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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

A POSSIBLE PROBE OF NEUTRINOLESS DOUBLE-BETA DECAY NUCLEAR MATRIX ELEMENTS

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Future experiments on the search for the $0\nu\beta\beta$ decay will be sensitive to the effective Majorana mass in the region of the inverted mass hierarchy. If a positive signal is observed, a possibility to test models of calculation of nuclear matrix elements of the process will appear. We discuss this possibility in some detail.

Будущие эксперименты по поиску $0\nu\beta\beta$-распада будут чувствительны к эффективной майорановской массе в области иерархии обратных масс. Если будет наблюден положительный сигнал, то возникает возможность проверить модели, в которых вычисляются ядерные матричные элементы данного процесса. В настоящей статье мы обсуждаем эту возможность.

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INTRODUCTION

The discovery of neutrino oscillations in atmospheric [1], solar [2], reactor [3] and accelerator [4, 5] neutrino experiments is one of the most important aspects in particle physics.

Small neutrino masses and neutrino mixing cannot be generated by the Standard Higgs mechanism. A new mechanism is necessary. Many such mechanisms have been proposed. In order to reveal the true nature of neutrino masses and mixing, first of all we need to know whether *neutrinos with definite masses are Majorana or Dirac particles.*

Neutrino oscillations $\nu_l \rightleftarrows \nu_{l'}$ $(l, l' = e, \mu, \tau)$ are an interference phenomenon. Thus, an investigation of neutrino oscillations allows us to determine values of very small neutrino mass-squared differences. However, in neutrino oscillations the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved and their study cannot resolve the problem of the nature of neutrinos ν_i with definite masses m_i [6]. In order to reveal the nature of neutrinos with definite masses, it is necessary to search for processes in which L is violated.

An investigation of the neutrinoless double β decay ($0\nu\beta\beta$ decay),

$$(A, Z) \to (A, Z + 2) + e^- + e^-, \tag{1}$$

of some even–even nuclei (^{76}Ge, ^{130}Te, ^{136}Xe and others) is the most sensitive way to search for a small violation of the total lepton number induced by the exchange of virtual Majorana neutrinos with small masses (see recent reviews [7–9])[1].

In the case of the Majorana neutrino mass mechanism, the half-life of the $0\nu\beta\beta$ decay takes the form [10]

$$\frac{1}{T_{1/2}^{0\nu}(A,Z)} = |m_{\beta\beta}|^2 |M^{0\nu}(A,Z)|^2 G^{0\nu}(E_0,Z). \tag{2}$$

Here,

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \tag{3}$$

is the effective mass of Majorana neutrinos (U_{ei} is an element of the unitary 3×3 Pontecorvo–Maki–Nakagawa–Sakata neutrino mixing matrix [11, 12]), $M^{0\nu}(A,Z)$ is the nuclear matrix element (NME), $G^{0\nu}(E_0,Z)$ is the known phase space factor, which includes also the Fermi function describing the Coulomb interaction of the emitted electrons with the final nucleus and E_0 is the released energy.

The value of the effective Majorana mass depends on the absolute values of the neutrino masses m_i, which are determined by the lightest neutrino mass, the type of the neutrino mass spectrum, the modules $|U_{ei}|$, and the Majorana CP phase differences. Let us stress that because Majorana phases do not enter into the $\nu_l \to \nu_{l'}$ transition probabilities, the measurement of $|m_{\beta\beta}|$ in the $0\nu\beta\beta$-decay experiment is a unique source of information.

The decay rate of the $0\nu\beta\beta$ decay is proportional to the product of the effective Majorana neutrino mass and the nuclear matrix element, which is determined by nuclear properties and does not depend on neutrino masses and mixing. Thus, from the measurement of the $0\nu\beta\beta$-decay half-life only *the product of the effective Majorana neutrino mass and the nuclear matrix element* can be deduced. In order to determine the value of the effective Majorana mass $m_{\beta\beta}$, we need to know the value of NME.

The calculation of the $0\nu\beta\beta$-decay NMEs is a complicated nuclear many-body problem. We briefly discuss this subject below. At present, five different approaches have been applied to such calculations. The results of these calculations differ significantly (for some nuclei by a factor of 2–3). There is no model independent approach to evaluate NMEs. From our point of view, it will be very important to find a way to test the results of different calculations of the $0\nu\beta\beta$-decay NMEs. Such a possibility is discussed in this paper.

1. NUCLEAR MATRIX ELEMENTS OF THE $0\nu\beta\beta$ DECAY

The standard seesaw mechanism of the neutrino mass generation [13] connects the smallness of the neutrino masses with a violation of the total lepton number L at $\sim 10^{14}$ GeV scale. There are three general consequences of the standard seesaw mechanism:

1. Neutrinos with definite masses are Majorana particles.

[1]The sensitivity of the $0\nu\beta\beta$-decay experiments is ensured by a large amount of radioactive isotope (about 1 t or more in future experiments), by possibilities to use low-background underground laboratories, by the high-energy resolution of ^{76}Ge and other detectors, etc.

2. The Majorana neutrino mass mechanism is the only mechanism of the $0\nu\beta\beta$ decay.

3. Heavy Majorana leptons, seesaw partners of neutrinos, must exist.

Possibilities to test the Majorana mass mechanism of the $0\nu\beta\beta$ decay were considered in [14]. The CP-violating decays of Majorana leptons with mass $\sim 10^{14}$ GeV could lead to the baryon asymmetry of the Universe (see [15] and references therein).

Here, we assume that the $0\nu\beta\beta$ decay is due to the exchange of virtual light Majorana neutrinos and consider a possibility to test different models of the calculation of NMEs.

The Majorana mass mechanism of the $0\nu\beta\beta$ decay is based on the Standard Model weak charged current interaction

$$\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} 2 \sum_i \bar{e}_L(x)\gamma_\alpha\ \nu_{eL}(x)\ j^\alpha(x) + \text{h.c.} \tag{4}$$

($j^\alpha(x)$ is the hadronic charged current and G_F is the Fermi constant) and the Majorana neutrino mixing

$$\nu_{eL}(x) = \sum_{i=1}^{3} U_{ei}\nu_{iL}(x). \tag{5}$$

Here,

$$\nu_i^c(x) = C\bar{\nu}_i^T(x) = \nu_i(x) \tag{6}$$

is the Majorana field with the mass m_i[1].

From (4) and (5) it follows that the $0\nu\beta\beta$ decay is a second-order weak process with virtual Majorana neutrinos. For the matrix element of the process we have (see, for example, [8])

$$\langle f|S^{(2)}|i\rangle = i\left(\frac{G_F}{\sqrt{2}}\right)^2 N\bar{u}(p_1)\gamma_\alpha(1-\gamma_5)\gamma_\beta\ C\bar{u}^T(p_2)\int \sum_i U_{ei}^2 m_i\, \mathrm{e}^{ip_1x_1+ip_2x_2}\times$$
$$\times\frac{1}{(2\pi)^4}\int\frac{\mathrm{e}^{-iq\ (x_1-x_2)}d^4q}{p^2-m_i^2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\ d^4x_1\, d^4x_2 - (p_1 \rightleftarrows p_2). \tag{7}$$

Here, p_1 and p_2 are electron momenta; $J^\alpha(x)$ is the weak charged current in the Heisenberg representation; $|N_i\rangle$ and $|N_f\rangle$ are, respectively, the states of the initial and the final nuclei, and $N = N_{p_1}N_{p_2}$ is the product of standard normalization factors $\left(N_p = \dfrac{1}{(2\pi)^{3/2}\sqrt{2p^0}}\right)$ of electron wave functions.

In (7) the integrations over x_1^0, x_2^0 and q^0 can be easily performed. The following well-justified approximations are standard:

1. Long-wave approximation[2] $\mathrm{e}^{-i\mathbf{p}_i\mathbf{x}_i} \simeq 1$, $i = 1, 2$. Thus, only S-states of the emitted electrons are taken into account.

2. In the neutrino propagator neutrino, mass can be neglected[3].

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[1] In Eq. (6) the charge conjugation matrix C satisfies the condition $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha,\ C^T = -C$.

[2] In fact, $|\mathbf{p}_i\mathbf{x}_i| \leqslant p_iR$, where $R \simeq 1.2\cdot 10^{-13}A^{1/3}$ cm is the radius of a nucleus ($i = 1, 2$). Taking into account that $p_i \lesssim$ MeV we have $p_iR \simeq 0.6(p_i/\text{MeV})\cdot 10^{-2}A^{1/3} \ll 1$.

[3] In fact, from uncertainty relation it follows that $\bar{q} \simeq 1/\bar{r} \simeq 100$ MeV, where $\bar{q}$ is the average momentum of the virtual neutrino and $\bar{r} \simeq 1/m_\pi$ is the average distance between nucleons in a nucleus.

3. The closure approximation.

This approximation is based on the fact that the momentum of the virtual neutrino q is much larger than the excitation energy of a nucleus $(E_n - M_{i,f})$ (E_n is the total energy of an excited state, M_i and M_f are masses of initial and final nuclei, respectively). Within the closure approximation we get

$$\sum_n \frac{\langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2))|N_i\rangle}{E_n + p_2^0 + q - M_i} \simeq \frac{\langle N_f|J^\alpha(\mathbf{x}_1)J^\beta(\mathbf{x}_2))|N_i\rangle}{\bar{E} + q - \left(\dfrac{M_i + M_f}{2}\right)}, \tag{8}$$

where $\bar{E}$ is the average energy of the intermediate nuclear states.

Using these approximations, we obtain the following expression for the matrix element of the $0\nu\beta\beta$ decay:

$$\langle f|S^{(2)}|i\rangle = 8\pi i \left(\frac{G_F}{\sqrt{2}}\right)^2 m_{\beta\beta} N_{p_1} N_{p_2} \bar{u}(p_1)(1+\gamma_5)C\bar{u}^T(p_2)\times$$
$$\times \int d^3x_1\, d^3x_1 \langle N_f|J^\alpha(\mathbf{x}_1)K(|\mathbf{x}_1 - \mathbf{x}_2|)J^\alpha(\mathbf{x}_2))|N_i\rangle\delta(E_f + p_1^0 + p_2^0 - E_i), \tag{9}$$

where

$$K(|\mathbf{x}_1 - \mathbf{x}_2|) = \frac{1}{(2\pi)^3}\int \frac{e^{i\mathbf{q}(\mathbf{x}_1 - \mathbf{x}_2)}}{q(\bar{E} + q - \left(\dfrac{M_i + M_f}{2}\right)} d^3q. \tag{10}$$

The effective weak nuclear current $J^\alpha(\mathbf{x}_1) = (J^0(\mathbf{x}_1, \mathbf{J}(\mathbf{x}_1))$ is given by the following approximate expressions [16]:

$$J^0(\mathbf{x}) = \sum_{n=1}^{A} \tau_n^+ \delta(\mathbf{x} - \mathbf{x}_n) g_V(q^2) \tag{11}$$

and

$$\mathbf{J}(\mathbf{x}) = -\sum_{n=1}^{A} \tau_n^+ \delta(\mathbf{x} - \mathbf{x}_n)\left[g_A(q^2)\boldsymbol{\sigma}_n + g_M(q^2) i\frac{\boldsymbol{\sigma}_n \times \mathbf{q}}{2m_p} - g_P(q^2)\frac{(\boldsymbol{\sigma}_n\mathbf{q})\mathbf{q}}{2m_p}\right]. \tag{12}$$

Here, $g_V(q^2), g_A(q^2), g_M(q^2)$ and $g_P(q^2)$ are vector, axial-vector, (weak) magnetic and induced pseudoscalar form factors of the nucleon. From the conserved vector current (CVC) and the partially conserved axial-vector current (PCAC) hypothesis it follows that

$$g_V(q^2) = F_1^p(q^2) - F_1^n(q^2), \quad g_M(q^2) = F_2^p(q^2) - F_2^n(q^2), \quad g_P(q^2) = \frac{2m_p g_A}{q^2 + m_\pi^2}, \tag{13}$$

where $F_1^{p(n)}$ and $F_2^{p(n)}$ are the Dirac and Pauli electromagnetic form factors of the proton (neutron) and g_A is the axial-vector coupling constant of the nucleon.

The expressions (11) and (12) are derived from the one-nucleon matrix element of the weak charged hadronic current. For the number-density of nucleons in a nucleus the following approximate expression is used:

$$\bar{\Psi}(\mathbf{x})\gamma^0\Psi(\mathbf{x}) = \rho(\mathbf{x}) = \sum_{n=1}^{A} \delta(\mathbf{x} - \mathbf{x}_n). \tag{14}$$

Then, from (9) one can determine the nuclear matrix element $M^{0\nu}$, which involves integration over coordinates of nucleons $\mathbf{x}_1$ and $\mathbf{x}_2$ with the integrand containing a product of two hadronic charged currents and the neutrino propagator $K(|\mathbf{x}_1 - \mathbf{x}_2|)$. $M^{0\nu}$ can be written as a sum of the Fermi (F), Gamow–Teller (GT) and the tensor (T) contributions:

$$M^{0\nu} = \langle 0_i^+ | \sum_{kl} \tau_k^+ \tau_l^+ \left[\frac{H_F(r_{kl})}{g_A^2} + H_{\rm GT}(r_{kl})\sigma_{kl} - H_T(r_{kl}) S_{kl} \right] |0_f^+\rangle. \quad (15)$$

Here, $S_{kl} = 3(\boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kl})(\boldsymbol{\sigma}_l \cdot \hat{\mathbf{r}}_{kl}) - \sigma_{kl}$, $\sigma_{kl} = \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l$. The radial parts of the exchange potentials are

$$H_{F,{\rm GT},T}(r_{kl}) = \frac{2}{\pi} R \int\limits_0^\infty \frac{j_{0,0,2}(q r_{kl}) h_{F,{\rm GT},T}(q^2) q}{q + \bar{E} - (M_i + M_f)/2} dq. \quad (16)$$

The functions $h_{F,{\rm GT},T}(q^2)$ are combinations of different form factors and can be found in [17].

The nuclear matrix elements $M^{0\nu}$ must be evaluated using tools of nuclear structure theory. Five different many-body approximate methods have been applied for the calculation of the $0\nu\beta\beta$-decay NME:

1. Interacting Shell Model (ISM). The ISM allows one to consider only a limited number of orbits close to the Fermi level, but all possible correlations within the space are included. Proton–proton, neutron–neutron and proton–neutron (isovector and isoscalar) pairing correlations in the valence space are treated exactly. Proton and neutron numbers are conserved and angular momentum conservation is preserved. Multiple correlations are properly described in the laboratory frame. Monopole corrected G-matrices are used. The Strasbourg–Madrid codes can deal with problems involving basis of 10^{11} Slater determinants, using relatively modest computational resources. A good spectroscopy for parent and daughter nuclei is achieved.

2. Quasiparticle Random Phase Approximation (QRPA). The QRPA has the advantage of large valence space but is not able to comprise all the possible configurations. Usually, single particle states in the Wood–Saxon potential are considered. One is able to include in orbit in the QRPA model space also the spin-orbit partner, which guarantees that the Ikeda sum rule is fulfilled. This is crucial to describe correctly the Gamow–Teller strength. The proton–proton and neutron–neutron pairings are considered. They are treated in the BCS approximation. Thus, proton and neutron numbers are not exactly conserved. The many-body correlations are treated at the RPA level within the quasi-boson approximation. Two-body G-matrix elements, derived from realistic one-boson exchange potentials within the Brueckner theory, are used for the determination of nuclear wave functions.

3. Interacting Boson Model (IBM). In the IBM the low-lying states of the nucleus are modeled in terms of bosons. The bosons are in either $L = 0$ (s boson) or $L = 2$ (d boson) states. Thus, one is restricted to 0^+ and 2^+ neutron pairs transferring into two protons. The bosons interact through one- and two-body forces, giving rise to bosonic wave functions.

4. Projected Hartree–Fock–Bogoliubov Method (PHFB). In the PHFB, wave functions of good particle number and angular momentum are obtained by projection on the axially symmetric intrinsic HFB states. In applications to the calculation of the $0\nu\beta\beta$-decay NMEs the nuclear Hamiltonian was restricted only to quadrupole interaction. The PHFB is restricted in its scope. With a real Bogoliubov transformation without parity mixing, one can describe only neutron pairs with even angular momentum and positive parity.

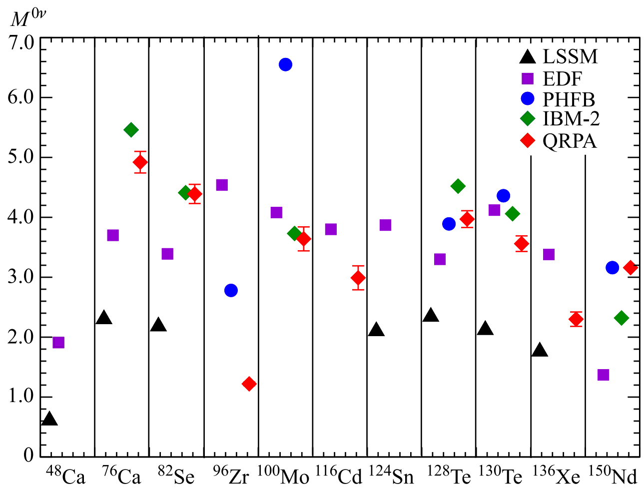

Fig. 1. (Color online) The $0\nu\beta\beta$-decay NMEs calculated within different nuclear structure approaches: interacting Shell Model (ISM) [18], (Renormalized) Quasiparticle Random Phase Approximation (R)QRPA [19, 20], Projected Hartree–Fock–Bogoliubov approach (P-HFB) [21], Interacting Boson Model (IBM) [22], and by Energy Density Functional Method (EDF) [23]. (The EDF results are multiplied by 0.80 in order to account for difference between UCOM and Jastrow.) The Miller–Spencer Jastrow two-nucleon short-range correlations are taken into account. $g_A = 1.25$ and $R = 1.2\,A^{1/3}$ fm are assumed

5. Energy Density Functional Method (EDF). The EDF is considered to be an improvement with respect to the PHFB. The density functional methods based on the Gogny functional are taken into account. The particle number and angular momentum projection for mother and daughter nuclei is performed and configuration mixing within the generating coordinate method is included. A large single particle basis (11 major oscillator shells) is considered.

The main differences among these approaches are in the mean field, residual interaction, size of the model space and in the character of the many-body approximation.

The $0\nu\beta\beta$-decay NMEs calculated within five different approaches are presented in Fig. 1. The values of the MNEs have been obtained with unquenched axial-vector coupling constant g_A ($g_A = 1.25$), Spencer–Miller Jastrow short-range correlations (the EDF values are multiplied by 0.80 in order to account for the difference between UCOM and Jastrow [17]), the same nucleon form factors of dipole shape, higher order corrections to nucleon current and the nuclear radius $R = r_0 A^{1/3}$, with $r_0 = 1.2$ fm (the QRPA values for $r_0 = 1.1$ fm are rescaled with factor $1.2/1.1$).

From Fig. 1 we can make the following conclusions:

1. The ISM values of NMEs, with the exception of the NME for the double magic nucleus $^{48}\mathrm{Ca}$, practically do not depend on the nucleus. In addition, they are significantly smaller when compared with NMEs of other approaches.

2. In the case of $^{130}\mathrm{Te}$ all the discussed methods, with the exception of the ISM, give practically the same result.

3. Ratios of the maximal and minimal values of NMEs calculated in different approaches are equal to 3.1, 2.4, 2.0, 3.7, 1.8, 1.3, 1.8, 1.9, 2.1, 1.9 and 2.3 for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te, ^{136}Xe and ^{150}Nd, respectively.

It is worth noting that due to the theoretical efforts made over the last years the disagreement among different NMEs is now much less severe than it was five years ago. Nevertheless, the present-day situation with the calculation of $0\nu\beta\beta$-decay NMEs cannot be considered as satisfactory. Further progress is required and it is believed that the situation will be improved with time. Nevertheless, taking into account the complexity of the problem of a reliable treatment of many-body nuclear system, we believe that it is important to find a way to cross-check different calculations of the $0\nu\beta\beta$-decay NMEs.

2. EXPERIMENTS ON THE SEARCH FOR THE $0\nu\beta\beta$ DECAY

Numerous experiments have been carried out to search for neutrinoless double-beta decay of many nuclei. No evidence for the $0\nu\beta\beta$ decay was found[1]. The most stringent lower bounds on the half-lives of the $0\nu\beta\beta$ decay were obtained in the Heidelberg–Moscow experiment [26] (10.96 kg germanium 86% enriched in the isotope ^{76}Ge), in the CUORICINO experiment [27] (40.7 kg of TeO_2, 11.2 kg of the isotope ^{130}Te) and in the NEMO experiment [28] (6914 g of enriched ^{100}Mo).

From the data of these experiments the following lower limits on the half-life of the $0\nu\beta\beta$ decay were obtained:

$$\begin{aligned} T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) &> 1.9\cdot 10^{25}\ \mathrm{y},\\ T^{0\nu}_{1/2}(^{130}\mathrm{Te}) &> 1.0\cdot 10^{24}\ \mathrm{y},\\ T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) &> 5.8\cdot 10^{23}\ \mathrm{y}. \end{aligned} \tag{17}$$

Taking into account NMEs of different calculations, the following upper bounds on the effective Majorana mass $|m_{\beta\beta}|$ can be inferred:

$$\begin{aligned} |m_{\beta\beta}| &< 0.20{-}0.32\ \mathrm{eV}, \quad \text{Heidelberg–Moscow}\\ |m_{\beta\beta}| &< 0.30{-}0.71\ \mathrm{eV}, \quad \text{CUORICINO,}\\ |m_{\beta\beta}| &< 0.47{-}0.96\ \mathrm{eV}, \quad \text{NEMO3.} \end{aligned} \tag{18}$$

At present many new experiments on the search for the $0\nu\beta\beta$ decay of ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, and ^{136}Xe are under construction or preparation (see reviews [7, 29]). Some of them will be sensitive only to the quasi-degenerate neutrino mass spectrum. However, according to the analysis made in [30], in the experiments CUORE [31], EXO [32], KamLAND–Zen [33] and NEXT [34] the inverted hierarchy of neutrino masses will be probed.

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[1]Let us notice, however, that some participants of the Heidelberg–Moscow experiment claim the observation of the $0\nu\beta\beta$ decay of ^{76}Ge with half-life in the range $T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) = (1.30{-}3.55)\cdot 10^{25}$ y [24]. The estimated value of the effective Majorana mass is $|m_{\beta\beta}| \simeq 0.17{-}0.45$ eV. This result will be checked by the ^{76}Ge GERDA experiment [25].

3. POSSIBLE TEST OF NME CALCULATION

If we have information about the effective Majorana mass, then from the measurement of the half-life of the $0\nu\beta\beta$ decay we can obtain a value of the corresponding NME. The effective Majorana mass depends on the mixing angles θ_{12} and θ_{13}, the values of the neutrino masses m_i, and the Majorana phase differences.

In the case of mixing of three massive neutrinos the results of neutrino oscillation experiments are compatible with two types of the neutrino mass spectrum:

1. Normal spectrum (NS)

$$m_1 < m_2 < m_3, \quad \Delta m_{12}^2 \ll \Delta m_{23}^2. \tag{19}$$

2. Inverted spectrum (IS)

$$m_3 < m_1 < m_2, \quad \Delta m_{12}^2 \ll |\Delta m_{13}^2|. \tag{20}$$

From the neutrino oscillation data two mass-squared differences are known: the solar $\Delta m_{12}^2 = \Delta m_S^2$ and the atmospheric $\Delta m_{23}^2 = |\Delta m_{13}^2| = \Delta m_A^2$. Thus, absolute values of all neutrino masses depend only on the lightest neutrino mass $m_0 = m_1(m_3)$. Taking into account that $\Delta m_S^2 \ll \Delta m_A^2$, we find

$$\begin{aligned} \text{NS}: &\quad m_2 = \sqrt{m_0^2 + \Delta m_S^2}, \quad m_3 \simeq \sqrt{m_0^2 + \Delta m_A^2}. \\ \text{IS}: &\quad m_1 = \sqrt{m_0^2 + \Delta m_A^2} \simeq m_2. \end{aligned} \tag{21}$$

For the cases of the normal and inverted neutrino mass spectra the dependence of the effective Majorana mass $|m_{\beta\beta}|$ on m_0 is presented in Fig. 2[1].

The present-day experiments are sensitive to the values of $|m_{\beta\beta}|$ that correspond to $m_0 \gg \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV. In this case neutrino masses are practically equal (quasi-degenerate neutrino mass spectrum):

$$m_1 \simeq m_2 \simeq m_3 \simeq m_0 \tag{22}$$

and the effective Majorana mass is given by the expression

$$|m_{\beta\beta}| \simeq m_0 |\cos^2\theta_{13}\cos^2\theta_{12} + \cos^2\theta_{13}\sin^2\theta_{12}\,\mathrm{e}^{2i\alpha_{12}} + \sin^2\theta_{13}\,\mathrm{e}^{2i\alpha_{13}}|, \tag{23}$$

where $\alpha_{1i} = \phi_i - \phi_1$ are (unknown) Majorana phase differences. From (23) we find the region

$$m_0(\cos^2\theta_{13}\cos^2 2\theta_{12} - \sin^2\theta_{13}) \leqslant |m_{\beta\beta}| \leqslant m_0. \tag{24}$$

The region (24) is presented by the area between two parallel lines in the upper part of Fig. 2.

Information about m_0 can be inferred from the data of the future tritium β-decay experiment KATRIN [38] and future cosmological observations (see [37, 39, 40]). The KATRIN experiment will be sensitive to $m_0 \simeq 2 \cdot 10^{-1}$ eV. Cosmological observations allow one to infer the value of the sum of the neutrino masses $\sum_i m_i$. From existing cosmological data

[1]In this plot we took into account the latest T2K value of the parameter $\sin^2\theta_{13}$ [35].

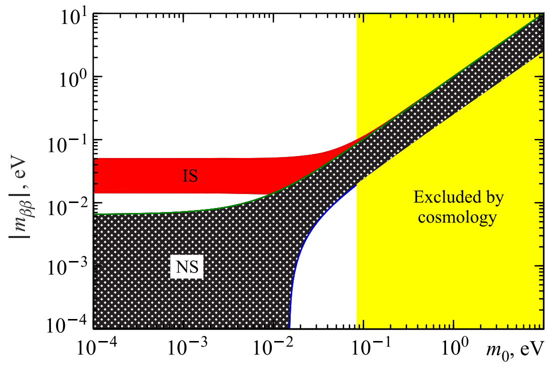

Fig. 2. (Color online) Effective Majorana neutrino mass $|m_{\beta\beta}|$ as a function of the lightest neutrino mass m_0 for the cases of normal (NS, $m_0 = m_1$) and inverted (IS, $m_0 = m_3$) spectrum of neutrino masses. The following values of the neutrino mass-squared differences and mixing angles are used: $\Delta m_A^2 = (2.43 \pm 0.13) \cdot 10^{-3}$ eV^2 [5], $\Delta m_S^2 = (7.65^{+0.13}_{-0.20}) \cdot 10^{-5}$ eV^2 [36], $\tan^2\theta_{12} = 0.452^{+0.035}_{-0.033}$ [3] and $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ [35] for NS (IS). The current limit of $\sum_{i=1}^{3} m_i \leqslant 0.28$ eV [37] for the sum of neutrino masses excludes values of m_0 larger than 0.084 eV

the bound $\sum_i m_i \lesssim 0.5$ eV was obtained [37, 39]. Future different cosmological observations will be sensitive to $\sum_i m_i$ in the range $6 \cdot 10^{-3}$–10^{-1} eV (see, for example, [40]).

If it happens that the neutrino mass spectrum is quasi-degenerate, neutrino oscillation data in combination with data from cosmological and β-decay endpoint measurements will establish the range for $|m_{\beta\beta}|$ and, consequently, will allow one to test models for the calculation of the $0\nu\beta\beta$-decay NMEs.

Future experiments on the search for $0\nu\beta\beta$ decay will probe the inverted mass hierarchy

$$m_3 \ll m_1 < m_2, \quad m_0 \ll \sqrt{\Delta m_A^2}, \quad \sum_i m_i \simeq 2\sqrt{\Delta m_A^2} \simeq 1 \cdot 10^{-1} \text{ eV}. \tag{25}$$

Neglecting a small contribution of the term $U_{e3}^2 m_3$, for the effective Majorana mass we get

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2} \cos^2\theta_{13} (1 - \sin^2 2\,\theta_{12} \sin^2\alpha_{12})^{1/2}. \tag{26}$$

Thus, in the case of the inverted mass hierarchy, the scale of the effective Majorana mass is determined by $\sqrt{\Delta m_A^2}$. From (26) we get

$$\sqrt{\Delta m_A^2} \cos^2\theta_{13} \cos 2\,\theta_{12} \leqslant |m_{\beta\beta}| \leqslant \sqrt{\Delta m_A^2} \cos^2\theta_{13}. \tag{27}$$

For the neutrino oscillation values $\Delta m_A^2 = 2.32^{+0.12}_{-0.08} \cdot 10^{-3}$ eV^2 [5] and $\tan^2\theta_{12} = 0.452^{+0.035}_{-0.033}$ [3] the range of effective Majorana mass in (27) is given by

$$1.7 \cdot 10^{-2} \leqslant |m_{\beta\beta}| \leqslant 5.1 \cdot 10^{-2} \text{ eV}. \tag{28}$$

If it is established by future cosmological measurements that the inverted mass hierarchy is realized in nature ($\sum_i m_i \simeq 1 \cdot 10^{-1}$ eV) and that the Majorana neutrino mass mechanism is the dominant mechanism of the $0\nu\beta\beta$ decay, *the value of* $|m_{\beta\beta}|$ *must be within the range* (28).

From (2) and (27) it follows that the value of the nuclear matrix element $M^{0\nu}$ is restricted by the condition

$$|M^{0\nu}(A,Z)|_{\min} \leqslant |M^{0\nu}(A,Z)| \leqslant |M^{0\nu}(A,Z)|_{\max}, \tag{29}$$

where

$$\begin{aligned} |M^{0\nu}(A,Z)|_{\min} &= \frac{1}{(\Delta m_A^2 \cos^4\theta_{13} T_{1/2}^{0\nu}(A,Z) G^{0\nu})^{1/2}}, \\ |M^{0\nu}(A,Z)|_{\max} &= \frac{1}{(\Delta m_A^2 \cos^4\theta_{13} \cos^2 2\theta_{12} T_{1/2}^{0\nu}(A,Z) G^{0\nu})^{1/2}}. \end{aligned} \tag{30}$$

The inequality (29) will allow us to check models of the calculation of NMEs if in future experiments the $0\nu\beta\beta$ decay is observed.

The range (28) is presented by the horizontal band in Fig. 2. We see that this band is widespread from the inverted mass hierarchy region ($m_0 \ll \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV) up to $m_0 \leqslant 5 \cdot 10^{-2}$ eV (which corresponds to $\sum_i m_i \lesssim 2 \cdot 10^{-1}$ eV). This means that the inequality (29) can be applied to the test of models of the calculation of NMEs even in the case that the sensitivity of the future cosmological data is worse than $\sum_i m_i \simeq 1 \cdot 10^{-1}$ eV.

The inequality (29) can be exploited in the case that the $0\nu\beta\beta$ decay of only one kind of nucleus is observed. More severe test of models of the calculation of NMEs could be performed, if the $0\nu\beta\beta$ decay of several kinds of nuclei is detected. It could happen that an NME calculated in the framework of some model satisfies the inequality (29) for one kind of nucleus, but not for another. Such a situation will disfavor the considered nuclear structure model.

Our final remarks are as follows:

1. If it is found in future cosmological observations that $\sum_i m_i \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV, it would mean that the normal neutrino mass hierarchy

$$m_1 < m_2 \ll m_3, \quad m_0 \ll \sqrt{\Delta m_S^2} \simeq 9 \cdot 10^{-3}\ \text{eV} \tag{31}$$

is realized in nature. For the effective Majorana mass we have in this case

$$|m_{\beta\beta}| \simeq \left| \sqrt{\Delta m_S^2} \cos^2\theta_{13} \sin^2\theta_{12} + \sqrt{\Delta m_A^2} \sin^2\theta_{13}\, \mathrm{e}^{2i\alpha_{23}} \right| \leqslant 3.9 \cdot 10^{-3}\ \text{eV}. \tag{32}$$

We will not discuss this case. Apparently a new technology is needed in order to reach such a small value of $|m_{\beta\beta}|$.

2. According to the standard seesaw mechanism of the neutrino mass generation, which is based on the assumption that the lepton number L is violated at a GUT scale, the only mechanism of the $0\nu\beta\beta$ decay is the Majorana mass mechanism. The inequality (29) is based on this assumption.

Many other mechanisms of the $0\nu\beta\beta$ decay, caused by a possible violation of L at a scale which is much smaller than the standard seesaw GUT scale, were discussed in the literature [41–46]. If L is violated at a $\sim$TeV scale, it can be shown that contributions of such additional mechanisms to the amplitude of the $0\nu\beta\beta$ decay can be comparable with the contribution of the Majorana mass mechanism.

If the contribution of additional mechanisms to the $0\nu\beta\beta$-decay amplitude is significant, the allowed region for the effective Majorana mass will depend on many (unknown) parameters and it is natural to expect that it will be different from the region presented in Fig. 2. In this case we cannot come to the inequality (29). Let us stress, however, that the assumption of the violation of the total lepton number at a relatively small $\sim$TeV scale can be tested in LHC experiments (see [47]), in experiments on the search for $\mu \to e + \gamma$ decay (see [44]) and in other experiments.

CONCLUSION

After the discovery of neutrino masses and mixing, the problem of the nature of neutrinos with definite masses has become the most important aspect of the physics of massive and mixed neutrinos. The standard seesaw mechanism, which is based on the assumption that the total lepton number is violated at a very large (GUT) scale, is commonly considered as the most natural mechanism of the generation of small Majorana neutrino masses.

The main way of investigation of the problem of the Majorana nature of neutrinos with definite masses is to study the $0\nu\beta\beta$ decay of ^{76}Ge, ^{82}Se, ^{130}Te, ^{136}Xe and other even-even nuclei. The measurement of half-lives of the $0\nu\beta\beta$ decay allows us to determine only *the product* of the effective Majorana mass and the nuclear matrix element.

The calculation of the $0\nu\beta\beta$-decay NMEs is a complicated nuclear many-body problem. Different methods (Shell Model, Quasiparticle Random Phase Approximation, The Interacting Boson Model, and others) are applied for the calculation of the $0\nu\beta\beta$-decay NMEs at present. Results of different calculations differ by a factor of 2–3.

In this paper we proposed a possible test of the calculations of the $0\nu\beta\beta$-decay NMEs. This test is based on the assumption that in future experiments sensitive to the inverted mass hierarchy, half-lives of the $0\nu\beta\beta$ decay will be measured. We assume that the light Majorana neutrino mass mechanism is the dominant mechanism of the $0\nu\beta\beta$ decay and information about the lightest neutrino mass will be available from future high-precision cosmological measurements.

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