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THREE-PION CORRELATIONS FOR STUDYING PARTIAL COHERENCE IN NUCLEAR COLLISIONS

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Use of three-particle correlations provides a unique tool for studying partial coherence in relativistic heavy-ion collisions. Here a theory is presented for multiple coherent source components of partially coherent pion radiation. Results are given for the relative probabilities of emission of one, two, or three pions. The calculations on the relation between two- and three-pion correlators give some evidence, when compared with available experimental data, for the existence of partial coherence and multiple coherent components in heavy-ion collisions.

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INTRODUCTION

The possibility of observing partial coherence in nuclear collisions has been discussed for over thirty years [1,2]. In principle, the effect could be demonstrated by the zero-momentum-difference intercept of the two-pion correlation function, when observing normalized values below two. However, such an experimental observation of partial coherence is demanded because of problems due to long-lived resonances and particle misidentification, making extrapolation of the data to the zero momentum difference a very challenging problem in practice.

A proposed solution to the particle misidentification problem is measurement of genuine correlations of three pions and normalization of the result by the genuine two-pion correlation function to the power $3/2$ [3]. Long-lived resonances and particle misidentification then affect similarly both the numerator and denumerator and thus the normalized three-particle correlator r_3 obtained in this way offers an unbiased estimate of partial coherence: a measured value of $r_3/2 < 1$ would provide the long-searched signature of partial coherence in the pion radiation from heavy-ion collisions. In fact, there already exist experimental data from which values $r_3/2 < 1$ have been extrapolated to zero momentum difference [4–7]. However, the extrapolation method used in these analyses has been questioned with a conclusion that there is not yet evidence of partial coherence in heavy-ion collisions, but it might be obtained by three-pion correlations and proper analysis of either existing or future experimental data [8].

If the partial coherence in pion radiation is eventually demonstrated, the source properties producing this radiation will become under study. This work contributes to the understanding of properties of such particle radiation. The conventional analysis, which assumes one coherent source current, is extended here to cover the case of multiple coherent source currents.

There is no a priori reason why only one coherent component should exist. On the contrary, results on photon radiation in the research field of free-electron lasers, analogous to pion radiation in heavy-ion collisions, suggest the existence of multiple coherent components [9–12]. Although the extension of the theory to multiple coherent source components is straightforward in principle, the analysis should carefully account a number of contributions and provide proper observables to reveal details of the searched coherence effects [13]. Finally, it should be emphasized that tentative application of the developed theory, assuming that the extrapolation biases of [8] are negligible, suggests existence of multiple coherent source components in partially coherent pion radiation.

1. MULTIPLE COHERENT COMPONENTS

Emission of photons from a synchrotron radiation source provides an example of a collision process with known interactions between the colliding constituents consisting of a bunch of electrons and a spatially varying magnetic field. Transverse acceleration of the electrons causes a source current producing an emitted radiation field which can be coupled back to the motion of the electrons by self-amplified spontaneous emission. The free-electron laser (FEL) is operated according to that scheme, where it has been shown by theoretical calculations that a definite number N of independent processes takes place as given by the ratio of the electron-bunch length to the electron-cooperation length [9]. As predicted, the measured spectra of single FEL pulses contain approximately N peaks, although their positions and intensities vary [11, 12]. Furthermore, it has been shown that the latter finding is a general property of pulsed radiation which is not depending on laser operation [14].

For the analysis of the radiation source with multiple coherent components, it is assumed that the source current is described by [10]

$$J(x) = \sum_{j=1}^{N} n_j J_0(x - x_j) + \sum_{j'=1}^{N'} J_0'(x - x_{j'}), \tag{1}$$

where the first term consists of N independent elementary processes of n_j $(j = 1, 2, \ldots, N)$ particles radiating coherently. The second term with primed symbols describes incoherent radiation from N' elementary processes of single radiating particles. The elementary source currents J_0 $[J_0']$ are localized around the space-time points x_j $[x_{j'}]$ with a characteristic probability distribution $\varrho(x_j)$ $[\varrho'(x_{j'})]$ for coherent [incoherent] processes.

In analogy with photon correlation experiments, probabilities $P_1(p_1)$, $P_2(p_1, p_2)$, and $P_3(p_1, p_2, p_3)$ describe emission of one, two, and three pions or photons with momenta p_i $(i = 1, 2, 3)$ [2, 10, 15]. Their explicit form is calculated by averaging products of the four-dimensional Fourier transforms $\tilde{J}_0(p), \tilde{J}_0'(p)$ of the elementary source currents over the distributions of the source coordinates $x_j, x_{j'}$, the number of independent processes N, N', and the intensities n_j^2. For the sake of completeness, the results are reproduced here for the lowest order probabilities [13]:

$$P_1(p) = \left\langle \sum_{j=1}^{N} n_j^2 \right\rangle |\tilde{J}_0(p)|^2 + \langle N' \rangle |\tilde{J}_0'(p)|^2, \tag{2}$$

where the averages over N, N', and n_j^2 are denoted by $\langle \ldots \rangle$, and

$$P_2(p_1,p_2) = P_1(p_1)P(p_2) + \left\langle \sum_{j_1} \sum_{j_2 \neq j_1} n_{j_1}^2 n_{j_2}^2 \right\rangle |\tilde{J}_0(p_1)|^2 |\tilde{J}_0(p_2)|^2 |\tilde{\varrho}(q_{12})|^2 + \\ + \left\{ \left\langle \sum_{j=1}^{N} n_j^2 \right\rangle \langle N' \rangle \tilde{J}_0(p_1)\tilde{J}_0(p_2)^* \tilde{J}_0'(p_1)^* \tilde{J}_0'(p_2) \tilde{\varrho}(q_{12}) \tilde{\varrho}'(q_{12})^* + \text{c.c.} \right\} + \\ + \langle N' \rangle^2 |\tilde{J}_0'(p_1)|^2 |\tilde{J}_0'(p_2)|^2 |\tilde{\varrho}'(q_{12})|^2, \quad (3)$$

where $\tilde{\varrho}(q_{12}) = \int d^4x_j \exp(iq_{12}x_j)\varrho(x_j)$, $q_{12} = p_1 - p_2$ and +c.c. stands for complex conjugate of the term inside the braces. The corresponding equation for the three-particle probability is

$$P_3(p_1,p_2,p_3) = P_1(p_1)P_1(p_2)P_1(p_3) + \{P_2(p_1,p_2)P_1(p_3) - P_1(p_1)P_1(p_2)P_1(p_3) + \text{c.p.}\} + \\ + \left\{ \left\langle \sum_{j_1} \sum_{j_2 \neq j_1} \sum_{j_3 \neq j_1, j_2} n_{j_1}^2 n_{j_2}^2 n_{j_3}^2 \right\rangle |\tilde{J}_0(p_1)|^2 |\tilde{J}_0(p_2)|^2 |\tilde{J}_0(p_3)|^2 \tilde{\varrho}(q_{12}) \tilde{\varrho}(q_{23}) \tilde{\varrho}(q_{31}) + \text{c.c.} \right\} + \\ + \left\{ \left\langle \sum_{j_1} \sum_{j_2 \neq j_1} n_{j_1}^2 n_{j_2}^2 \right\rangle \langle N' \rangle \tilde{J}_0(p_1) |\tilde{J}_0(p_2)|^2 \times \right. \\ \left. \times \tilde{J}_0(p_3)^* \tilde{J}_0'(p_1)^* \tilde{J}_0'(p_3) \tilde{\varrho}(q_{12}) \tilde{\varrho}(q_{23}) \tilde{\varrho}'(q_{31}) + \text{c.p.} + \text{c.c.} \right\} + \\ + \left\{ \left\langle \sum_{j} n_j^2 \right\rangle \langle N' \rangle^2 \tilde{J}_0(p_1) \tilde{J}_0(p_2)^* \tilde{J}_0'(p_1)^* \tilde{J}_0'(p_2) |\tilde{J}_0'(p_3)|^2 \times \right. \\ \left. \times \tilde{\varrho}(q_{12}) \tilde{\varrho}'(q_{23}) \tilde{\varrho}'(q_{31}) + \text{c.p.} + \text{c.c.} \right\} + \\ + \{\langle N' \rangle^3 |\tilde{J}_0'(p_1)|^2 |\tilde{J}_0'(p_2)|^2 |\tilde{J}_0'(p_3)|^2 \tilde{\varrho}'(q_{12}) \tilde{\varrho}'(q_{23}) \tilde{\varrho}'(q_{31}) + \text{c.c.}\}, \quad (4)$$

where +c.p. stands for two cyclically permuted terms $(p_1 \to p_2,\ p_2 \to p_3,\ p_3 \to p_1) + (p_1 \to p_3,\ p_2 \to p_1,\ p_3 \to p_2)$ inside the braces. Equation (4) has been simplified as in deriving Eq. (3) where all contributions related to the coherent part were kept and the incoherent part was approximated by $\langle N'^2 \rangle = \langle N'(N'-1) \rangle = \langle N' \rangle^2$.

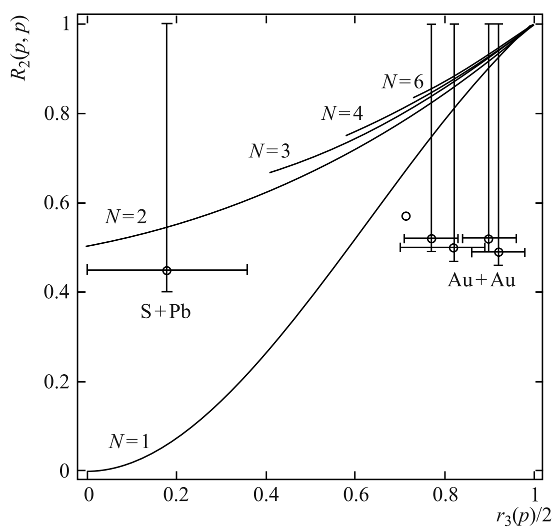
2. NORMALIZED TWO- AND THREE-PION CORRELATORS

The zero-momentum-difference intercept of the two-pion correlation function is obtained as the two-pion correlator $R_2(p,p) = [P_2(p,p) - P_1(p)^2]/P_1(p)^2$, where the contribution of random single-pion events $P_1(p)^2$ is removed from $R_2(p,p)$ to include only the genuine two-pion effects. Similarly the normalized three-pion correlator is given by $r_3(p) = R_3(p,p,p)/[R_2(p,p)]^{3/2}$, where the contributions by single- and two-pion events are removed in $R_3(p,p,p)$ [3]. The latter contributions consist of the two first terms of Eq. (4). If the distribution of the number of coherent processes is sufficiently narrow, it can be assumed that $N \simeq \langle N \rangle$ is a fixed integer. Then [13]

$$\frac{r_3(p)}{2} = \frac{3R_2(p,p) - 2 + 2[1 - R_2(p,p)]^{3/2}\sqrt{N}}{[R_2(p,p)]^{3/2}} \tag{5}$$

gives the quantitative relation between the normalized three-pion correlator and the two-pion correlator.

The two-pion correlator $R_2(p,p)$ cannot be obtained directly from two-pion correlation measurements, since biasing by long-lived resonances, particle misidentification, experimental binning effects, and final-state interactions reduces the experimentally observed two-pion correlation. Nevertheless, the experimental two-pion correlations provide important information on the available parameter space by giving a lower limit for the value of $R_2(p,p)$ as shown in the figure. The upper part of the vertical uncertainty bars of the data points extends up to the value $R_2(p,p) = 1$ due to the uncertainty related to the above-mentioned biasing effects. In the normalized three-pion correlator $r_3(p)/2$, the effects of long-lived resonances, particle misidentification, and experimental binning are cancelled [3]. Final-state interactions are the most problematic effect in reliable experimental determination of $r_3(p)/2$. Here theoretical calculations have to be used to estimate the correction. The correction adds some uncertainty to the experimental results, but meaningful data can still be obtained from three-pion cor-



Relation between $r_3(p)/2$ and $R_2(p,p)$ for different numbers N of coherent source components (solid lines) and data from S + Pb [4] and Au + Au [7] collisions. Parameter $R_2(p,p)$ is the zero-momentum-difference intercept of the two-pion correlation function. In the normalized three-pion correlator $r_3(p)/2$, the effects of long-lived resonances, particle misidentification, and experimental binning are cancelled [3]. The result with a single coherent component, used in the analysis by Adams et al. [7], is shown by the curve labeled $N = 1$. Experimental data from S + Pb collisions are in agreement with the curve $N = 2$ [13]

relation measurements [7]. If the normalized three-pion correlator $r_3(p)/2$ and the number of coherent processes are known, Eq. (5) can be used to determine the value of $R_2(p,p)$, as shown in the figure.

3. CONCLUDING REMARKS

Existence of partial coherence in pion radiation produced by relativistic heavy-ion collisions is an interesting research question, which can be tackled using the principles of the analysis presented in this paper. By the analogy of the FEL operation, it can be expected that the partial coherence, if eventually observed, is related to multiple coherent source currents in the heavy-ion collision. Furthermore, a tentative analysis of the presently available experimental data supports the existence of multiple coherent components in $S+Pb$ collisions. For more quantitative results of such a situation, a careful extrapolation method of the experimental data to zero momentum difference should be developed and the theoretical analysis might be improved in such a way that a suitable average of the fixed N curves of the figure is taken.

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