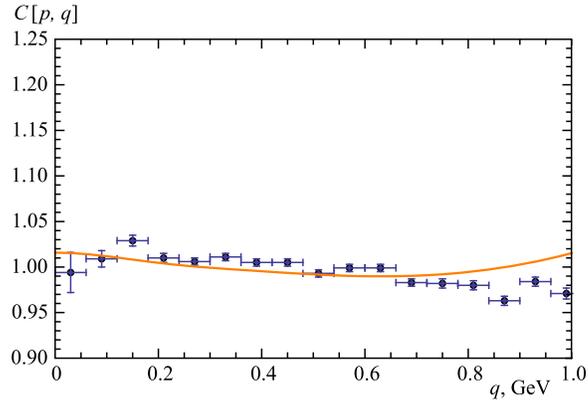


Fig. 3. The same as Fig. 1 but in  $0.4 < p_T < 0.55$  GeV bin

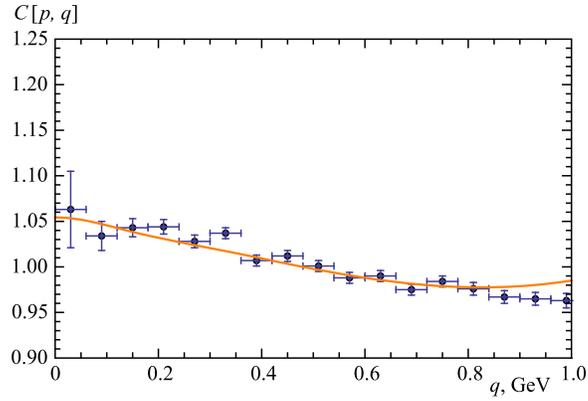


Fig. 4. The same as Fig. 1 but in  $0.55 < p_T < 0.7$  GeV bin

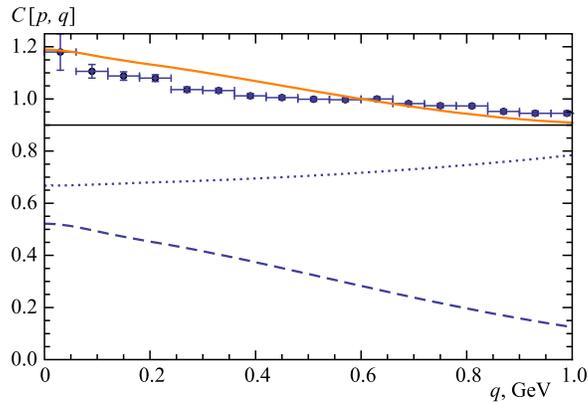


Fig. 5. Above thin horizontal line: non-femtoscopic two-pion correlation functions in  $0.7 < p_T < 1.05$  GeV bin from a simulation using PHOJET [3] (solid dots) and that calculated from the analytical model (solid line). Below thin horizontal line: relative contribution to non-femtoscopic correlation function by  $C_{\text{NF}}^{2\text{jet}}$  (dotted line) and  $C_{\text{NF}}^{1\text{jet}}$  (dashed line)

non-femtoscopic correlation functions at midrapidity for  $N \geq 12$  charged particle multiplicities bin in  $p + p$  collisions at  $\sqrt{s} = 900$  GeV are taken from [3] and [8]. To minimize the number of fit parameters, we fixed  $T_T = \alpha$  for all calculations. Our results are obtained for  $N = 17$ ,  $T_T = \alpha = 0.75$  GeV; note that with these parameter values the mean transverse momentum,  $\langle p_T \rangle$ , is  $\langle p_T \rangle \approx 0.67$  GeV. One can see from the figures that the main trends of the PHOJET correlation functions are reproduced well despite a simplicity of our model. The behavior of the non-femtoscopic correlation function is a result of the competition of the two trends: an increase of the correlation function with  $q_{inv}$  because of the momentum conservation and a decrease of it due to the fragmentation of one minijet into the registered pion pair. Figure 5 demonstrates also the relative contribution of first and second terms in Eq.(22) to the non-femtoscopic correlation function.

The baseline behavior can significantly affect the  $p_T$ -dependence of the interferometry radii [3,4]. In particular, the decreasing trend in  $q_{inv}$  of the non-femtoscopic correlation functions generated in PHOJET simulations, which manifests itself stronger at larger  $p_T$ , makes the HBT radii become flatter in  $p_T$  [3,4] as compared to HBT radii calculated with  $C_{NF} = 1$ . This could influence interpretation of  $p + p$  collisions. Note, however, that hydrodynamic models, which are very successful in description of heavy-ion collisions and give reasonable description of elementary particle collisions (see, e.g., the recent paper [9] where it was demonstrated that EPOS model<sup>1</sup>, which includes hydrodynamic stage, can describe  $p + p$  collisions at LHC energies), typically do not demonstrate noticeable minijets production at relatively low  $p_T$ . Then the question arises of whether the non-femtoscopic effects are ultimately caused by minijets and conservation laws only, or the similar behavior can be attributed to hydrodynamics also. On the other hand, if such behavior is ultimately related with minijets, it strongly restricts the area of applicability of the corresponding correlation baseline to the processes where the matter, in great extent, is produced through minijets emission. Note that if thermalization takes place in  $p + p$  collisions, then the PHOJET model cannot give adequate description of matter evolution there and utilization of the PHOJET generated correlation baseline for  $p + p$  collisions can be in doubt.

In our opinion, adequate description of unlike pion correlations by the non-femtoscopic correlation functions obtained from PHOJET event generator, see [3], demonstrates that utilization of the PHOJET generator for calculations of the correlation baseline for identical pions could be justified, even while the physics behind PHOJET is, perhaps, not completely adequate for description of  $p + p$  collisions at LHC energies, in particular, for description of particle momentum spectra and multiplicities. We shall demonstrate now that similar behavior of the non-femtoscopic correlation functions in  $p + p$  collisions can also appear in hydrodynamic models if one accounts for event-by-event fluctuating initial conditions for hydrodynamic stage (in hybrid models this stage is matched with subsequent hadronic cascade stage).

Let us give here the illustrative example as for such a possibility. First, note that there are no correlations induced by the exact global energy momentum conservation in hydrodynamic/hybrid models, and corresponding conservation laws are satisfied only in average for particles that are produced at some hypersurface where hydrodynamics is switched off. Then,

---

<sup>1</sup>EPOS model calculates flux tubes that are utilized as initial conditions for hydrodynamic expansion, and the later rare hadronic stage is calculated by means of hadronic cascade model (UrQMD).

one can expect that the only source of the non-femtoscopic correlations in such models is event-by-event fluctuations of initial conditions for hydrodynamic stage. These fluctuations result in fluctuations of the two-particle and single-particle momentum spectra, and, as usual, the effect of fluctuations is more pronounced for small systems. Then

$$|M_N(p_1, p_2, \dots, p_N)|^2 = \sum_i |M_N(p_1, p_2, \dots, p_N; u_i)|^2, \quad (25)$$

where  $M_N(p_1, p_2, \dots, p_N; u_i)$  is  $N$ -particle production amplitude for some  $u_i$ -type of the initial conditions. Let us assume, for the sake of simplicity, uncorrelated particle emissions for each specific initial condition, then

$$|M_N(p_1, p_2, \dots, p_N; u_i)|^2 = H(u_i) f(p_1; u_i) f(p_2; u_i) \cdots f(p_{N-1}; u_i) f(p_N; u_i), \quad (26)$$

where  $H(u_i)$  denotes distribution over initial conditions,  $\sum_i H(u_i) = 1$ . It is convenient to

normalize  $f(p; u_i)$  as follows:  $\int \frac{d^3p}{E} f(p; u_i) = 1$ , then  $K = 1$ , see Eq. (7). Accounting for  $\delta(p_1, \dots, p_N) = 1$  in Eq. (6), the two-particle non-femtoscopic correlation function,  $C_{\text{NF}}$ , then reads

$$C_{\text{NF}}(p_1, p_2) = \frac{\sum_i H(u_i) f(p_1; u_i) f(p_2; u_i)}{\sum_i H(u_i) f(p_1; u_i) \sum_j H(u_j) f(p_2; u_j)}. \quad (27)$$

Evidently, the different type of fluctuations, i.e, the form of distribution  $H(u_i)$ , leads to the different behavior of the non-femtoscopic correlations. To illustrate that fluctuations can lead to the non-femtoscopic correlation functions that are similar to ones induced by minijets, let us consider the toy model where  $H(\mathbf{u}_T) = (\alpha^2/\pi) \exp(-\mathbf{u}_T^2 \alpha^2)$ , and  $f(p; \mathbf{u}_T) = \frac{\beta^2 \gamma}{\pi^{3/2}} E \exp(-(\mathbf{p}_T - \mathbf{u}_T)^2 \beta^2) \exp(-p_L^2 \gamma^2)$ , normalization is chosen in such a way that  $\int d^2u_T H(\mathbf{u}_T) = 1$  and  $\int \frac{d^3p}{E} f(p; \mathbf{u}_T) = 1$ <sup>1</sup>. One can easily see that in this case  $C_{\text{NF}}$  decreases with  $q_T^2$ ,

$$C_{\text{NF}}(p, q) \sim \exp\left(-\frac{\beta^4}{2(\alpha^2 + \beta^2)} q_T^2\right), \quad (28)$$

and it means (after taking into account (20) and (21)) that  $C_{\text{NF}}$  decreases with  $q_{\text{inv}}^2$  too, which is similar to the behavior of  $C_{\text{NF}}$  if the non-femtoscopic correlations are induced by minijets.

## CONCLUSIONS

We can conclude that noticeable non-femtoscopic two-pion correlations can appear for small systems as a result of cluster (minijet) structures in final momentum space of produced

---

<sup>1</sup>Note that such momentum spectra fluctuations can take place if, e.g., zero transverse flow does not coincide with maximal energy density.

particles, or as a result of event-by-event fluctuating initial conditions for hydrodynamic stage, another source of non-femtoscopic correlations is global energy–momentum conservation constraints. The latter typically results in an increase with  $q_{inv}$  for fairly high  $q_{inv}$  of the non-femtoscopic two-pion correlation functions of small systems, whereas the former mostly determines a decrease of the ones at relatively low  $q_{inv}$ . We presented here the simple analytical model that takes into account correlations induced by the transverse momentum conservation as well as minijets, and show that the model gives reasonable description of the correlation baseline for identical pions [3], the latter has been obtained from the PHOJET event generator simulations of proton–proton collision events at  $\sqrt{s} = 900$  GeV.

Although details of particle production processes affect the non-femtoscopic correlations, the different types of multiparticle production could result in qualitatively similar non-femtoscopic correlation functions. We presented some heuristic arguments that the non-femtoscopic correlation functions of identical pions calculated in hydrodynamics with event-by-event fluctuating initial conditions can be qualitatively similar at relatively low  $q_{inv}$  to ones calculated in PHOJET-like generators where the non-femtoscopic correlations for low  $q_{inv}$  are mainly caused by the minijets.

**Acknowledgements.** The research was carried out partially within the scope of the EUREA: European Ultra Relativistic Energies Agreement (European Research Group: «Heavy Ions at Ultrarelativistic Energies») and is supported by the State Fund for Basic Research of Ukraine (Agreement of 2011) and National Academy of Sciences of Ukraine (Agreement of 2011).

#### REFERENCES

1. Lisa M. et al. // *Ann. Rev. Nucl. Part. Sci.* 2005. V. 55. P. 357;  
Lisa M., Pratt S. arXiv:0811.1352 [nucl-ex];  
Chajęcki Z. // *Acta Phys. Polon. B.* 2009. V. 40. P. 1119.
2. Makhlin A. N., Sinyukov Yu. M. // *Z. Phys. C.* 1988. V. 39. P. 69;  
Sinyukov Yu. M. // *Nucl. Phys. A.* 1989. V. 498. P. 151.
3. Aamodt K. et al. (*ALICE Collab.*) // *Phys. Rev. D.* 2010. V. 82. P. 052001.
4. Aamodt K. et al. (*ALICE Collab.*). arXiv:1101.3665 [hep-ex].
5. Aggarwal M. M. et al. (*STAR Collab.*). arXiv:1004.0925 [nucl-ex].
6. Chajęcki Z., Lisa M. // *Phys. Rev. C.* 2008. V. 78. P. 064903;  
Chajęcki Z., Lisa M. // *Phys. Rev. C.* 2009. V. 79. P. 034908.
7. Engel R. // *Z. Phys. C.* 1995. V. 66. P. 203;  
Engel R., Ranft J. // *Phys. Rev. D.* 1996. V. 54. P. 4244.
8. *The Durham HepData Project.* <http://hepdata.cedar.ac.uk/>.
9. Werner K. et al. arXiv:1010.0400 [nucl-th].