

INITIAL TEMPERATURE AND EoS OF QUARK MATTER VIA DIRECT PHOTONS

M. Csanád, I. Májér

Eötvös University, Department of Atomic Physics, Budapest, Hungary

The time evolution of the quark–gluon plasma created in gold–gold collisions of the Relativistic Heavy Ion Collider (RHIC) can be described by hydrodynamical models. Distribution of hadrons reflects the freeze-out state of the matter. To investigate the time evolution, one needs to analyze penetrating probes, such as direct photon spectra. Distributions of low-energy photons were published in 2010 by PHENIX. In this paper we analyze a 3 + 1 dimensional solution of relativistic hydrodynamics and calculate momentum distribution of direct photons. Using earlier fits of this model to hadronic spectra, we compare photon calculations to measurements and find that the initial temperature of the center of the fireball is at least (519 ± 12) MeV, while for the equation of state we get $c_s = 0.36 \pm 0.02$.

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1. PERFECT FLUID HYDRODYNAMICS

In the last several years it has been revealed that the matter produced in the collisions of the Relativistic Heavy Ion Collider (RHIC) is a nearly perfect fluid [1]; i.e., it can be described with perfect fluid hydrodynamics. There was a long search for exact hydrodynamic models (solutions of the partial differential equations of hydrodynamics) and several models proved to be applicable. There are 1 + 3 dimensional models, as well as relativistic models — but no 1 + 3 dimensional *and* relativistic model has been tested yet. In this paper we extract observables from the relativistic, ellipsoidally symmetric model of [2]. Hadronic observables were calculated in [3]. Here we calculate momentum distribution and elliptic flow of direct photons.

Perfect fluid hydrodynamics is based on local conservation of entropy or number density (n), energy-momentum density (chosen as $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$) for perfect fluids. Here ϵ is energy density, p is pressure and $g^{\mu\nu}$ is the metric tensor, $\text{diag}(-1, 1, 1, 1)$. The conservation equations are closed by the equation of state, which gives the relationship between ϵ and p . Typically $\epsilon = \kappa p$ is chosen. Note that exact, analytic result for hydrodynamic models is that the hadronic observables do not depend on the initial state or the equation of state separately, just through the final state [4]. Thus, we fix the final state from the data and determine initial state parameters from direct photon spectra.

Even though many solve the above equations numerically, there are only a few exact solutions for these equations. See a small review of solutions in [3]. As far as we know, until now there was only one 1 + 3 dimensional relativistic solution investigated: the solution in [2]. Hadronic observables from this solution were computed and compared to data in [3]. The present paper calculates thermal photon observables from this realistic 1 + 3 dimensional model and compares them to data for the first time. Our method is different from numerical calculations: here one can determine the best values of the parameters of the solution by fitting the analytic model results to data.

2. THE ANALYZED SOLUTION

The analyzed solution, as described in [2,3], assumes self-similarity and ellipsoidal symmetry. The ellipsoidal symmetry means that at a given proper time the thermodynamical quantities are constant on the surface of expanding ellipsoids. The ellipsoids are given by constant values of the scale variable $s = r_x^2/X(t)^2 + r_y^2/Y(t)^2 + r_z^2/Z(t)^2$, where $X(t) = \dot{X}_0 t$, $Y(t) = \dot{Y}_0 t$, and $Z(t) = \dot{Z}_0 t$ are time proportional scale parameters (axes of the $s = 1$ ellipsoid), and spatial coordinates are r_x, r_y , and r_z . The velocity-field is a 3D Hubble-type expansion, $u^\mu(x) = x^\mu/\tau$. This means that the solution is accelerationless. Note that similarly to [3], we use transverse expansion u_t and eccentricity ϵ instead of x and y direction expansion rates \dot{X}_0 and \dot{Y}_0 .

The temperature is $T(x) = T_0 (\tau_0/\tau)^{3/\kappa} \nu(s)^{-1}$, the number density $n(x) = n_0 (\tau_0/\tau)^3 \times \nu(s)$ with τ being the proper time and $\nu(s)$ an arbitrary function of s . We choose τ_0 to be the time of the freeze-out, thus T_0 is the central freeze-out temperature. The function $\nu(s)$ is chosen as $\nu(s) = \exp(-bs/2)$, where $b = \left. \frac{\Delta T}{T} \right|_r$ is the temperature gradient. If the fireball is the hottest in the center, then $b < 0$. Note that for the momentum distribution of direct photons, we do not need any kind of density just the temperature distribution.

The picture used in hydro models is that the pre freeze-out (FO) medium is described by hydrodynamics, and the post freeze-out medium is that of observed hadrons. The hadronic observables can be extracted from the solution via the phase-space distribution at the FO. This will correspond to the hadronic final state or source distribution $S(x, p)$. See details about this topic in [3]. It is important to see that the same final state can be achieved with different equations of state or initial conditions [4]. However, as we will see below, the source function of photons is sensitive to the whole time evolution, thus both to initial conditions and equation of state.

For the source function of photon creation we have then

$$S(x, p)d^4x = \mathcal{N} \frac{p_\mu u^\mu(x)}{\exp(p_\mu u^\mu(x)/T(x)) - 1} d^4x, \quad (1)$$

where $\mathcal{N} = g/(2\pi)^3$ (with degeneracy factor g), $p_\mu u^\mu$ is the energy of the photon in the co-moving system (from the Cooper–Frye prefactor). We will use a second-order saddle-point approximation. In this approximation the point of maximal emittivity and Gaussian source widths can be calculated.

3. THERMAL PHOTON OBSERVABLES

The invariant single particle momentum distribution can then be calculated from the source function by integrating on space-time. Our calculated quantity will be $N_1(p_t)$, which is the invariant single particle momentum distribution taken at midrapidity and integrated on the azimuthal angle φ , similarly to [3]. The result on the invariant transverse momentum distribution depends on the initial and final times. We introduce the variable $\xi = t/\tau_0$ (with τ_0 as the freeze-out time), then in terms of ξ the time integration goes from i to 1, if $i = t_i/\tau_0$.

The result is

$$\begin{aligned}
 N_1(p_t) = & \sum_{n=0}^{\infty} (2\pi)^{3/2} \sqrt{\rho_x \rho_y \rho_z} \tau_0^4 T_0 \left(\frac{p_t}{T_0} \right)^{\frac{3-4\kappa}{3}} \frac{\kappa}{3} \frac{B^n}{A^{n+\frac{4\kappa}{3}-\frac{3}{2}}} \times \\
 & \times \left\{ \left[\frac{(\rho_x - 1)^2 + (\rho_y - 1)^2}{4} (a_{0n} + a_{1n}) + \frac{a_{0n} - a_{1n}}{4} \right] \Gamma \left(n + \frac{4\kappa}{3} - \frac{3}{2}, A \frac{p_t}{T_0} \xi^{3/\kappa} \right) \Big|_1^i + \right. \\
 & \left. + \frac{\rho_x^2 + \rho_y^2 + \rho_z^2}{2} a_{0n} A \Gamma \left(n + \frac{4\kappa}{3} - \frac{5}{2}, A \frac{p_t}{T_0} \xi^{3/\kappa} \right) \Big|_1^i \right\}, \quad (2)
 \end{aligned}$$

where we introduced the auxiliary quantities $\rho_x = \kappa / (\kappa - 3 - \kappa b / X_0^2)$, and ρ_y, ρ_z similarly, while $A = 1 - (\rho_x + \rho_y)/4$ and $B = (\rho_x - \rho_y)/4$. Furthermore, a_0, a_1 are the Taylor coefficients of the first two modified Bessel functions:

$$I_0(x) = \sum_{n=0}^{\infty} a_{0n} x^n \quad \text{with} \quad a_0 = \left(1, 0, \frac{1}{4}, 0, \frac{1}{64}, 0, \frac{1}{2304}, 0, \dots \right), \quad (3)$$

$$I_1(x) = \sum_{n=0}^{\infty} a_{1n} x^n \quad \text{with} \quad a_1 = \left(0, \frac{1}{2}, 0, \frac{1}{16}, 0, \frac{1}{384}, 0, \frac{1}{18432}, \dots \right). \quad (4)$$

As the coefficients are strongly decreasing, in real calculations we can restrict ourselves to use only first two of them; i.e., we can make the approximation of $I_0(x) = x$ and $I_1(x) = x^2/2$.

We can also calculate the elliptic flow, the second Fourier coefficient of the invariant single particle momentum distribution, similarly to [3]. The result is

$$\begin{aligned}
 v_2(p_t) = & \sum_{n=0}^{\infty} \frac{(2\pi)^{3/2} \sqrt{\rho_x \rho_y \rho_z}}{N_1(p_t)} \tau_0^4 T_0 \left(\frac{p_t}{T_0} \right)^{\frac{3-4\kappa}{3}} \frac{\kappa}{3} \frac{B^n}{A^{n+\frac{4\kappa}{3}-\frac{3}{2}}} \times \\
 & \times \left\{ \left[\frac{(\rho_x - 1)^2 + (\rho_y - 1)^2}{8} (a_{0n} + 2a_{1n} + a_{2n}) - \right. \right. \\
 & \left. \left. - \frac{a_{0n} - 2a_{1n} + a_{2n}}{4} \right] \Gamma \left(n + \frac{4\kappa}{3} - \frac{3}{2}, A \frac{p_t}{T_0} \xi^{3/\kappa} \right) \Big|_1^i + \right. \\
 & \left. + \frac{\rho_x^2 + \rho_y^2 + \rho_z^2}{2} a_{1n} A \Gamma \left(n + \frac{4\kappa}{3} - \frac{5}{2}, A \frac{p_t}{T_0} \xi^{3/\kappa} \right) \Big|_1^i \right\} / N_1(p_t). \quad (5)
 \end{aligned}$$

Here the I_2 modified Bessel function had to be introduced as well, with coefficients of $a_2 = (0, 0, 1/8, 0, 1/96, 0, 1/3072, 0, \dots)$.

4. COMPARISON TO PHENIX DATA

Hadronic data were described with this model in [3], determining freeze-out (final-state) parameters (expansion rates, freeze-out proper time and freeze-out temperature). We use the parameters of the hadronic fit and leave only the remaining as free parameters. The free parameters will be κ (the equation-of-state parameter) and t_i , the initial time of the evolution.

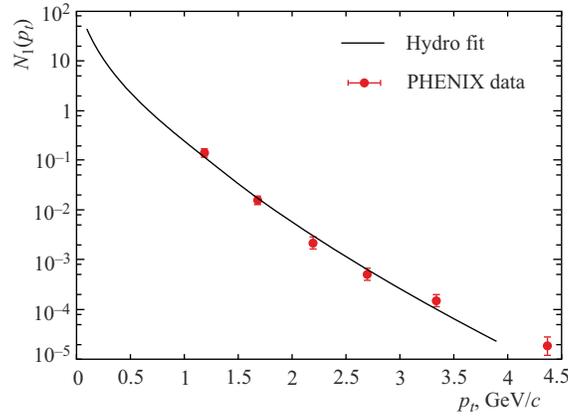
The first five fit parameters were taken from the hadronic fits of [3] (see more details about the parameters therein). The EoS parameter κ was fitted, while for the initial time we determined an interval of acceptability (with 95% confidence). The χ^2 of the fit was 7.0 here, the number of points was 5, so with two fitted parameters our confidence level is 7.2%

Parameter	Value	Type
Central FO temperature T_0 , MeV	204	Fixed
FO proper time τ_0 , fm/c	7.7	Fixed
Eccentricity ϵ	0.34	Fixed
Transverse expansion u_t^2/b	-0.34	Fixed
Longitudinal expansion \dot{Z}_0^2/b	-1.6	Fixed
Equation-of-state κ	7.7 ± 0.7	Free
Initial time t_i , fm/c	0–0.7	Free

We use direct photon data from the PHENIX collaboration [5], measured in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions. Note that we also utilized a normalizing factor to describe the data. Fit parameters are summarized in the table. The fit itself is shown in the figure. The equation-of-state result is $\kappa = 7.7 \pm 0.7$, or alternatively, using $\kappa = 1/c_s^2$:

$$c_s = 0.36 \pm 0.02, \quad (6)$$

which is in nice agreement with both lattice QCD calculations [6] and experimental results from hadronic data [7,8]. Note, however, that the spectrum is not very sensitive to the initial time as in early times the thermal photon emission is not in the region of the experimental data. Our model does not contain acceleration, it is a Hubble-flow type of model, but the initial acceleration does not play a large role in the thermal photon spectrum because of this insensitivity to the initial time. We determined an «interval of acceptability» for t_i . The maximum value for t_i within 95% probability is 0.7 fm/c. This can then be used to determine



Invariant transverse momentum of direct photons from our hydro model. The model validity goes until roughly 3 GeV in transverse momentum

a lower bound for the initial temperature. Thus, the initial temperature of the fireball (in its center) is

$$T_i > (519 \pm 12) \text{ MeV} \quad (7)$$

at 0.7 fm/c. The uncertainty comes from the uncertainty of κ . This is in accordance with other models' 300–600 MeV result [5].

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