

CALCULATING PARTICLE CORRELATORS WITH THE ACCOUNT OF DETECTOR EFFICIENCY

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The formulae for m -order correlators K_m of a given particle observable (e.g., energy, transverse momentum or a conserved discrete quantum number) accounting for the track reconstruction efficiencies in a real detector are presented. The calculation of second- to fourth-order correlators is considered in some detail. Similar to the case of an ideal detector, the correlators can be expressed through the event-by-event fluctuation measures of the observable single event mean, the pseudocorrelators (determined by the pseudocentral moments of the observable distribution) and their cross terms. It allows one to avoid the combinatorics and essentially reduce the computer time when calculating the higher-order correlators in high multiplicity events. Compared with the case of ideal detector, this reduction is somewhat smaller due to the increased number of pseudocorrelators and additional calculations of the moments of the distribution of the track weights. For a constant track reconstruction efficiency, the correlator formulae reduce to those for an ideal detector. However, in real experiments the efficiencies are usually essentially dependent on particle momenta and may lead to substantial corrections of momentum correlators on the level of tens of percent.

Представлены формулы для корреляторов m -го порядка K_m наблюдаемых для частиц величин (таких как энергия, поперечный импульс или сохраняющиеся дискретные квантовые числа) с учетом эффективности реконструкции треков. Приведены некоторые подробности вычислений корреляторов от второго до четвертого порядка. Подобно случаю идеального детектора корреляторы могут быть выражены через пособытийные характеристики флуктуации среднего значения наблюдаемой величины, псевдокорреляторы (обусловленные псевдоцентральными моментами распределения наблюдаемой величины) и их перекрестные члены. Это позволяет избежать комбинаторики и существенно уменьшить время вычислений корреляторов более высоких порядков. Однако в сравнении с идеальным случаем это уменьшение менее значительно из-за большего числа псевдокорреляторов и дополнительных вычислений моментов распределений весов. При постоянной эффективности реконструкции треков формулы для корреляторов сводятся к формулам, полученным для идеального детектора. Однако в реальных экспериментах эффективность реконструкции треков может существенно зависеть от изучаемой наблюдаемой величины и приводить к значительным поправкам к импульсным корреляторам на уровне десятков процентов.

PACS: 06.20.-f; 06.20.Dk

INTRODUCTION

The investigation of correlations is very important for hadron physics [1–4]. The integral correlation characteristics — the correlators of particle energies, transverse momenta or rapidities — have been suggested [5,6] to study the production mechanism of very high multiplicity events. It was shown [7] that the correlators are closely related with the event-by-event fluctuations of the event mean particle observables.

It should be noted that the higher-order fluctuation measures or the higher, non-Gaussian, moments of the event-by-event distribution of the observable mean (related with skewness and kurtosis for the orders $m = 3$ and 4 , respectively) are more sensitive signatures of the critical phenomena in multiparticle production (e.g., in the case of particle freeze-out near the critical endpoint) since they increase as powers $\xi^{5m/2-3}$ of the correlation length ξ [4].

In the case of an ideal 100% efficient detector, a fast and simple procedure to calculate the correlators with the help of the fluctuation measures and so-called event-wise pseudocorrelators has been suggested [7], exploiting the expressions of pseudocorrelators through the central moments of the observable distribution [8]. Using the PYTHIA generator, the multiplicity dependence of second- and third-order pseudocorrelators and their ratio have been studied in [9]. The correction terms generated in the correlator analysis due to the multiplicity-dependent observable mean have been investigated in [10]. The two-particle transverse momentum correlators have been used as a correlation measure and studied as a function of event centrality in Au + Au collisions at RHIC [11]. Both the analyses in [10] and [11] are valid on the assumption of a constant detector efficiency.

In this paper, we formulate a fast decomposition procedure to calculate the correlators, avoiding the combinatorics in the case of observable-dependent track reconstruction efficiencies.

1. PARTICLE CORRELATORS IN THE CASE OF IDEAL DETECTOR

In the case of an ideal detector, the m th order correlator in the events with a given charged hadron multiplicity n is defined as

$$K_m(n) = \left\langle \frac{1}{C_m^n} \sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n \Delta\varepsilon_{i_1}^{(l)} \cdots \Delta\varepsilon_{i_m}^{(l)} \right\rangle, \quad (1)$$

$$\Delta\varepsilon_{i_\lambda}^{(l)} = \varepsilon_{i_\lambda}^{(l)} - \langle \varepsilon \rangle. \quad (2)$$

Here $C_m^n = \frac{n!}{m!(n-m)!}$ is the normalization factor equal to the number of combinations;

$\varepsilon_{i_\lambda}^{(l)}$ is the observable (e.g., energy, momentum, strangeness, electric or baryon charge) of the i_λ th charged hadron ($i_1 < \dots < i_m$) in the l th event; n is the charged hadron multiplicity in an event. The observable mean

$$\langle \varepsilon \rangle = \langle \bar{\varepsilon}^{(l)} \rangle, \quad (3)$$

where $\bar{\varepsilon}^{(l)}$ is the observable average in the l th event:

$$\bar{\varepsilon}^{(l)} = \frac{1}{n} \sum_{i=1}^n \varepsilon_i^{(l)}, \quad (4)$$

and

$$\langle \rangle = \frac{1}{N(n)} \sum_{l=1}^{N(n)} \quad (5)$$

stands for the averaging over the $N(n)$ events with the charged hadron multiplicity n . Note that the correlator formula (1), when formally applied to one particle, yields $K_1(n) = 0$ according to definition of the observable means in (3) and (4).

Defining

$$\Delta\bar{\varepsilon}^{(l)} = \bar{\varepsilon}^{(l)} - \langle \varepsilon \rangle, \quad (6)$$

$$\Delta\tilde{\varepsilon}_i^{(l)} = \varepsilon_i^{(l)} - \bar{\varepsilon}^{(l)} \quad (7)$$

and using the equality $\Delta\varepsilon_i^{(l)} = \Delta\tilde{\varepsilon}_i^{(l)} + \Delta\bar{\varepsilon}^{(l)}$, one can decompose the correlator on the event-by-event fluctuations of the event mean observable $\Delta\bar{\varepsilon}^{(l)}$, event-wise pseudocorrelators $k_\lambda^{(l)}(n)$ and the corresponding cross terms [7]:

$$K_m(n) = \left\langle \sum_{\lambda=0}^m C_\lambda^m \Delta\bar{\varepsilon}^{(l)m-\lambda} k_\lambda^{(l)}(n) \right\rangle, \quad (8)$$

where $k_0^{(l)} = 1$. The event-wise pseudocorrelators are defined similarly to (1) up to the substitution $\Delta\varepsilon_i^{(l)} \rightarrow \Delta\tilde{\varepsilon}_i^{(l)}$:

$$k_m^{(l)}(n) = \frac{1}{C_m^n} \sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n \Delta\tilde{\varepsilon}_{i_1}^{(l)} \cdots \Delta\tilde{\varepsilon}_{i_m}^{(l)}. \quad (9)$$

Similar to the correlator, the first-order pseudocorrelator also vanishes by definition: $k_1^{(l)} = 0$. It is remarkable that one can avoid the combinatorics in (9), expressing the pseudocorrelators through the central moments of the observable distribution [7,8] (see also Sec. 3).

2. ACCOUNTING FOR TRACK RECONSTRUCTION EFFICIENCIES

In the case of a nonideal detector, one has to account for the track reconstruction efficiencies with the help of the weighting function

$$w_i^{(l)} = \frac{f_i^{(l)}}{\omega_i^{(l)}}, \quad (10)$$

where $\omega_i^{(l)}$ is the track reconstruction efficiency depending on particle pseudorapidity η and transverse momentum p_T associated with i th track in l th event, and $f_i^{(l)}$ is a function correcting for fake tracks, secondary and out of kinematic region particles.

The efficiency corrected average observable in the l th event is

$$\bar{\varepsilon}^{(l)} = \frac{\sum_{i=1}^n \varepsilon_i^{(l)} w_i^{(l)}}{\sum_{i=1}^n w_i^{(l)}}. \quad (11)$$

For the efficiency corrected m -particle correlator, we have

$$K_m(n) = \left\langle \frac{\sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n w_{i_1}^{(l)} \cdots w_{i_m}^{(l)} \Delta \varepsilon_{i_1}^{(l)} \cdots \Delta \varepsilon_{i_m}^{(l)}}{\sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n w_{i_1}^{(l)} \cdots w_{i_m}^{(l)}} \right\rangle. \quad (12)$$

The decomposition similar to (8) now takes the form

$$K_m(n) = \left\langle \sum_{\lambda=0}^m C_\lambda^m \Delta \bar{\varepsilon}^{(l) m-\lambda} k_\lambda^{(l,m)}(n) \right\rangle, \quad (13)$$

where $k_0^{(l,m)} = 1$. Note that now the pseudocorrelators $k_\lambda^{(l,m)}$ depend also on the correlator order m :

$$k_\lambda^{(l,m)}(n) = \frac{\sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n w_{i_1}^{(l)} \cdots w_{i_m}^{(l)} \Delta \tilde{\varepsilon}_{i_1}^{(l)} \cdots \Delta \tilde{\varepsilon}_{i_\lambda}^{(l)}}{\sum_{i_1=1}^{n-(m-1)} \cdots \sum_{i_m=i_{m-1}+1}^n w_{i_1}^{(l)} \cdots w_{i_m}^{(l)}}. \quad (14)$$

Obviously, such a pseudocorrelator coincides with the true one for $\lambda = m$ only: $k_m^{(l,m)} = k_\lambda^{(l)}$. Again, $K_1 = k_1^{(l)} = 0$ by definition. Note, however, that the pseudocorrelators $k_1^{(l,m)}$ do not vanish for $m > 1$.

Particularly, the second-order correlator can be decomposed as

$$K_2(n) = \langle \Delta \bar{\varepsilon}^{(l)2} + 2\Delta \bar{\varepsilon}^{(l)} k_1^{(l,2)}(n) + k_2^{(l,2)}(n) \rangle. \quad (15)$$

Here the first term $\langle \Delta \bar{\varepsilon}^{(l)2} \rangle$ is a quadratic measure of the fluctuation of the observable event-wise mean around the sample mean. The second term is a cross term which vanishes in the ideal case of unit reconstruction weights $w_i^{(l)}$ since the first-order event-wise pseudocorrelator for ideal detector $k_1^{(l)}$ vanishes by definition.

Similarly, the three-particle correlator is decomposed into four terms:

$$K_3(n) = \langle \Delta \bar{\varepsilon}^{(l)3} + 3\Delta \bar{\varepsilon}^{(l)2} k_1^{(l,3)}(n) + 3\Delta \bar{\varepsilon}^{(l)} k_2^{(l,3)}(n) + k_3^{(l,3)}(n) \rangle. \quad (16)$$

Here the first term $\langle \Delta \bar{\varepsilon}^{(l)3} \rangle$ is a cubic measure of the fluctuation of the observable event-wise mean around the sample mean. The second and third terms are cross terms, the first of them vanishing in the case of an ideal detector due to vanishing of the first-order event-wise pseudocorrelator $k_1^{(l)}$.

3. CALCULATING EVENT-WISE PSEUDOCORRELATORS THROUGH PSEUDOCENTRAL MOMENTS OF OBSERVABLE DISTRIBUTION

We will describe in some detail the calculation of the event-wise second- and third-order pseudocorrelators with the help of pseudocentral moments of the observable distribution.

In the case of an ideal detector, one may use the identity

$$\sum_{i=1}^n \Delta \tilde{\varepsilon}_i^{(l)} = 0 \quad (17)$$

and its powers to express the event-wise pseudocorrelators $k_m^{(l)}(n)$ through the central moments $S_\lambda^{(l)}$, $\lambda \leq m$, of the single-particle observable distribution,

$$S_\lambda^{(l)}(n) = \frac{1}{n} \sum_{i=1}^n \Delta \tilde{\varepsilon}_i^{(l)\lambda}. \quad (18)$$

Thus, for the second- and third-order pseudocorrelator, we have [7, 8]:

$$k_2^{(l)}(n) = -\frac{1}{n-1} S_2^{(l)}, \quad (19)$$

$$k_3^{(l)}(n) = \frac{2}{(n-1)(n-2)} S_3^{(l)}. \quad (20)$$

Using the identity

$$\sum_{i=1}^n \sum_{j=1, \neq i}^n w_i^{(l)} w_j^{(l)} f_{ij} = \sum_{i=1}^n w_i^{(l)} \left(\sum_{j=1}^n w_j^{(l)} f_{ij} - w_i^{(l)} f_{ii} \right), \quad (21)$$

where f_{ij} are arbitrary functions. After generalizing identity (17) and its second power for the case of a nonideal detector:

$$\sum_{i=1}^n w_i^{(l)} \Delta \tilde{\varepsilon}_i^{(l)} = 0, \quad (22)$$

$$\sum_{i=1}^n (w_i^{(l)} \Delta \tilde{\varepsilon}_i^{(l)})^2 + \sum_{i=1}^n \sum_{j=1, \neq i}^n w_i^{(l)} w_j^{(l)} \Delta \tilde{\varepsilon}_i^{(l)} \Delta \tilde{\varepsilon}_j^{(l)} = 0, \quad (23)$$

and substituting the sums over the ordered m -plets $\{i_1 < \dots < i_m\}$ in the pseudocorrelator definitions by the sums over the m -plets $\{i_1 \neq \dots \neq i_m\}$, one gets for the first- and second-order pseudocorrelators contributing to the second-order correlator:

$$k_1^{(l,2)}(n) = -\frac{\sum_{i=1}^n w_i^{(l)2} \Delta \tilde{\varepsilon}_i^{(l)}}{n(\overline{n\bar{w}^{(l)2}} - \overline{w^{(l)2}})}, \quad (24)$$

$$k_2^{(l,2)}(n) \equiv k_2^{(l)}(n) = -\frac{\sum_{i=1}^n w_i^{(l)2} \Delta \tilde{\varepsilon}_i^{(l)2}}{n(\overline{n\bar{w}^{(l)2}} - \overline{w^{(l)2}})}, \quad (25)$$

where

$$\overline{w^{(l)\lambda}} = \frac{1}{n} \sum_{i=1}^n w_i^{(l)\lambda}. \quad (26)$$

Formula (25) shows that the pseudocorrelator $k_2^{(l)}$ is negatively defined and does not explicitly depend on correlations of the observables of different particles.

Note that it can be rewritten as

$$k_2^{(l)}(n) = -\frac{\overline{w}^{(l)} S_2'^{(l,2)}}{n\overline{w}^{(l)2} - \overline{w^{(l)2}}}, \quad (27)$$

where $S_2'^{(l,2)}$ is the event-wise second pseudocentral moment:

$$S_2'^{(l,2)}(n) = \frac{\sum_{i=1}^n w_i^{(l)2} \Delta \tilde{\varepsilon}_i^{(l)2}}{\sum_{i=1}^n w_i^{(l)}}. \quad (28)$$

We use the prefix «pseudo» because of the quadratic weights in (28) for $S_2'^{(l,2)}$ contrary to the linear weights in the true efficiency corrected λ th central moment:

$$S_\lambda^{(l)}(n) = \frac{\sum_{i=1}^n w_i^{(l)} \Delta \tilde{\varepsilon}_i^{(l)\lambda}}{\sum_{i=1}^n w_i^{(l)}}. \quad (29)$$

Generally, the λ -order pseudocorrelators contributing to m -order correlator ($\lambda \leq m$) can be expressed through the λ th pseudocentral moments calculated with the powers $\mu \leq m$ of the weights:

$$S_\lambda'^{(l,\mu)}(n) = \frac{\sum_{i=1}^n w_i^{(l)\mu} \Delta \tilde{\varepsilon}_i^{(l)\lambda}}{\sum_{i=1}^n w_i^{(l)}}. \quad (30)$$

Of course, $S_\lambda'^{(l,\mu)} = S_\lambda$ in case of an ideal detector.

Thus, the first-order pseudocorrelator in (24) can be rewritten as

$$k_1^{(l,2)}(n) = -\frac{\overline{w}^{(l)} S_1'^{(l,2)}}{n\overline{w}^{(l)2} - \overline{w^{(l)2}}}. \quad (31)$$

As for the pseudocorrelators contributing to the the third-order correlator, using in addition the identity valid for arbitrary functions f_{ijk} :

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1, \neq i}^n \sum_{k=1, \neq i, j}^n w_i^{(l)} w_j^{(l)} w_k^{(l)} f_{ijk} &= \sum_{i=1}^n w_i^{(l)} \times \\ &\times \left[\sum_{j=1}^n w_j^{(l)} \left(\sum_{k=1}^n w_k^{(l)} f_{ijk} - w_i^{(l)} f_{iji} - w_j^{(l)} f_{ijj} \right) - \right. \\ &\left. - w_i^{(l)} \left(\sum_{k=1}^n w_k^{(l)} f_{iik} - 2w_i^{(l)} f_{iii} \right) \right] \quad (32) \end{aligned}$$

and the third power of identity (22):

$$\begin{aligned} \sum_{i=1}^n (w_i^{(l)} \Delta \tilde{\varepsilon}_i^{(l)})^3 + 3 \sum_{i=1}^n \sum_{j=1, \neq i}^n w_i^{(l)} \Delta \tilde{\varepsilon}_i^{(l)} (w_j^{(l)} \Delta \tilde{\varepsilon}_j^{(l)})^2 + \\ + \sum_{i=1}^n \sum_{j=1, \neq i}^n \sum_{k=1, \neq i, j}^n w_i^{(l)} w_j^{(l)} w_k^{(l)} \Delta \tilde{\varepsilon}_i^{(l)} \Delta \tilde{\varepsilon}_j^{(l)} \Delta \tilde{\varepsilon}_k^{(l)} = 0, \end{aligned} \quad (33)$$

one gets

$$k_1^{(l,3)}(n) = -2 \frac{\sum_{i=1}^n w_i^{(l)2} (n\bar{w}^{(l)} - w_i^{(l)}) \Delta \tilde{\varepsilon}_i^{(l)}}{n(n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}})}, \quad (34)$$

$$k_2^{(l,3)}(n) = -\frac{\sum_{i=1}^n w_i^{(l)2} (n\bar{w}^{(l)} - 2w_i^{(l)}) \Delta \tilde{\varepsilon}_i^{(l)2}}{n(n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}})}, \quad (35)$$

$$k_3^{(l,3)}(n) \equiv k_3^{(l)}(n) = 2 \frac{\sum_{i=1}^n w_i^{(l)3} \Delta \tilde{\varepsilon}_i^{(l)3}}{n(n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}})}. \quad (36)$$

Obviously, the pseudocorrelators in Eqs. (34), (35) and (36) can be expressed through the pseudocentral moments of the observable distribution as

$$k_1^{(l,3)}(n) = -2 \frac{\bar{w}^{(l)} (n\bar{w}^{(l)} S_1^{\prime(l,2)} - S_1^{\prime(l,3)})}{n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}}}, \quad (37)$$

$$k_2^{(l,3)}(n) = -\frac{\bar{w}^{(l)} (n\bar{w}^{(l)} S_2^{\prime(l,2)} - 2S_2^{\prime(l,3)})}{n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}}}, \quad (38)$$

$$k_3^{(l)}(n) = 2 \frac{\bar{w}^{(l)} S_3^{\prime(l,3)}}{n^2\bar{w}^{(l)3} - 3n\bar{w}^{(l)}\overline{w^{(l)2}} + 2\overline{w^{(l)3}}}. \quad (39)$$

The generalization of the expressions for the efficiency corrected pseudocorrelators $k_\lambda^{(l,m)}(n)$ for $m > 3$ is straightforward. To perform the corresponding rather lengthy analytical calculations, we have written a Maple [12] code, which is available under the request. Here we present only the results for $m = 4$:

$$k_1^{(l,4)} = -\frac{3\bar{w}^{(l)} (n^2 \bar{w}^{(l)2} S_1^{\prime(l,2)} - n \overline{w^{(l)2}} S_1^{\prime(l,2)} - 2n \bar{w}^{(l)} S_1^{\prime(l,3)} + 2 S_1^{\prime(l,4)})}{n^3 \bar{w}^{(l)4} - 6n^2 \bar{w}^{(l)2} \overline{w^{(l)2}} + 3n \overline{w^{(l)2}^2} + 8n \bar{w}^{(l)} \overline{w^{(l)3}} - 6 \overline{w^{(l)4}}}, \quad (40)$$

$$k_2^{(l,4)} = \frac{\bar{w}^{(l)} (4n \bar{w}^{(l)} S_2^{\prime(l,3)} - 2n \bar{w}^{(l)} S_1^{\prime(l,2)2} - n \overline{w^{(l)2}} S_2^{\prime(l,2)} - n^2 \bar{w}^{(l)2} S_2^{\prime(l,2)} - 6 S_2^{\prime(l,4)})}{n^3 \bar{w}^{(l)4} - 6n^2 \bar{w}^{(l)2} \overline{w^{(l)2}} + 3n \overline{w^{(l)2}^2} + 8n \bar{w}^{(l)} \overline{w^{(l)3}} - 6 \overline{w^{(l)4}}}, \quad (41)$$

$$k_3^{(l,4)} = \frac{\bar{w}^{(l)}(2n\bar{w}^{(l)}S_3^{\prime(l,3)} + 3n\bar{w}^{(l)}S_1^{\prime(l,2)}S_2^{\prime(l,2)} - 6S_3^{\prime(l,4)})}{n^3\bar{w}^{(l)4} - 6n^2\bar{w}^{(l)2}\bar{w}^{(l)2} + 3n\bar{w}^{(l)2^2} + 8n\bar{w}^{(l)}\bar{w}^{(l)3} - 6\bar{w}^{(l)4}}, \quad (42)$$

$$k_4^{(l,4)} = \frac{3\bar{w}^{(l)}(n\bar{w}^{(l)}S_2^{\prime(l,2)^2} - 2S_4^{\prime(l,4)})}{n^3\bar{w}^{(l)4} - 6n^2\bar{w}^{(l)2}\bar{w}^{(l)2} + 3n\bar{w}^{(l)2^2} + 8n\bar{w}^{(l)}\bar{w}^{(l)3} - 6\bar{w}^{(l)4}}. \quad (43)$$

It should be noted that all the formulae used to calculate correlators of a given observable ε reduce to those for an ideal detector in the case of ε -independent track reconstruction efficiencies.

4. CALCULATING CORRELATORS FOR MONTE-CARLO EVENTS

To estimate the computing time of the correlator calculations as well as the corrections of momentum correlators due to realistic momentum dependence of the track reconstruction efficiency, we have used the Monte-Carlo generator PYTHIA [13] to simulate events of pp interactions at 7 TeV with charged hadron multiplicity $n \geq 5$. The reconstructed tracks have been simulated with the help of the rejection method [14] assuming a similar p_T - and η -dependence of the track reconstruction efficiency as in the ATLAS experiment [15]: it depends only weakly on η and rapidly increases with p_T from $\sim 10\%$ at $p_T = 0.1$ GeV/ c and achieves a level of $\sim 80\%$ at $p_T = 0.8$ GeV/ c . Such a dependence is typical for high-energy multiparticle production experiments [16–19].

To estimate the acceleration of the correlator calculations, when substituting the direct formula (12) by the decomposition formula (13), we have used the Processor AMD Phenom(tm) II X6 1100T with CPU 3.3 GHz. We have found that the computation time of the correlator K_m according to (12) behaves in accordance with the corresponding combinatorics:

$$T_m(n, N) = 1.9 \frac{n^m}{m!} N(n) \quad [\text{ps}], \quad (44)$$

while the computation time according to the decomposition formula (13) is strongly reduced and depends on the multiplicity only linearly:

$$T'_m(n, N) = \left\{ 0.5 \left[m + \frac{1}{2}m(m+1) \right] n + 300 \right\} N(n) \quad [\text{ps}]. \quad (45)$$

The m -dependence in square brackets corresponds to the calculation of m averages \bar{w} , $\bar{w}^2, \dots, \bar{w}^m$ and $1 + 2 + \dots + m$ terms $\bar{\varepsilon}, S_1^{\prime(2)}, S_2^{\prime(2)}, \dots, S_1^{\prime(m)}, S_2^{\prime(m)}, \dots, S_m^{\prime(m)}$. This reduction is somewhat smaller than in the case of a constant track reconstruction efficiency, when the square bracket in (45) reduces to $[m+1]$ in correspondence with the calculation of $m+1$ terms $\bar{\varepsilon}, S_1, S_2, \dots, S_m$.

The reduction of the computation time is not critical for moderate multiplicities and not too high orders of the correlators. Thus, for the multiplicity $n = 100$, the computation time according to the direct formula (12) is reasonable even for the fifth-order correlator and for $N = 10^6$ events it composes about 160 s. The decomposition formula (13) becomes of principle importance for calculation of higher-order correlators in central heavy-ion collisions at high energies. Thus, for a typical charged hadron multiplicity $n = 1000$ the computation

time of the fifth-order correlator in $N = 10^6$ events comprises half a year according to the direct formula (12), compared with 10 ms when using the decomposition formula (13).

As for the corrections of the momentum correlators K_m or the fluctuation measures $\langle \Delta \bar{\varepsilon}^m \rangle$ due to a typical momentum-dependent track reconstruction efficiency, for $m \leq 4$ they amount up to several tens of percent.

We have not considered here the corrections due to possible multiplicity dependence of the observable mean which may be on the level of several tens of percent [10].

CONCLUSIONS

The formulae for the m -order correlators K_m of a given particle observable ε (e.g., energy, transverse momentum, rapidity or a conserved discrete quantum number) accounting for the track reconstruction efficiencies are presented with some calculation details for $m = 2, 3, 4$. Similar to the case of an ideal detector, one can reduce the computation time by avoiding the combinatorics and expressing the correlators through the event-by-event fluctuation measures of the observable single event mean, the pseudocorrelators (determined by the pseudocentral moments of the observable distribution) and their cross terms. The number of the terms to be calculated is however higher due to increased number of pseudocorrelators and additional calculations of the moments of the distribution of the track weights. The correlators are affected by detector inefficiency in the case of substantial ε -dependence of track reconstruction efficiencies. The reduction of the correlator computation time with the help of the decomposition formula as well as the corrections of momentum correlators due to a typical momentum-dependent track reconstruction efficiency have been estimated with the help of Monte-Carlo events of pp collisions at 7 TeV for particles with $p_T > 0.1$ GeV/ c . It was found that the reduction of the computation time is of principle importance for calculation of the higher-order ($m > 4$) correlators in the events with charged hadron multiplicities of the order of several hundreds or higher. The estimated corrections of the momentum correlators K_m or the fluctuation measures $\langle \Delta \bar{\varepsilon}^m \rangle$ for $m \leq 4$ are up to several tens of percent and should be taken into account.

Acknowledgements. We are grateful to J. A. Budagov, J. D. Manjavidze and E. Sarkisyan-Grinbaum for useful discussions and comments. We are also thankful to V. P. Gerdt for the help with the Maple analytical calculations.

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Received on May 28, 2012.