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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

A NOVEL MATHEMATICAL MECHANISM OF THE CRITICAL ENDPOINT GENERATION

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We develop a novel model of the QCD matter critical endpoint by matching the deconfinement phase transition curve with the nil line of the bag surface tension coefficient. As a result, this leads to a new structure of the leading singularities of isobaric partition, and in contrast to all previous studies of such models, the deconfined phase in our approach is defined not by an essential singularity of the isobaric partition function but its simple pole. As an unexpected result, we find out that the first order phase transition in this model is the surface tension induced transition. The sufficient conditions of its existence are analyzed and the possible physical consequences are discussed.

С помощью совмещения кривой фазового перехода деконфайнмента с кривой нулевых значений коэффициента поверхностного натяжения мешков получена новая модель критической точки КХД-материи. В результате это ведет к новой структуре лидирующих сингулярностей изобарической статсуммы, и, в отличие от всех предыдущих исследований таких моделей, фаза деконфайнмента в нашем подходе определена не с помощью существенно особой точки изобарической статсуммы, а с помощью ее простого полюса. Неожиданно найдено, что фазовый переход первого рода в данной модели индуцирован поверхностным натяжением. Проанализированы достаточные условия существования такого фазового перехода, и обсуждены его возможные физические следствия.

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INTRODUCTION

One of the most important recent discoveries of the quark–gluon plasma (QGP) phenomenology is a realization of the fact that in and above the cross-over region the QGP is a strongly interacting liquid [1]. Such behavior is observed in the lattice QCD simulations at the temperatures as high as three values of the cross-over temperature $T_{\rm co}$ at vanishing baryonic density [2, 3]. Also it is strongly supported by the enormous elliptic flow measured by the recent LHC heavy-ion experiments [4, 5]. However, the model equations of state which are used to describe the QGP in this region have nothing to do with the properties of liquid. The only exception is the quark–gluon bag with surface tension model (QGBSTM) [6, 7] which is a successor of a famous Fisher droplet model (FDM) [8] and the statistical multifragmentation model (SMM) [9–11] and, as its predecessors, it exploits the physical mechanism of the tricritical endpoint appearance which is typical for ordinary liquids. Besides that, the QGBSTM includes two other popular statistical models: thus, its discrete mass spectrum corresponds

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to the hadron gas model [12] whereas its continuous mass spectrum not only generalizes the gas of QGP bags model [13], but it also accounts for such typical properties of the liquids as the surface tension and Fisher power law [8]. These properties make the QGBSTM rather realistic and allow us to describe the lattice QCD thermodynamics at vanishing baryonic densities rather well [7]. Moreover, very recently the QGBSTM critical exponents were calculated and analyzed [14]. It was also demonstrated [14] that the critical exponents of the 3-dimensional $O(4)$ spin model, which are believed to match the universality class of QCD, can be easily reproduced by the QGBSTM.

However, this model has the tricritical endpoint only which is an intersection of the deconfinement phase transition (PT) and the surface induced PT (for more detail, see [6]). A few recent attempts [15,16] to generate the critical endpoint within the gas of QGP bags model framework were successful only in producing the line of endpoints along which the order of PT gradually increases (for a detailed critique, see [6]). In contrast to these attempts, here we suggest a solution of this long standing problem and formulate the quark–gluon bag model with critical endpoint (QGBSTM2). This is achieved by matching the deconfinement PT and surface induced PT [6] critical lines of the QGBSTM. Below we demonstrate that although in this case the surface tension vanishes in the critical endpoint like in the QGBSTM, in the FDM, in the SMM and in ordinary liquids [17], the analytical structure of singularities that describe the deconfinement PT is absolutely new. Clearly, the novel mathematical mechanism to generate the deconfinement PT and its critical endpoint are important not only for the QGP phenomenology, but also for the nuclear liquid–gas PT described by the SMM and for the liquid–gas PTs in ordinary liquids.

The work is organized as follows. In the next section we present the basic elements of the QGBSTM2. Section 2 is devoted to the analysis of the sufficient conditions for the critical endpoint existence in the present model, while the last section contains our conclusions.

1. QUARK–GLUON BAGS WITH SURFACE TENSION MODEL

The most convenient way to study the QGBSTM2 phase structure is to use the isobaric partition [6] for analyzing its rightmost singularities. It allows one to solve exactly a number of models in thermodynamic limit [7, 18] and for finite volumes [19–21]. Therefore, we assume that after the Laplace transform its grand canonical partition $Z(V,T,\mu)$ generates the following isobaric one:

$$\hat{Z}(s,T,\mu) \equiv \int\limits_0^\infty dV\, \mathrm{e}^{-sV}\, Z(V,T,\mu) = \frac{1}{[s - F(s,T,\mu)]}, \qquad (1)$$

where the function $F(s,T,\mu)$ includes [6] the discrete F_H and continuous F_Q volume spectra of the bags

$$F(s,T,\mu) \equiv F_H(s,T,\mu) + F_Q(s,T,\mu) = \sum_{j=1}^{n} g_j \exp\left(\frac{\mu}{T} b_j - v_j s\right) \phi(T, m_j) + \qquad (2)$$

$$+ u(T) \int\limits_{V_0}^{\infty} \frac{dv}{v^\tau} \exp\left[(s_Q(T,\mu) - s)v - \Sigma(T,\mu) v^\kappa\right]; \qquad (3)$$

$u(T)$ and $s_Q(T,\mu)$ are continuous and, at least, double differentiable functions of their arguments (see [6,7] for detail). The density of bags with mass m_k, eigen volume v_k, baryon charge b_k and degeneracy g_k is given by $\phi_k(T) \equiv g_k \phi(T, m_k)$ where

$$\phi_k(T) \equiv \frac{g_k}{2\pi^2} \int\limits_0^\infty p^2 dp \, \exp\left[-\frac{\sqrt{p^2+m_k^2}}{T}\right] = g_k \frac{m_k^2 T}{2\pi^2} K_2\Big(\frac{m_k}{T}\Big). \tag{4}$$

The continuous part of the volume spectrum (3) generalizes the exponential mass spectrum introduced by Hagedorn [22] and it can be derived in both the MIT bag model [13] and finite width model of QGP bags [7]. The presence of e^{-sv} term accounts for the hard-core repulsion of the Van der Waals type in (3). $\Sigma(T,\mu)$ denotes the ratio of the T- and μ-dependent surface tension coefficient and T (the reduced surface tension coefficient hereafter) which has the form

$$\Sigma(T,\mu) = \begin{cases} \Sigma^- > 0, & T \to T_\Sigma(\mu) - 0, \\ 0, & T = T_\Sigma(\mu), \\ \Sigma^+ < 0, & T \to T_\Sigma(\mu) + 0. \end{cases} \tag{5}$$

In choosing such a simple surface free energy parameterization, we follow the original Fisher idea [8] which allows one to account for the surface free energy by considering a mean bag of volume v and surface extent v^κ. As discussed in [6] and shown in [14], the power $\kappa < 1$ in (3) is a constant which, in principle, may differ from the usual FDM and SMM value $2/3$.

It has to be emphasized that we do not require the precise disappearance of $\Sigma(T,\mu)$ above the critical endpoint as in FDM and SMM. It has already been noticed [6] and is argued here again that this point is of crucial importance in formulating the statistical model with deconfining cross-over. We would like also to mention the negative value of the reduced surface tension coefficient $\Sigma(T,\mu)$ above the $T_\Sigma(\mu)$-line in the (μ,T)-plane should not be taken surprising. It is the well-known fact that the surface tension coefficient in the grand canonical ensemble includes the energy and entropy contributions which have the opposite signs [8,20]. Therefore, $\Sigma(T,\mu) < 0$ does not mean that the surface energy changes the sign, but rather signals that the surface entropy contribution simply exceeds the surface energy part and results in the negative values of surface free energy. In other words, the number of nonspherical bags of fixed volumes becomes so big that the Boltzmann exponent which accounts for the energy «costs» of these bags does not provide their suppression any more. Such a situation is standard for the statistical ensembles with the fluctuating extensive characteristics (the surface of fixed volume bag fluctuates around its mean value) and can be studied rigorously by considering the surface deformations [20,21]. Moreover, the recently established relation between the bag surface tension coefficient and the string tension of confining color tube [23] indicates that the surface tension coefficient is inevitably negative in the cross-over region.

By construction the isobaric partition (1) has two types of singularities: the simple pole $s^* = s_H(T,\mu)$ determined by the equation

$$s^* = F(s^*, T, \mu), \tag{6}$$

and in addition there appears an essential singularity $s^* = s_Q(T,\mu)$ which is defined by the point $s = s_Q(T,\mu) - 0$ in which the continuous part of spectrum $F_Q(s,T,\mu)$ in Eq. (3)

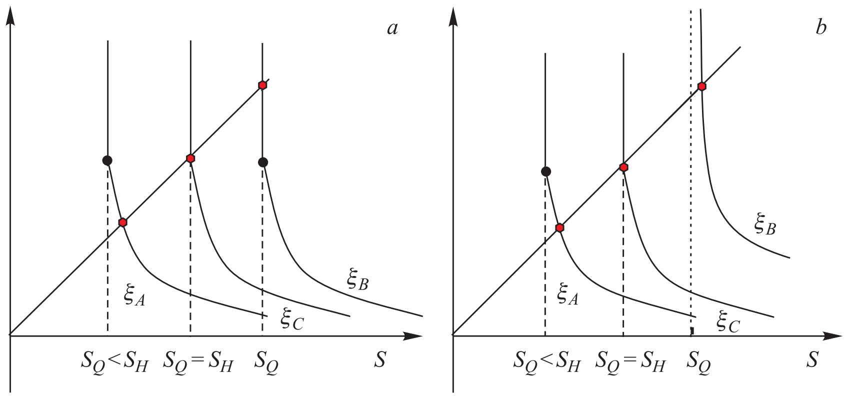

Fig. 1. *a*) Singularities of the isobaric partition (1) and the corresponding graphical solution of Eq. (6) which describes a PT in the models similar to QGBSTM. The solution of Eq. (6) is shown by a filled hexagon. $F(s,\xi)$ is shown by a solid curve for a few values of the parameter sets ξ. $F(s,\xi)$ diverges for $s < s_Q(\xi)$ (shown by dashed lines), but is finite at $s = s_Q(\xi)$ (shown by black circle). At low values of the parameters $\xi = \xi_A$, which can be either $\xi \equiv \{T, \mu = \text{const}\}$ or $\xi \equiv \{T = \text{const}, \mu\}$, the simple pole s_H is the rightmost singularity and it corresponds to hadronic phase. For $\xi = \xi_B \gg \xi_A$ the rightmost singularity is an essential singularity $s = s_Q(\xi_B)$, which describes QGP. At intermediate value $\xi = \xi_C$ both singularities coincide $s_H(\xi_C) = s_Q(\xi_C)$ and this condition is a Gibbs criterion (7). At transition from the low energy density phase to the high density one the rightmost singularity changes from the simple pole to the essential singularity. *b*) Singularities of the isobaric partition (1) and the corresponding graphical solution of Eq. (6) which describes a PT in the QGBSTM2. The notations are the same as for the panel *a*. In this case, however, the rightmost singularity for each phase is the simple pole, whereas at the PT the essential singularity matches the simple pole due to the vanishing surface tension coefficient

becomes divergent. This singularity is also defined by Eq. (6). Usually the statistical models similar to QGBSTM [6, 7, 10, 11, 15, 16, 19, 24–26] are dealing with the following structure of singularities. The pressure of low energy density phase (confined) $p_H(T,\mu)$ is described by the simple pole $s = s_H(T,\mu) = p_H(T,\mu)/T$ which is the rightmost singularity of the isobaric partition (1), whereas the pressure of high energy density phase (deconfined) $p_Q(T,\mu)$ fixes the system pressure, if the essential singularity $s = s_Q(T,\mu) = p_Q(T,\mu)/T$ of this partition becomes the rightmost one (see Fig. 1, *a*). In this model we consider $p_Q(T,\mu)$ as a parameter which can be taken either directly from the lattice QCD data or from the microscopic models which study the pressure of quark–gluon phase in an infinite volume.

The deconfining PT occurs at the equilibrium line $T_c(\mu)$ where both singularities match each other

$$s_H(T,\mu) = s_Q(T,\mu) \Rightarrow T = T_c(\mu). \quad (7)$$

And in this equation one can easily recognize the Gibbs criterion for phase equilibrium. Similar behavior of the rightmost singularities is just depicted in Fig. 1.

According to [6] the deconfining PT occurs, if the phase equilibrium temperature (7) is lower than the temperature of the null surface tension line (5) for the same value of baryonic chemical potential, i.e., $T_c(\mu) < T_\Sigma(\mu)$, whereas at low values of μ the PT is transformed into a cross-over because the line $T = T_\Sigma(\mu)$ leaves the QGP phase smoothly to appear in the hadronic phase. The intersection point $(\mu_{\rm end}; T_c(\mu_{\rm end}))$ of these two lines $T_c(\mu) = T_\Sigma(\mu)$ is just the tricritical endpoint since for $\mu \geqslant \mu_{\rm end}$ and $T > T_c(\mu_{\rm end})$ at the null surface tension line $T = T_\Sigma(\mu)$ there exists the surface induced PT [6].

The new element of principle importance which is responsible for the critical endpoint generation in the QGBSTM2 is a matching of the surface induced PT with the deconfining one in order to get rid of the former and to «hide» it inside the mixed phase. To demonstrate the possible result, let us assume that the surface tension coefficient changes its sign exactly at the deconfining PT line, i.e., for $\max\{\mu(T_c)\} \geqslant \mu \geqslant \mu_{\rm end}$ and $T \leqslant T_c(\mu_{\rm end})$, one has $T_c(\mu) = T_\Sigma(\mu)$ while keeping the cross-over transition for $\mu < \mu_{\rm end}$. The idea to match these two PT lines is almost evident, but a nontriviality is seen in the fact that an existence of both the critical endpoint at $(\mu_{\rm end}; T_c(\mu_{\rm end}))$ and the 1st order deconfining PT at $T_c(\mu) = T_\Sigma(\mu)$ is deeply rooted in an entire change of the rightmost singularity pattern and, as we show in the next section, in another mechanism of the deconfining PT.

An important physical consequence of this new PT mechanism is that it leads to the power law in the distribution of large bags with respect to their volumes in the entire mixed phase. This can be seen from the volume spectrum of large bags (3), if one substitutes $s = s_Q(T,\mu) = p_Q(T,\mu)/T$ and $\Sigma(T,\mu) = 0$ in it. As one can see from the exact solutions of the FDM [8] and simplified SMM [10, 11], the power law with respect to the droplet volume exists at the critical or tricritical endpoint only. In the QGBSTM [6] the power law in the volume attenuation of bags exists at the curve of the vanishing surface tension coefficient, which is located above the phase diagram of the deconfining PT in the $\mu - T$ plane except for the tricritical endpoint. Therefore, there is a possibility that the lattice QCD simulations can verify an existence of the power law and, thus, can help to distinguish the QGBSTM and QGBSTM2. Also we stress that, if the presently assumed framework along with the parameterization of surface tension of bags (5) are applied to the finite width model of QGP bags [7] or, alternatively, if one uses the pressure of model [7] as the parameter $p_Q(T,\mu)$ in QGBSTM2, then the corresponding power law should exist for the mass attenuation of heavy QGP bags in its mixed phase.

2. THE SUFFICIENT CONDITIONS FOR THE CRITICAL ENDPOINT EXISTENCE

Under adopted assumption the rightmost singularity in the QGBSTM2 is always the simple pole since in the right-hand side vicinity of $s \to s_Q(T,\mu) + 0$ the value of $F_Q(s,T,\mu) \to \infty$ for $\Sigma = \Sigma^+ < 0$. Then the motion of singularities corresponds to the example shown in Fig. 1, *b*. The question, however, appears whether such behavior corresponds to PT indeed, since this is not the case for other known models. To clarify the point, it is convenient to introduce the variable $\Delta^\pm \equiv \Delta(T_\Sigma \pm 0, \mu) = s^\pm - s_Q(T_\Sigma \pm 0, \mu)$ and to compare the T derivative of the rightmost singularity $s^- \equiv s^*(T_\Sigma - 0, \mu)$ below and $s^+ \equiv s^*(T_\Sigma + 0, \mu)$ above the PT line $T_c(\mu) = T_\Sigma(\mu)$ for the same magnitudes of μ. Due to the relation between the system pressure $p(T,\mu)$ and the rightmost singularity $s^*(T,\mu) = p(T,\mu)/T$, the difference of T derivatives, $\partial(\Delta^+ - \Delta^-)/\partial T$, if revealed on both sides of the PT line, is defined by the difference of the corresponding entropy densities. Therefore, according to the standard classification of the PT order, the nonzero values of $\partial(\Delta^+ - \Delta^-)/\partial T \neq 0$ signal about the 1st order PT.

Now using the auxiliary functions

$$\mathcal{K}_a(x) \equiv \int\limits_{V_0\Delta}^{\infty} dz \frac{\exp\left[-z + x z^{\kappa}\right]}{z^a}, \tag{8}$$

$$g_\tau(\Delta^\pm, \Sigma^\pm) \equiv \frac{\exp\left[-\Delta^\pm V_0 - \Sigma^\pm V_0^\kappa\right]}{(\tau-1) V_0^{\tau-1}}, \tag{9}$$

and calculating the following integral

$$I_\tau(\Delta^\pm, \Sigma^\pm) \equiv \int\limits_{V_0}^{\infty} dv \frac{\exp\left[-\Delta^\pm v - \Sigma^\pm v^\kappa\right]}{v^\tau} = \Bigg[g_\tau(\Delta^\pm, \Sigma^\pm) - \frac{\Delta^\pm}{\tau - 1} g_{\tau-1}(\Delta^\pm, \Sigma^\pm) - \\ - \frac{\kappa \Sigma^\pm}{\tau-1} g_{\tau-\kappa}(\Delta^\pm, \Sigma^\pm) + \frac{(\Delta^\pm)^{\tau-1}}{\tau-1} \Phi\left(-\frac{\Sigma^\pm}{(\Delta^\pm)^\kappa}\right)\Bigg], \tag{10}$$

$$\Phi(x) \equiv \mathcal{K}_{\tau-2}(x) - \frac{\kappa(2\tau - 3 - \kappa)x}{(\tau-2)(\tau-1-\kappa)} \mathcal{K}_{\tau-1-\kappa}(x) + \frac{\kappa^2 x^2}{\tau - 1 - \kappa} \mathcal{K}_{\tau-2\kappa}(x) \tag{11}$$

by parts, it is possible to rewrite the continuous part of volume spectrum (3) as $F_Q(s^\pm, T, \mu) = u(T) I_\tau(\Delta^\pm, \Sigma^\pm)$. From Eqs. (10) and (11) one can show the necessary condition of deconfining PT existence at $\Sigma^\pm \to 0$ becomes $s_Q(T_\Sigma, \mu) = F_H(s_Q(T_\Sigma, \mu), T_\Sigma, \mu) + u(T_\Sigma) g_\tau(0,0)$ and it provides $\Delta^\pm \to +0$, indeed. For $\tau < 1 + 2\kappa$ such a statement follows directly from the present form of (11), whereas for larger values of τ exponent one needs to integrate $\mathcal{K}_a(x)$ functions in (11) while they converge at the lower integration limit for $\Delta^\pm \to +0$.

Analyzing Eqs. (8)–(11) one can easily find

$$\frac{\partial \Delta^\pm}{\partial T} = \frac{\dfrac{\partial F_H}{\partial T} + \dfrac{\partial s_Q}{\partial T}\left[\dfrac{\partial F_H}{\partial s} - 1\right] + \dfrac{\partial u}{\partial T} I_\tau(\Delta^\pm, \Sigma^\pm) - u I_{\tau-\kappa}(\Delta^\pm, \Sigma^\pm) \dfrac{\partial \Sigma^\pm}{\partial T}}{1 + u I_{\tau-1}(\Delta^\pm, \Sigma^\pm) - \dfrac{\partial F_H}{\partial s}}, \tag{12}$$

which in the limit $\Delta^\pm, \Sigma^\pm \to 0$ gives

$$\frac{\partial \Delta^+}{\partial T} - \frac{\partial \Delta^-}{\partial T} \to - \frac{u I_{\tau-\kappa}(0,0)\left[\dfrac{\partial \Sigma^+}{\partial T} - \dfrac{\partial \Sigma^-}{\partial T}\right]}{1 + u I_{\tau-1}(0,0) - \dfrac{\partial F_H}{\partial s}}. \tag{13}$$

This is quite a remarkable result because it clearly shows that the 1st order deconfining PT does exist in the present model only, if the T derivative of reduced surface tension coefficient has a discontinuity at the phase equilibrium line. Thus, a discontinuity of the first derivative of system pressure is generated by a discontinuity of the derivative of surface tension coefficient. In other words, within the QGBSTM2 the deconfining 1st order PT is just a surface induced one. The necessary condition for its existence is the finiteness of integrals $I_{\tau-\kappa}(0,0)$ and $I_{\tau-1}(0,0)$ in (13), i.e., $\tau > 2$.

Moreover, in order to realize a PT from hadronic matter to QGP it is necessary to have at the PT line $\frac{\partial \Delta^+}{\partial T} - \frac{\partial \Delta^-}{\partial T} = \frac{1}{T}\frac{\partial}{\partial T}\left[p_Q(T,\mu) - p_H(T,\mu)\right] > 0$ and, hence, at this line

$$\frac{\partial \Sigma^+}{\partial T} - \frac{\partial \Sigma^-}{\partial T} < 0. \tag{14}$$

Now it is clear that at the critical endpoint $(\mu_{\rm end}; T_c(\mu_{\rm end}))$ the entropy density gap vanishes due to the disappearing difference $\frac{\partial \Sigma^+}{\partial T} - \frac{\partial \Sigma^-}{\partial T} = 0$. With the general parameterization of reduced surface tension coefficient which is consistent with (5), we have

$$\Sigma(T,\mu) = \frac{1}{T}\begin{cases} \sigma^- \left[\frac{T_\Sigma(\mu) - T}{T_\Sigma(\mu)}\right]^{\zeta^-}, & T \to T_\Sigma(\mu) - 0, \\ -\sigma^+ \left[\frac{T - T_\Sigma(\mu)}{T_\Sigma(\mu)}\right]^{\zeta^+}, & T \to T_\Sigma(\mu) + 0, \end{cases} \tag{15}$$

and it allows us to conclude about the values of $\zeta^\pm$ and the values of coefficients $\sigma^\pm \geqslant 0$. It is obvious from (12) that $\zeta^\pm \geqslant 1$, otherwise the corresponding entropy density is divergent at the PT line. If, for instance, $\zeta^+ = 1$, as predicted by the Hills and Dales model [20], then $\zeta^- = 1$, and according to (14) one has $\sigma^+ > \sigma^-$. If, however, $\zeta^- > 1$, then from (14) it follows that $\sigma^+\zeta^+(T - T_\Sigma(\mu))^{\zeta^+ - 1} > 0$ for $T \to T_\Sigma(\mu) + 0$. The latter is consistent with the equality $\zeta^+ = 1$.

It is not difficult to show that in accordance with (7) the inequalities

$$\frac{\partial F_H}{\partial T} + \frac{\partial s_Q}{\partial T}\left[\frac{\partial F_H}{\partial s} - 1\right] + \frac{\partial u}{\partial T} g_\tau(0,0) > u g_{\tau-\kappa}(0,0)\frac{\partial \Sigma^+}{\partial T}, \tag{16}$$

$$\frac{\partial F_H}{\partial T} + \frac{\partial s_Q}{\partial T}\left[\frac{\partial F_H}{\partial s} - 1\right] + \frac{\partial u}{\partial T} g_\tau(0,0) < u g_{\tau-\kappa}(0,0)\frac{\partial \Sigma^-}{\partial T} \tag{17}$$

are the sufficient conditions of the 1st order PT existence that provide (14) and guarantee the uniqueness of solutions $\Delta^\pm \to +0$ on both sides of the PT line.

The critical endpoint $(\mu_{\rm end}; T_c(\mu_{\rm end}))$ exists, if in its vicinity the difference $\sigma^+ - \sigma^-$ vanishes as $d^{\zeta_{\rm end}}$ with

$$d \equiv T - T_c(\mu_{\rm end}) - \left.\frac{\partial T_\Sigma}{\partial \mu}\right|_{\mu_{\rm end}} (\mu - \mu_{\rm end}) \tag{18}$$

and $\zeta_{\rm end} \geqslant 1$. By construction in the $\mu - T$ plane d as defined by (18) vanishes at the tangent line to the PT curve at $(\mu_{\rm end}; T_c(\mu_{\rm end}))$. As one can easily see from either T or μ derivative of (12), any second derivative of the difference $\Delta^+ - \Delta^- = 0$ at the critical endpoint $(\mu_{\rm end}; T_c(\mu_{\rm end}))$, if $\zeta^+ = \zeta^- = \zeta_{\rm end} = 1$ only, which provides the 2nd order PT available at this point. The higher order PT at the critical endpoint may exist for $\zeta_{\rm end} = 2$.

Thus, the QGBSTM2 is able to describe three phases, which are the hadronic phase, the mixed phase of hadrons and QGP, and the QGP cross-over phase [6], and two ways for phase transformation: either the deconfining PT for $\mu \geqslant \mu_{\rm end}$ or the cross-over at $\mu < \mu_{\rm end}$.

CONCLUSIONS

Here we present a novel solvable model, QGBSTM2, which develops critical endpoint at $(\mu_{\rm end}; T_c(\mu_{\rm end}))$. This model naturally explains the transformation of the 1st order deconfining PT into a weaker PT at the endpoint and into a cross-over at low baryonic densities as driven by negative surface tension coefficient of the QGP bags at high energy densities. This conclusion is also supported by the recently derived relation between the string tension of confining color tube and the surface tension coefficient of QGP bags [23].

An important finding of QGBSTM2 is that a solvable model of the QCD critical endpoint can be formulated for $\tau > 2$. This is obtained by matching the deconfining PT line $T_c(\mu)$ with the line of vanishing surface tension coefficient $T_\Sigma(\mu)$ for $\mu \geqslant \mu_{\rm end}$ and $T \leqslant T_c(\mu_{\rm end})$. Such a step unexpectedly leads to a new strong claim that the 1st order PT in QGBSTM2 should not be accompanied by change of the leading singularity type as was argued earlier in [6, 7, 10, 11, 15, 16, 19, 24–26]. Then, in contrast to all previous findings, the high density QGP phase is defined by a simple pole of the isobaric partition (1) and not by its essential singularity (compare the panels *a* and *b* of Fig. 1). As a consequence, for the first time we discover that the critical endpoint in this model with the constituents of nonzero proper volume exists not for $\tau \leqslant 1$ as in the SMM [10, 11] and not for $1 < \tau \leqslant 2$ as the tricritical endpoints in the SMM and in the QGBSTM [6], but for $\tau > 2$, i.e., as in the FDM [8] in which the eigen volume of constituents is zero. We believe that this new mechanism of the first order PT is important not only for the formulation of the phenomenological exactly solvable model for the QCD critical endpoint, but is also significant for the critical endpoint in usual liquids.

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