Письма в ЭЧАЯ. 2013. Т. 10, № 1(178). С. 51–57

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

DKP EQUATION UNDER A VECTOR YUKAWA-TYPE POTENTIAL

S. Hassanabadi, A. A. Rajabi, S. Zarrinkamar, H. Hassanabadi [1]

Physics Department, Shahrood University of Technology, Shahrood, Iran

Approximate analytical solutions of the Duffin–Kemmer–Petiau equation are obtained for a vector Yukawa potential. The solutions are reported for any J state. The results are obtained based on an exponential approximation as well as the NU analytical technique. Energy solutions are numerically discussed for various quantum numbers and different η, α.

Получены приближенные аналитические решения уравнения Duffin–Kemmer–Petiau для векторного юкавского потенциала. Решения обсуждаются для любых J-состояний. Результаты получены с помощью экспоненциальной аппроксимации и аналитического метода Никифорова–Уварова. Полученные численные решения для энергий обсуждаются для различных квантовых чисел и параметров η, α.

PACS: 03.65.Ca; 03.65.Pm; 03.65.Nk

INTRODUCTION

Based on the elegant idea of Duffin, Kemmer, and Petiau, the equation named after them, i.e., Duffin–Kemmer–Petiau, hence abbreviated DKP equation, provides us with a rich theoretical background to study most common integer spin systems — spin-0 and spin-1 ones [1–4]. On the contrary to Dirac and Klein–Gordon (KG) equation, there have been only few studies on the DKP equation, as for many years a large community of physicists thought that KG and DKP equations are exactly the same. Recently, however, the interest in the latter has been increased as the equivalence is known to be generally violated [5–14]. Furthermore, DKP equation enables us to study spin-1 particles and provides us with a richer background to study interactions. Interestingly, in some cases, the present experimental data vote in favor of DKP equation rather than KG or Proca equations [15–20]. As a final merit of this equation, it should be that not only in the annals of cosmology and gravity, but also particle and nuclear physics seek for such a rich wave equation [21–25]. DKP equation for Coulomb and quadratic terms has been studied in [26–28]. Woods–Saxon and Hulthen equations are investigated by different approaches as well [29–34]. Nevertheless, within the present work, we use NU analytical technique for Yukawa potential [35, 37] which is much easier and richer. In the present work, we obtain approximate analytical energy eigenvalues, normalized wave functions for the Yukawa potential of the form

$$U(r) = -\frac{\eta}{r}\mathrm{e}^{-\alpha r}, \tag{1}$$

[1] E-mail: h.hasanabadi@shahroodut.ac.ir

where α is the screening parameter and η is the strength of the potential. It is well known that Yukawa potential was used in various fields of physics. In high-energy physics, it is applied to consider the interaction of hadrons in short-range gauge theories [38–40], in atomic and molecular physics [41, 42], solid state [43]. This potential also describes the shielding effect of ions embedded in plasmas where it is called the Debye–Huckel potential [44]. We first review the DKP equation. Next, a thorough introduction to the Nikiforov–Uvarov method is presented, and finally energy spectra for various quantum numbers and different η, α are reported.

DKP EQUATION

The DKP Hamiltonian for scalar and vector interactions is

$$(\beta \mathbf{p} c + mc^2 + U_s + \beta^0 U_v^0)\psi(\mathbf{r}) = \beta^0 E\psi(\mathbf{r}), \tag{2}$$

where

$$\psi(\mathbf{r}) = \begin{pmatrix} \psi_{\text{upper}} \\ i\psi_{\text{lower}} \end{pmatrix}. \tag{3}$$

The upper and lower components, respectively, are

$$\psi_{\text{upper}} \equiv \begin{pmatrix} \varphi \\ \phi \end{pmatrix}, \quad \psi_{\text{lower}} \equiv \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}. \tag{4}$$

β^0 is the usual 5×5 matrix and U_s, U_v^0, respectively, represent the scalar and vector interactions. The equation, in $(3 + 0)$-dimensions, is written as [1–4]

$$(mc^2+U_s)\phi = (E-U_v^0)\varphi+\hbar c\boldsymbol{\nabla}\mathbf{A}, \quad \boldsymbol{\nabla}\phi = (mc^2+U_s)\mathbf{A}, \quad (mc^2+U_s)\varphi = (E-U_v^0)\phi, \tag{5}$$

where $\mathbf{A} = (A_1, A_2, A_3)$. In Eq. (3), ψ is a simultaneous eigenfunction of J^2 and J_3, i.e.,

$$\begin{aligned} J^2 \begin{pmatrix} \psi_{\text{upper}} \\ \psi_{\text{lower}} \end{pmatrix} &= \begin{pmatrix} L^2\psi_{\text{upper}} \\ (L+S)^2\psi_{\text{lower}} \end{pmatrix} = J(J+1) \begin{pmatrix} \psi_{\text{upper}} \\ \psi_{\text{lower}} \end{pmatrix}, \\ J_3 \begin{pmatrix} \psi_{\text{upper}} \\ \psi_{\text{lower}} \end{pmatrix} &= \begin{pmatrix} L_3\psi_{\text{upper}} \\ (L_3+s_3)\psi_{\text{lower}} \end{pmatrix} = M \begin{pmatrix} \psi_{\text{upper}} \\ \psi_{\text{lower}} \end{pmatrix}, \end{aligned} \tag{6}$$

and the general solution is considered as

$$\psi_{JM}(r) = \begin{pmatrix} f_{nJ}(r)Y_{JM}(\Omega) \\ g_{nJ}(r)Y_{JM}(\Omega) \\ i\sum\limits_{L} h_{nJL}(r)Y_{JL1}^{M}(\Omega) \end{pmatrix}, \tag{7}$$

where spherical harmonics $Y_{JM}(\Omega)$ are of order J; $Y_{JL1}^{M}(\Omega)$ are the normalized vector spherical harmonics; and f_{nJ}, g_{nJ}, and h_{nJL} represent the radial wavefunctions. The above equations yield the coupled differential equations [26, 35]

$$
\begin{aligned}
&(E_{n,J} - U_v^0)F_{n,J}(r) = (mc^2 + U_s)G_{n,J}(r), \\
&\left(\frac{dF_{n,J}(r)}{dr} - \frac{J+1}{r}F_{n,J}(r)\right) = -\frac{1}{\alpha_J}(mc^2 + U_s)H_{1,n,J}(r), \\
&\left(\frac{dF_{n,J}(r)}{dr} + \frac{J}{r}F_{n,J}(r)\right) = \frac{1}{\zeta_J}(mc^2 + U_s)H_{-1,n,J}(r), \\
&-\alpha_J\left(\frac{dH_{1,n,J}(r)}{dr} + \frac{J+1}{r}H_{1,n,J}(r)\right) + \zeta\left(\frac{dH_{-1,n,J}(r)}{dr} - \frac{J}{r}H_{-1,n,J}(r)\right) = \\
&\qquad = \frac{1}{\hbar c}\left((mc^2 + U_s)F_{n,J}(r) - (E_{n,J} - U_v^0)G_{n,J}(r)\right),
\end{aligned}
\tag{8}
$$

which gives

$$
\begin{aligned}
&\frac{d^2F_{n,J}(r)}{dr^2}\left[1 + \frac{\zeta_J^2}{\alpha_J^2}\right] - \frac{dF_{n,J}(r)}{dr}\left[\frac{U_s'}{(m+U_s)}\left(1 + \frac{\zeta_J^2}{\alpha_J^2}\right)\right] + \\
&\quad + F_{n,J}(r)\left[-\frac{J\,(J+1)}{r^2}\left(1 + \frac{\zeta_J^2}{\alpha_J^2}\right) + \frac{U_s'}{(m+U_s)}\left(\frac{J+1}{r} - \frac{\zeta_J^2}{\alpha_J^2}\frac{J}{r}\right) - \right. \\
&\qquad \left. -\frac{1}{\alpha_J^2}\left((m+U_s)^2 - \left(E_{n,J} - U_v^0\right)^2\right)\right] = 0,
\end{aligned}
\tag{9}
$$

where $\alpha_J = \sqrt{(J+1)/(J+1)}$ and $\zeta_J = \sqrt{J/(2J+1)}$. When $U_s = 0$, we recover the well-known formula [26, 35]

$$
\left(\frac{d^2}{dr^2} - \frac{J(J+1)}{r^2} + (E_{n,J} - U_v^0)^2 - m^2\right)F_{n,J}(r) = 0. \tag{10}
$$

Then we have subsisting Yukawa potential in Eq. (10) yielding in

$$
\left(\frac{d^2}{dr^2} - \frac{J(J+1)}{r^2} + \left(E_{n,J} + \frac{\eta}{r}\mathrm{e}^{-\alpha r}\right)^2 - m^2\right)F_{n,J}(r) = 0. \tag{11}
$$

Now, using the approximations [45, 50]

$$
\frac{1}{r^2} \approx \frac{4\alpha^2\mathrm{e}^{-2\alpha r}}{(1 - \mathrm{e}^{-2\alpha r})^2}, \tag{12}
$$

Eq. (11) reduces to

$$
\left(\frac{d^2}{dr^2} - \frac{4J(J+1)\alpha^2\mathrm{e}^{-2\alpha r}}{(1 - \mathrm{e}^{-2\alpha r})^2} + E_{n,J}^2 + \frac{4\eta^2\alpha^2\mathrm{e}^{-4\alpha r}}{(1 - \mathrm{e}^{-2\alpha r})^2} + \frac{4E_{n,J}\eta\alpha\,\mathrm{e}^{-2\alpha r}}{(1 - \mathrm{e}^{-2\alpha r})} - m^2\right)F_{n,J}, \tag{13}
$$

as well as the transformation $s = \mathrm{e}^{-2\alpha r}$ brings Eq. (13) into the form

$$
\begin{aligned}
&\frac{d^2}{ds^2} + \frac{(1-s)}{s(1-s)}\frac{d}{ds} + \frac{1}{s^2(1-s)^2}\left\{s^2\left[\frac{E^2}{4\alpha^2} + \eta^2 - \frac{m^2}{4\alpha^2} - \frac{E\eta}{\alpha}\right] + \right. \\
&\qquad \left. + s\left[-J(J+1) - \frac{E^2}{2\alpha^2} + \frac{E\eta}{\alpha} + \frac{m^2}{2\alpha^2}\right] + \frac{E^2}{4\alpha^2} - \frac{m^2}{4\alpha^2}\right\}F_{n,J}(r) = 0.
\end{aligned}
\tag{14}
$$

THE NIKIFOROV–UVAROV METHOD

The NU method, which was based on the elegant idea of Nikiforov and Uvarov, enables one to analytically solve a wide class of second-order differential equations into mathematical. Let us consider an ordinary differential equation of the form [51, 52]

$$\left[\frac{d^2}{ds^2} + \frac{\alpha_1 - \alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds} + \frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{[s(1-\alpha_3 s)]^2}\right]\psi(s) = 0. \tag{15}$$

Based on the NU technique, the eigenfunctions are

$$\psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} p_n^{\left(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \tag{16}$$

and the energy eigenvalues satisfy

$$\alpha_2 n - (2n+1)\alpha_5 + (2n+1)\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right) + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0, \tag{17}$$

where

$$\alpha_4 = \frac{1}{2}(1-\alpha_1), \quad \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3), \quad \alpha_6 = \alpha_5^2 + \xi_1, \quad \alpha_7 = 2\alpha_4\alpha_5 - \xi_2, \tag{18a}$$

$$\alpha_8 = \alpha_4^2 + \xi_3, \tag{18b}$$

$$\alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6, \tag{18c}$$

$$\alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}, \tag{18d}$$

$$\alpha_{11} = \alpha_2 - 2\alpha_5 + 2\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right), \tag{18e}$$

$$\alpha_{12} = \alpha_4 + \sqrt{\alpha_8}, \tag{18f}$$

$$\alpha_{13} = \alpha_5 - \left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right). \tag{18g}$$

EXACT BOUND-STATE SOLUTIONS

Therefore, using Eqs. (14)–(18), the compact form of the energy relations can be found as

$$n(n+1) + \frac{1}{2} + (2n+1)\sqrt{\frac{1}{4} + J(J+1)} + (2n+1)\sqrt{\frac{-E_{n,J}^2}{4\alpha^2} + \frac{m^2}{4\alpha^2}} + J(J+1) - \frac{E_{n,J}\eta}{\alpha} + 2\sqrt{\left(\frac{1}{4} + J(J+1)\right)\left(-\frac{E_{n,J}^2}{4\alpha^2} + \frac{m^2}{4\alpha^2}\right)} = 0, \tag{19}$$

and eigenfunctions are obtained as

$$F_{n,J} = s^{\sqrt{-\frac{E_{n,J}^2}{4\alpha^2} + \frac{m^2}{4\alpha^2}}}(1-s)^{\frac{1}{2}+\sqrt{\frac{1}{4}+J(J+1)}} P_n^{\left(2\sqrt{-\frac{E_{n,J}^2}{4\alpha^2}+\frac{m^2}{4\alpha^2}},\, 2\sqrt{\frac{1}{4}+J(J+1)}\right)}(1-2s). \tag{20}$$

CONCLUSION

We have solved the DKP equation for Yukawa potential by using new approximations to the centrifugal and Coulombic terms, and closed forms of energy spectra and corresponding wave functions are obtained by using NU method. Energy spectra for various quantum numbers and different $\eta, \alpha, m = 1$ are depicted in Tables 1–3. The results are applicable to many branches of physics where such types of potentials are present.

Table 1. **Energy spectra for various quantum numbers and** $\alpha = 0.001$**,** $\eta = 0.7$**,** $m = 1$

$\|n, J\rangle$	$E_{0,J}$ for $\alpha = 0.001$, $\eta = 0.7$	$\|n, J\rangle$	$E_{1,J}$ for $\alpha = 0.001$, $\eta = 0.7$	$\|n, J\rangle$	$E_{2,J}$ for $\alpha = 0.001$, $\eta = 0.7$	$\|n, J\rangle$	$E_{3,J}$ for $\alpha = 0.001$, $\eta = 0.7$
$\|0,0\rangle$	0.974500	$\|1,0\rangle$	0.985702	$\|2,0\rangle$	0.991016	$\|3,0\rangle$	0.993936
$\|0,1\rangle$	0.985702	$\|1,1\rangle$	0.991016	$\|2,1\rangle$	0.993936	$\|3,1\rangle$	0.995706
$\|0,2\rangle$	0.991016	$\|1,2\rangle$	0.993936	$\|2,2\rangle$	0.995706	$\|3,2\rangle$	0.996856
$\|0,3\rangle$	0.993936	$\|1,3\rangle$	0.995706	$\|2,3\rangle$	0.996856	$\|3,3\rangle$	0.997644
$\|0,4\rangle$	0.995706	$\|1,4\rangle$	0.996856	$\|2,4\rangle$	0.997644	$\|3,4\rangle$	0.998205
$\|0,5\rangle$	0.996856	$\|1,5\rangle$	0.997644	$\|2,5\rangle$	0.998205	$\|3,5\rangle$	0.998618
$\|0,6\rangle$	0.997644	$\|1,6\rangle$	0.998205	$\|2,6\rangle$	0.998618	$\|3,6\rangle$	0.998928
$\|0,7\rangle$	0.998205	$\|1,7\rangle$	0.998618	$\|2,7\rangle$	0.998928	$\|3,7\rangle$	0.999167
$\|0,8\rangle$	0.998618	$\|1,8\rangle$	0.998928	$\|2,8\rangle$	0.999172	$\|3,8\rangle$	0.999352
$\|0,9\rangle$	0.998928	$\|1,9\rangle$	0.999167	$\|2,9\rangle$	0.999352	$\|3,9\rangle$	0.999499
$\|0,10\rangle$	0.999167	$\|1,10\rangle$	0.999352	$\|2,10\rangle$	0.999499	$\|3,10\rangle$	0.999615

Table 2. **Energy spectra for various quantum numbers and** $\alpha = 0.0015$, $\eta = 0.9$**,** $m = 1$

$\|n, J\rangle$	$E_{0,J}$ for $\alpha = 0.0015$, $\eta = 0.9$	$\|n, J\rangle$	$E_{1,J}$ for $\alpha = 0.0015$, $\eta = 0.9$	$\|n, J\rangle$	$E_{2,J}$ for $\alpha = 0.0015$, $\eta = 0.9$	$\|n, J\rangle$	$E_{3,J}$ for $\alpha = 0.0015$, $\eta = 0.9$
$\|0,0\rangle$	0.913040	$\|1,0\rangle$	0.959055	$\|2,0\rangle$	0.976877	$\|3,0\rangle$	0.985464
$\|0,1\rangle$	0.959055	$\|1,1\rangle$	0.976877	$\|2,1\rangle$	0.985464	$\|3,1\rangle$	0.990217
$\|0,2\rangle$	0.976877	$\|1,2\rangle$	0.985464	$\|2,2\rangle$	0.990217	$\|3,2\rangle$	0.993110
$\|0,3\rangle$	0.985464	$\|1,3\rangle$	0.990217	$\|2,3\rangle$	0.993110	$\|3,3\rangle$	0.994993
$\|0,4\rangle$	0.990217	$\|1,4\rangle$	0.993110	$\|2,4\rangle$	0.994993	$\|3,4\rangle$	0.996284
$\|0,5\rangle$	0.993110	$\|1,5\rangle$	0.994993	$\|2,5\rangle$	0.996284	$\|3,5\rangle$	0.997202
$\|0,6\rangle$	0.994993	$\|1,6\rangle$	0.996284	$\|2,6\rangle$	0.997202	$\|3,6\rangle$	0.997875
$\|0,7\rangle$	0.996284	$\|1,7\rangle$	0.997202	$\|2,7\rangle$	0.997875	$\|3,7\rangle$	0.998381
$\|0,8\rangle$	0.997202	$\|1,8\rangle$	0.997875	$\|2,8\rangle$	0.998381	$\|3,8\rangle$	0.998766
$\|0,9\rangle$	0.997875	$\|1,9\rangle$	0.998381	$\|2,9\rangle$	0.998766	$\|3,9\rangle$	0.999065
$\|0,10\rangle$	0.998381	$\|1,10\rangle$	0.998766	$\|2,10\rangle$	0.999065	$\|3,10\rangle$	0.999298

Table 3. **Energy spectra for various quantum numbers and** $\alpha = 0.002$, $\eta = 0.8$, $m = 1$

$\|n, J\rangle$	$E_{0,J}$ for $\alpha = 0.002$, $\eta = 0.8$	$\|n, J\rangle$	$E_{1,J}$ for $\alpha = 0.002$, $\eta = 0.8$	$\|n, J\rangle$	$E_{2,J}$ for $\alpha = 0.002$, $\eta = 0.8$	$\|n, J\rangle$	$E_{3,J}$ for $\alpha = 0.002$, $\eta = 0.8$
$\|0, 0\rangle$	0.929849	$\|1, 0\rangle$	0.967712	$\|2, 0\rangle$	0.982088	$\|3, 0\rangle$	0.988952
$\|0, 1\rangle$	0.967712	$\|1, 1\rangle$	0.982088	$\|2, 1\rangle$	0.988952	$\|3, 1\rangle$	0.992729
$\|0, 2\rangle$	0.982088	$\|1, 2\rangle$	0.988952	$\|2, 2\rangle$	0.992729	$\|3, 2\rangle$	0.995015
$\|0, 3\rangle$	0.988952	$\|1, 3\rangle$	0.992729	$\|2, 3\rangle$	0.995015	$\|3, 3\rangle$	0.996495
$\|0, 4\rangle$	0.992729	$\|1, 4\rangle$	0.995015	$\|2, 4\rangle$	0.996495	$\|3, 4\rangle$	0.997499
$\|0, 5\rangle$	0.995015	$\|1, 5\rangle$	0.996495	$\|2, 5\rangle$	0.997499	$\|3, 5\rangle$	0.998206
$\|0, 6\rangle$	0.996495	$\|1, 6\rangle$	0.997499	$\|2, 6\rangle$	0.998206	$\|3, 6\rangle$	0.998717
$\|0, 7\rangle$	0.997499	$\|1, 7\rangle$	0.998206	$\|2, 7\rangle$	0.998717	$\|3, 7\rangle$	0.999091
$\|0, 8\rangle$	0.998206	$\|1, 8\rangle$	0.998717	$\|2, 8\rangle$	0.999091	$\|3, 8\rangle$	0.999369
$\|0, 9\rangle$	0.998717	$\|1, 9\rangle$	0.999091	$\|2, 9\rangle$	0.999369	$\|3, 9\rangle$	0.999575
$\|0, 10\rangle$	0.999091	$\|1, 10\rangle$	0.999369	$\|2, 10\rangle$	0.999575	$\|3, 10\rangle$	0.999728

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Received on October 9, 2011.