Письма в ЭЧАЯ. А. 2012. Т. 9, № 9–10. С. 204–211

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ON THE ORIGIN OF SUPERSELECTION RULES AND DIFFERENT SOLUTIONS OF THIRRING MODEL

S. E. Korenblit [1], *V. V. Semenov*

Irkutsk State University, Irkutsk, Russia

The normal forms of different one- and two-parametric solutions of Thirring model are connected with each other by making use of generalized conformal shift transformations. New alternative sources of superselection rules are shown.

Нормальные формы для различных одно- и двухпараметрических решений модели Тирринга связаны между собой обобщенным преобразованием конформного сдвига. Указаны альтернативные источники правил суперотбора.

PACS: 03.70.+k; 11.10.-z; 11.10.Kk; 11.15.Tk

INTRODUCTION

We have shown in the recent works [1] that Thirring model is exactly solvable [2–4] in fact due to intrinsic hidden exact linearizability of its Heisenberg equation (HEq) $2\partial_\xi \Psi_\xi(x) = -igJ^{-\xi}_{(\Psi)}(x)\Psi_\xi(x)$, and that the bosonization rules [5] can acquire an operator sense only for $Z_{(\chi)}(a) = 1$ among the free fields operators with unambiguously defined normal ordering procedure. For Heisenberg currents with current's–field's renormalization constant $Z_{(\Psi)}(a) = (-\Lambda^2 a^2)^{-\overline{\beta}^2/4\pi}$ these rules take place only in a weak sense:

$$\widehat{J}^\mu_{(\Psi)}(x) \overset{\mathrm{w}}{=} \frac{\beta}{2\sqrt{\pi}}\,\widehat{J}^\mu_{(\chi)}(x) \overset{\mathrm{w}}{=} -\frac{\beta}{2\pi}\,\epsilon^{\mu\nu}\partial_\nu\phi(x) = \frac{\beta}{2\pi}\,\partial^\mu\varphi(x), \text{ for} \tag{1}$$

$$\widehat{J}^0_{(\Psi)}(x) = \lim_{\widetilde{\varepsilon}\to 0}\widehat{J}^0_{(\Psi)}(x;\widetilde{\varepsilon}), \quad \widehat{J}^1_{(\Psi)}(x) = \lim_{\varepsilon\to 0}\widehat{J}^1_{(\Psi)}(x;\varepsilon), \text{ with} \tag{2}$$

$$\widehat{J}^\nu_{(\Psi)}(x;a) = Z^{-1}_{(\Psi)}(a)\left[\overline{\Psi}(x+a)\gamma^\nu\Psi(x) - \langle 0|\overline{\Psi}(x+a)\gamma^\nu\Psi(x)|0\rangle\right], \tag{3}$$

[1] E-mail: korenb@ic.isu.ru

and $\widetilde{\varepsilon}^{\mu} = -\epsilon^{\mu\nu}\varepsilon_{\nu}$, for the first $\widetilde{\varepsilon}^{0} = \varepsilon^{1} \to 0$, but fixed $\widetilde{\varepsilon}^{1} = \varepsilon^{0}$, $\varepsilon^{2} = -\widetilde{\varepsilon}^{2} > 0$. The following variant of Oksak solution [6, 7] of this model was obtained [1]:

$$\Psi_{\xi}^{Ok}(x) = \mathcal{N}_{\varphi}\left\{\exp\left(-i2\sqrt{\pi}\left[\varrho^{-\xi}(x) + \frac{\xi}{4}W^{\xi}\right]\right)\right\} v_{\xi}, \text{ with } \xi = \pm, \tag{4}$$

$$2\sqrt{\pi}\varrho^{-\xi}(x) = \overline{\alpha}\varphi^{-\xi}\left(x^{-\xi}\right) + \overline{\beta}\varphi^{\xi}\left(x^{\xi}\right), \quad 2\sqrt{\pi}\,W^{\xi} = \overline{\alpha}Q^{\xi} - \overline{\beta}Q^{-\xi}, \tag{5}$$

$$v_{\xi} = \widehat{v}_{\xi}\exp\left\{-a_0\frac{\pi}{8}\cosh 2\eta\right\}, \quad \widehat{v}_{\xi} = \left(\frac{\overline{\mu}}{2\pi}\right)^{1/2}\left(\frac{\overline{\mu}}{\Lambda}\right)^{\overline{\beta}^2/4\pi} \mathrm{e}^{i\varpi - i\xi\Theta/4}, \tag{6}$$

$$\overline{\alpha}^2 - \overline{\beta}^2 = 4\pi, \quad \overline{\beta} = \frac{\beta g}{2\pi}, \quad \mathrm{e}^{\eta} = \frac{2\sqrt{\pi}}{\beta} = \sqrt{1 + \frac{g}{\pi}}, \quad \overline{\alpha} \pm \overline{\beta} = 2\sqrt{\pi}\mathrm{e}^{\pm\eta}, \tag{7}$$

defined for $x^{\xi} = x^0 + \xi x^1$ in the space of massless pseudoscalar field $\phi(x)$: $\mathcal{P}c(k^1)\mathcal{P}^{-1} = -c(-k^1)$, $[c(k^1), c^{\dagger}(q^1)] = 4\pi k^0\delta(k^1 - q^1)$, $c(k^1)|0\rangle = 0$, for $O = \phi(-\infty, x^0) - \phi(\infty, x^0)$, $2\varphi^{\xi}\left(x^{\xi}\right) = \varphi(x) - \xi\phi(x)$, $2Q^{\xi} = O - \xi O_5$, with

$$k^0 = |k^1|, \quad \mathrm{P}\frac{1}{k^1} = \frac{\varepsilon\left(k^1\right)}{|k^1|}, \quad \frac{1}{4}\left(\mathrm{P}\frac{1}{k^1} - \frac{\xi}{k^0}\right) = \frac{-\xi\theta\left(-\xi k^1\right)}{2k^0} \text{ and} \tag{8}$$

$$\phi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dk^1}{2k^0}\left[c\left(k^1\right)\mathrm{e}^{-i(kx)} + c^{\dagger}\left(k^1\right)\mathrm{e}^{i(kx)}\right] \equiv \phi^{(+)}(x) + \phi^{(-)}(x), \tag{9}$$

$$\varphi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk^1\,\mathrm{P}\frac{1}{k^1}\left[c\left(k^1\right)\mathrm{e}^{-i(kx)} + c^{\dagger}\left(k^1\right)\mathrm{e}^{i(kx)}\right], \tag{10}$$

$$\varphi^{\xi(+)}(s) = -\frac{\xi}{2\pi}\int_{-\infty}^{\infty}\frac{dk^1}{2k^0}\theta\left(-\xi k^1\right)c(k^1)\,\mathrm{e}^{-ik^0 s}, \quad \varphi^{\xi(-)}(s) = \left[\varphi^{\xi(+)}(s)\right]^{\dagger}, \tag{11}$$

$$Q^{\xi(+)}(\widehat{x}^0) = \lim_{L\to\infty}\frac{i\xi}{2}\int_{-\infty}^{\infty} dk^1\theta\left(-\xi k^1\right)c(k^1)\,\mathrm{e}^{-ik^0\widehat{x}^0}\delta_L(k^1), \tag{12}$$

$$[\varphi(x), O] = [\phi(x), O_5] = i, \quad 2iO_5 = c(0) - c^{\dagger}(0), \tag{13}$$

$$[\varphi^{\xi}(s), \varphi^{\xi'}(\tau)] = -\frac{i}{4}\varepsilon(s-\tau)\delta_{\xi,\xi'}, \quad [\varphi^{\xi}(s), Q^{\xi'}] = \frac{i}{2}\delta_{\xi,\xi'}, \tag{14}$$

$$[\varphi^{\xi(\pm)}(s), \varphi^{\xi'(\mp)}(\tau)] = \mp\frac{\delta_{\xi,\xi'}}{4\pi}\ln\left(i\overline{\mu}\left\{\pm(s-\tau) - i0\right\}\right), \tag{15}$$

$$[\varphi^{\xi(\pm)}(s), Q^{\xi'(\mp)}] = \frac{i}{4}\delta_{\xi,\xi'}, \quad [Q^{\xi(\pm)}, Q^{\xi'(\mp)}] = \pm a_0\delta_{\xi,\xi'}, \tag{16}$$

$$a_0 = a_0(L) = \pi\int_0^{\infty} dk^1\,k^0\left(\delta_L(k^1)\right)^2, \quad L\to\infty, \quad \lim_{L\to\infty}\delta_L(k^1) = \delta(k^1). \tag{17}$$

For Eq. (4), that was constructed in [1] as dynamical mapping (DM), $\mathcal{N}_\varphi$ means the normal ordering with respect to $c(k^1)$, and the ultraviolet cut-off Λ [4] is introduced for the field regularization (6). The volume cut-off function $\delta_L(k^1)$ was invented for the charge regularization (12), leading to nonnegative constant a_0 (17) for the commutator (16), connected with the structure of vacuum state of chosen field representation [1, 3, 4, 6–8]. The observed weak linearization of HEq together with the nonlinearity of DM (4) and the initial conditions at $x^0 = 0$, generalized in a weak sense, have allowed us [1] to overcome the restrictions of Haag theorem, by removing the problems again into the representation construction of physical fields: at first as reducible massless free Dirac fields $\chi(x)$ (corresponding to $g = 0$ in the above formulas), and then as irreducible massless (pseudo)scalar fields $(\phi(x))$, $\varphi(x)$, taken here mutually dual and coupled by the relations ($\varepsilon(x^1) = \mathrm{sign}\,(x^1)$):

$$\left.\begin{matrix}\phi(x)\\ \varphi(x)\end{matrix}\right\} = -\frac{1}{2}\int\limits_{-\infty}^{\infty} dy^1\,\varepsilon\left(x^1 - y^1\right)\partial_0 \left\{\begin{matrix}\varphi\left(y^1, x^0\right),\\ \phi\left(y^1, x^0\right).\end{matrix}\right. \tag{18}$$

These free fields arisen in [1] as Schrödinger physical fields, in fact play a role of the asymptotic ones [6]. Due to automatic elimination of zero mode's contributions for (10) and for generating functional [4], the chosen here representation space [1] relaxes the problem of nonpositivity of its inner product induced by Wightman function (15) [6–8].

1. OTHER SOLUTIONS AND SUPERSELECTION RULES

Now we wish to connect the Oksak solution (4)–(6) with another known solutions of Thirring model [5, 8]. To this end, we use the properly unitary transformation of conformal shift [7] for the fields φ^ξ, generalized in the following way. By making use of the relations (13)–(16), we consider the family of solutions $\Psi(x,\sigma) = K_\sigma^{-1}\Psi^{Ok}(x)K_\sigma$, marked by arbitrary real parameter σ:

$$K_\sigma = \exp X_\sigma, \quad X_\sigma = i\sigma\frac{\overline{\xi}}{4}\left(Q^{-\overline{\xi}}Q^{-\overline{\xi}} - Q^{\overline{\xi}}Q^{\overline{\xi}}\right) = i\frac{\sigma}{4}OO_5, \quad \overline{\xi}, \xi = \pm, \tag{19}$$

$$\Psi_\xi(x,\sigma) = K_\sigma^{-1}\Psi_\xi^{Ok}(x)K_\sigma = \mathcal{N}_\varphi\left\{\mathrm{e}^{R_\xi(x,\sigma)}\right\}v_\xi(\sigma), \tag{20}$$

$$R_\xi(x,\sigma) = -i\left[2\sqrt{\pi}\,\varrho^{-\xi}(x) + \frac{\xi}{4}(\overline{\alpha} + \sigma\overline{\beta})Q^\xi - \frac{\xi}{4}(\overline{\beta} + \sigma\overline{\alpha})Q^{-\xi}\right], \tag{21}$$

$$v_\xi(\sigma) = v_\xi \exp\left\{-a_0\frac{\pi}{8}\left[\sigma^2\cosh 2\eta + 2\sigma\sinh 2\eta\right]\right\}. \tag{22}$$

For arbitrary σ this solution obeys the same canonical anticommutation relations (CAR) and the same bosonization rule (1)–(3) with the same renormalization constant $Z_{(\Psi)}(a)$. The parameter a_0 may be adsorbed into the regularization parameter $\overline{\mu}$ by the rescaling substitution, which unlike the Oksak (6) and free cases, now depends on Thirring coupling constant g (7):

$$\overline{\mu} \longmapsto \overline{\overline{\mu}}\exp\left\{a_0\frac{\pi}{4}\left(\sigma^2 + 1 + 2\sigma\tanh 2\eta\right)\right\}. \tag{23}$$

By using (18) and topological definitions both of O, O_5, it is a simple matter to check that $\sigma = \pm 1$ gives the two types of Mandelstam solution [5], while $\sigma = -\coth 2\eta$ corresponds to normal form of solution of Morchio et al. [8]. This again demonstrates the advantages of normal ordered form of DM:

$$\Psi_\xi(x,1) = \mathcal{N}_\varphi \left\{ \mathrm{e}^{R_\xi(x,1)} \right\} \widehat{v}_\xi \exp\left\{ -a_0 \frac{\pi}{4} \mathrm{e}^{2\eta} \right\}, \quad \sigma = 1, \tag{24}$$

$$R_\xi(x,1) = -i \left[\xi \frac{\beta}{2} \phi(x^1,x^0) - \frac{2\pi}{\beta} \int\limits_{-\infty}^{x^1} dy^1\, \partial_0 \phi(y^1,x^0) \right]; \tag{25}$$

$$\Psi_\xi(x,-1) = \mathcal{N}_\varphi \left\{ \mathrm{e}^{R_\xi(x,-1)} \right\} \widehat{v}_\xi \exp\left\{ -a_0 \frac{\pi}{4} \mathrm{e}^{-2\eta} \right\}, \quad \sigma = -1, \tag{26}$$

$$R_\xi(x,-1) = -i \left[\frac{2\pi}{\beta} \varphi(x^1,x^0) + \xi \frac{\beta}{2} \int\limits_{x^1}^{\infty} dy^1\, \partial_0 \varphi(y^1,x^0) \right]; \tag{27}$$

$$\Psi_\xi(x,-\coth 2\eta) = \mathcal{N}_\varphi \left\{ \exp\left[R_\xi(x,-\coth 2\eta) \right] \right\} v_\xi(\sigma = -\coth 2\eta), \tag{28}$$

$$R_\xi(x,-\coth 2\eta) = -i \left[2\sqrt{\pi}\, \varrho^{-\xi}(x) + \xi \frac{\pi}{2} \left(\frac{Q^\xi}{\overline{\alpha}} + \frac{Q^{-\xi}}{\overline{\beta}} \right) \right], \tag{29}$$

$$v_\xi(\sigma = -\coth 2\eta) = \widehat{v}_\xi \exp\left\{ -a_0 \frac{\pi}{8} \frac{\cosh 2\eta}{\sinh^2 2\eta} \right\}. \tag{30}$$

We would like to point out that $\sigma = 1$ corresponds to DM (24), (25) «onto» the pseudoscalar field $\phi(x)$, while $\sigma = -1$ gives another form (26), (27) of Mandelstam solution with bosonization «onto» the scalar field $\varphi(x)$, and that unlike (28), (29), the original solution of Morchio et al. [8] has $a_0 = 0$, as well as the original Oksak solution [6,7], but contains all Klein factors outside the normal form, so its renormalization constant remains to be unknown. We have used here that $2\varrho^{-\xi}(x) = \mathrm{e}^{\eta}\varphi(x) + \xi\,\mathrm{e}^{-\eta}\phi(x)$ and have utilized for brevity both definitions of last identity (7) read also as $\overline{\alpha} + \overline{\beta} = 4\pi/\beta$, $\overline{\alpha} - \overline{\beta} = \beta$.

There is another important maybe improper unitary transformation of the solutions (20), which introduces the two-parametric extension of Oksak solution (4) and for arbitrary σ, ρ obeys again the same CAR and the bosonization rule (1) with the same renormalization constant $Z_{(\Psi)}(a)$:

$$\Psi(x,\sigma,\rho) = \mathcal{L}_\rho^{-1} \Psi(x,\sigma) \mathcal{L}_\rho = K_\sigma^{-1} \Psi(x,0,\rho) K_\sigma, \quad \text{and for } \ \overline{\xi}, \xi = \pm : \tag{31}$$

$$\mathcal{L}_\rho = \exp Y, \quad Y = -\frac{i}{2} \rho\, Q^{\overline{\xi}} Q^{-\overline{\xi}} = -\frac{i}{8} \rho \left(O^2 - O_5^2 \right), \tag{32}$$

$$\Psi_\xi(x,\sigma,\rho) = \mathcal{L}_\rho^{-1} \Psi_\xi(x,\sigma) \mathcal{L}_\rho = \mathcal{N}_\varphi \left\{ \mathrm{e}^{R_\xi(x,\sigma,\rho)} \right\} v_\xi(\sigma,\rho), \tag{33}$$

$$R_\xi(x,\sigma,\rho) = -i 2\sqrt{\pi} \left[\varrho^{-\xi}(x) + \frac{\Sigma_+^\xi}{8} Q^\xi + \frac{\Sigma_-^\xi}{8} Q^{-\xi} \right], \tag{34}$$

$$v_\xi(\sigma,\rho) = \widehat{v}_\xi \exp\left\{ -a_0 \frac{\pi}{32} \left[\left(\Sigma_-^\xi \right)^2 + \left(\Sigma_+^\xi \right)^2 \right] \right\}, \text{ with} \tag{35}$$

$$\Sigma_\pm^\xi = \mathrm{e}^{-\eta} \left[\xi(1-\sigma) + \rho \right] \pm \mathrm{e}^{\eta} \left[\xi(1+\sigma) + \rho \right]. \tag{36}$$

For divergent value of a_0 at $L \to \infty$, e.g., for usual box [1], the ξ — dependence of the last exponential of c — number spinor (35) leads to nonphysical in general nonrenormalizable infrared divergences of every component of the field (33). For arbitrary ρ this ξ-dependence eliminates only for the solution (28) with $\sigma = -\coth 2\eta$, that transcribes (33)–(36) as

$$R_\xi(x, -\coth\, 2\eta, \rho) = -i\left[2\sqrt{\pi}\,\varrho^{-\xi}(x) + \xi\frac{\pi}{2}\left(\frac{Q^\xi}{\overline{\alpha}} + \frac{Q^{-\xi}}{\overline{\beta}}\right) + \rho\frac{\sqrt{\pi}}{2}W^\xi\right], \qquad (37)$$

$$v_\xi(-\coth\, 2\eta, \rho) = \widehat{v}_\xi \exp\left\{-a_0\frac{\pi}{8}\cosh 2\eta\left[\frac{1}{\sinh^2 2\eta} + \rho^2\right]\right\}. \qquad (38)$$

Let us turn to the vacuum expectation value (VEV) of the strings of these fields (33). Following [6] it is enough to consider only the product

$$\left\langle 0\left|\prod_{i=1}^{p}\Psi_{\xi_i}^{(l_i)}(x_i, \sigma\cdot\rho)\right|0\right\rangle, \text{ with } l_i = +1, \text{ for } \Psi_i, \quad l_i = -1, \text{ for } \Psi_i^\dagger. \qquad (39)$$

By virtue of the known relations for any numbers γ_i, $i = 1 \div p$, and for any linearly dependent on $c, c^\dagger$ operators $l_i R_{\xi_i}(x_i, \sigma, \rho) \mapsto \mathcal{R}_i = \mathcal{R}_i^{(+)} + \mathcal{R}_i^{(-)}$:

$$\sum_{i<k}^{p}\gamma_i\gamma_k = \frac{1}{2}\sum_{i\neq k}^{p}\gamma_i\gamma_k = \frac{1}{2}\left(\sum_{i=1}^{p}\gamma_i\right)^2 - \frac{1}{2}\sum_{i=1}^{p}\gamma_i^2, \qquad (40)$$

$$\prod_{i=1}^{p}\mathcal{N}\left\{\exp\left[\mathcal{R}_i\right]\right\} = \exp\left\{\sum_{i<k}^{p}[\mathcal{R}_i^{(+)}, \mathcal{R}_k^{(-)}]\right\}\mathcal{N}\left\{\exp\left(\sum_{j=1}^{p}\mathcal{R}_j\right)\right\}, \qquad (41)$$

the corresponding VEV of the string of the fields (33)–(36) reads [1]

$$\begin{aligned}\left(\Lambda^{\overline{\beta}^2/4\pi}\sqrt{2\pi}\right)^p \left\langle 0\right|\prod_{i=1}^{p}\Psi_{\xi_i}^{(l_i)}(x_i, \sigma, \rho)\left|0\right\rangle = \langle 0|\mathcal{N}_\varphi\left\{\exp\left(\sum_{j=1}^{p}\mathcal{R}_j\right)\right\}|0\rangle\times \\ \times\exp\left\{i\varpi\mathcal{S}_p - i\frac{\Theta}{4}\mathcal{S}_{p5}\right\}\exp\left\{\frac{1}{4}\left[\mathrm{e}^{2\eta}\mathcal{S}_p^2 + \mathrm{e}^{-2\eta}\mathcal{S}_{p5}^2\right]\ln\overline{\mu}\right\}\times \\ \times\exp\left\{-a_0\frac{\pi}{16}\left(\mathrm{e}^{2\eta}\Big[(1+\sigma)\mathcal{S}_p + \rho\mathcal{S}_{p5}\Big]^2 + \mathrm{e}^{-2\eta}\Big[(1-\sigma)\mathcal{S}_{p5} + \rho\mathcal{S}_p\Big]^2\right)\right\}\times \\ \times\prod_{i<k}^{p}\left\{\mathrm{e}^{i\pi(\xi_i-\xi_k)}\left[\frac{i(x_i^- - x_k^-) + 0}{i(x_i^+ - x_k^+) + 0}\right]^{\xi_i+\xi_k}\times\right. \\ \left.\times\left[i0\,\varepsilon(x_i^0 - x_k^0) - (x_i - x_k)^2\right]^{\mathrm{e}^{2\eta}+\xi_i\xi_k\,\mathrm{e}^{-2\eta}}\right\}^{l_i l_k/4}. \qquad (42)\end{aligned}$$

[1]Here for $\xi_i \pm \xi_k$, we use $\xi_i, \xi_k = +1, -1$, and $\delta_{\xi_i,\xi_k} = (1+\xi_i\xi_k)/2$, $\delta_{\xi_i,\pm 1} = (1\pm\xi_i)/2$.

This expression provides at least five independent sources of superselection rules [2, 6] that usually are associated only with conservation of scalar field's (vector current's) charge O and pseudoscalar field's (pseudovector current's) charge O_5, respectively. For the p-point Wightman function (42) there are

$$\mathcal{S}_p \equiv \sum_{i=1}^{p} l_i \Rightarrow 0, \quad \mathcal{S}_{p5} \equiv \sum_{i=1}^{p} l_i \xi_i \Rightarrow 0. \tag{43}$$

The first one defined by Oksak and Morchio et al., due to the above-mentioned conservation of both charges, originates from the VEV of normal exponential in the r.h.s., taken, instead of $|0\rangle$, for the vacuum state $|\widehat{\upsilon}\rangle$ averaged with respect to the field-translation gauge group like (45) below, leading to [6–8]:

$$\langle\widehat{\upsilon}|\mathcal{N}_{\varphi}\left\{\exp\left(\sum_{j=1}^{p}\mathcal{R}_j\right)\right\}|\widehat{\upsilon}\rangle \Longrightarrow \delta_{\mathcal{S}_p,0}\,\delta_{\mathcal{S}_{p5},0}. \tag{44}$$

For the usual nondegenerate vacuum state $|0\rangle$ this VEV is equal to 1 identically [5]. Nevertheless, these rules arise from the second lines at the limit $\overline{\mu} \to 0$ as the natural conditions of nonzero result [4]. We can suggest now three additional sources of these rules. The third one is the ϖ- and Θ-independence condition for the VEV (39), (42), the fourth one follows from its independence of the regularization parameter a_0, and the fifth one follows from its independence of σ and ρ, whenever the corresponding transformations (19), (32) leave the vacuum invariant. Obviously, the a_0-independence of (42) automatically means its σ-, ρ-independence, and vice versa.

The independence on the initial values of overall and relative phases has purely fermionic nature and does not reduce to the (pseudo)scalar field-translation gauge symmetry (45), which can only shift their random initial values. It is worth noting that both superselection rules (43) leave necessary for VEV (39) only the ultraviolet renormalization. Whence, only the last product of (42) over $i < k$ survives, which does not depend on the parameters $\overline{\mu}$, σ, ρ, a_0, ϖ, Θ, and gives the well-known expression for the p-point functions [2, 5, 6] with correct dynamical dimensions.

2. DISCUSSION AND CONCLUSIONS

Since a_0 (17) itself makes sense of regularization parameter, it should disappear in physical quantities. But for any real υ it defines the VEV of the operator of the field-translation gauge transformation [6, 9], e.g.:

$$\exp\{i\upsilon O_5\}\,\phi(x)\exp\{-i\upsilon O_5\} = \phi(x) + \upsilon, \tag{45}$$

$$\langle 0|\exp\{i\upsilon O_5\}|0\rangle \equiv \langle 0|\upsilon\rangle = \exp\left\{-a_0\upsilon^2\right\}, \tag{46}$$

which is well known in quantum theory of free massless (pseudo)scalar field [9]. The state $|\upsilon\rangle$ is a coherent state of harmonic oscillator, which like (13) corresponds to zero mode $k^1 = 0$, for $L \to \infty$, and whenever $a_0 \to +\infty$ simultaneously, then $\langle 0|\upsilon\rangle \Rightarrow 0$ and the state $|\upsilon\rangle$ defines another orthogonal vacuum state of the degenerate family [4, 9], e.g., for the usual box of the length $2L$ [1]. The finite a_0 for any continuous piecewise-smooth function of volume cut-off regularization means the charge definition, which has nothing to do with previous thermodynamic limit and corresponds to another vacuum structure of the representation space of (pseudo)scalar field [1]. A randomization of function $k^0\delta_L(k^1)$ and corresponding value of a_0 may be used for infrared «stabilization» of Wightman function (15), which replaces the regularization parameter $\overline{\mu}$ to some finite scale M [4].

We see that, unlike Schwinger model [6, 10], as long as we deal with the solutions (33)–(36) of the above «phase-decoupled» HEqs for $\xi = \pm$, that preserve the ϖ- and Θ-arbitrariness, both of superselection rules (43) and the conservation of both of currents should be fulfilled independently of chosen phase of the theory, including the phase with spontaneously broken chiral symmetry [4]. Formally, from this viewpoint, the breaking of the second rule (43) can be achieved either by introducing the mass term into HEq «by hand» [4], or otherwise, by excluding a_0 via taking Mandelstam solution with $\sigma = 1$, $\rho = 0$, supplemented with fixing of the values $\overline{\mu} \mapsto M$ and Θ [4]. However, since one of the gauge symmetries remains unbroken: $O|0\rangle \Rightarrow 0$, all the solutions (24)–(30) being connected by transformation (19) refer to the same vacuum state $|0\rangle$ regardless of the value σ.

For $\rho \neq 0$ it is impossible to remove the a_0-dependence for any σ. It can be only adsorbed into the parameter $\overline{\mu}$, whenever the first role (43) is fulfilled, by the rescaling: $\overline{\mu} \mapsto M \exp\left\{a_0(\pi/4)[\mathrm{e}^{4\eta}\rho^2 + (1-\sigma)^2]\right\}$.

Only when both rules (43) are fulfilled, the VEV (42) does not depend on the regularization and transformation parameters: $\overline{\mu}$, a_0, and σ, ρ, ϖ, Θ, and on the choice of volume cut-off regularization. Whereas the discarding of second superselection rule (43) inevitably spoils the σ-, ρ-, and Θ-invariance of this p-point fermionic Wightman function and its independence of the parameters $\overline{\mu}$ and a_0. So, they should be fixed by some additional conditions [4], what seems impossible for the regularization-dependent value a_0. The DM onto the free massive fields [4, 11] will be free from those parameters.

Acknowledgements. The authors thank Y. Frishman, A. N. Vall, S. V. Lovtsov, V. M. Leviant, and the participants of the seminar in BLTP JINR for useful discussions.

This work was supported in part by the RFBR (project No. 09-02-00749) and by the program «Development of Scientific Potential in Higher Schools» (project No. 2.2.1.1/1483, 2.1.1/1539).

REFERENCES

1. *Korenblit S. E., Semenov V. V.* Massless Thirring Model in Canonical Quantization Scheme // J. Nonlin. Math. Phys. 2011. V. 18. P. 65; arXiv:1003.1439 v.2[hep-th];
   Korenblit S. E., Semenov V. V. On Fermionic Tilde Conjugation Rules and Thermal Bosonization. Hot and Cold Thermofields // Phys. Part. Nucl. Lett. 2011. V. 8, No. 7. P. 779; arXiv:1108.5392[hep-th];
   Semenov V. V., Korenblit S. E. Finite Temperature Thirring Model: From Linearization through Canonical Transformations to Correct Normal Form of Thermofield Solution. arXiv:1109.2278[hep-th].
2. *Klaiber B.* Lectures in Theoretical Physics. Boulder: University of Colorado, 1967; Ed. by A. Barut and W. Brittin. New York: Gordon and Breach, 1968. V. X, part A. P. 141–176.
3. *Dell'Antonio G. F., Frishman Y., Zwanziger D.* Thirring Model in Terms of Currents: Solution and Light-Cone Expansions // Phys. Rev. D. 1972. V. 6. P. 988.

4. *Faber M., Ivanov A. N.* On the Equivalence between Sine-Gordon Model and Thirring Model in the Chirally Broken Phase of the Thirring Model // Eur. Phys. J. C. 2001. V. 20. P. 723; On Free Massless (Pseudo-) Scalar Quantum Field Theory in $(1+1)$-Dimensional Space-Time // Eur. Phys. J. C. 2002. V. 24. P. 653;
   Fujita T. et al. New Vacuum of Bethe Ansatz Solutions in Thirring Model // J. Phys. Soc. Japan. 2005. V. 74. P. 1143;
   Fujita T., Hiramoto M., Takahashi H. Reinterpretation of Spontaneous Symmetry Breaking in Quantum Field Theory and Goldstone Theorem. arXiv:hep-th/0510151.
5. *Mandelstam S.* Soliton Operators for the Quantized Sine-Gordon Equation // Phys. Rev. D. 1975. V. 11. P. 3026;
   Coleman S. Quantum Sine-Gordon Equation as the Massive Thirring Model // Ibid. P. 2088.
6. *Bogoliubov N. N. et al.* General Principles of Quantum Field Theory. Boston: Kluwer Acad. Publ., 1990.
7. *Oksak A. I.* Non-Fock Linear Boson Systems and Their Applications in Two-Dimensional Models // Teor. Mat. Fiz. 1981. V. 48. P. 297.
8. *Morchio G., Pierotti D., Strocchi F.* Infrared and Vacuum Structure in Two-Dimensional Local Quantum Field Theory Models. Fermion Bosonization // J. Math. Phys. 1992. V. 33. P. 777.
9. *Itzykson C., Zuber J.-B.* Quantum Field Theory. V. 1, 2. New York: McGraw-Hill Inc., 1980.
10. *Rubakov V. A.* Classical Gauge Fields. Theories with Fermions. Noncommutative Theories. M.: URSS, 2005.
11. *Chang S. J., Rajaraman R.* Chiral Vertex Operators in Off-Conformal Theory: The Sine-Gordon Example // Phys. Rev. D. 1996. V. 53. P. 2102.