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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

EXACT SOLUTIONS OF SPIN-ONE DKP EQUATION UNDER KRATZER POTENTIAL IN $(1+2)$ DIMENSIONS

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We study the relativistic spin-one Duffin–Kemmer–Petiau equation in the presence of Kratzer potential in $(1+2)$-dimensional Minkowski space-time. To obtain the energy eigenvalues and the corresponding eigenfunctions, the analytical Nikiforov–Uvarov method is used.

Представлено исследование релятивистского уравнения Даффина–Кеммера–Петьяу со спином один с потенциалом Кратцера в $(1+2)$-мерном пространстве-времени Минковского. Для получения собственных значений энергии и собственных функций использовался аналитический метод Никифорова–Уварова.

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INTRODUCTION

The exact solutions of the first-order relativistic Duffin–Kemmer–Petiau (DKP) equation are very important to study spin-zero and spin-one bosons [1–4]. The equation becomes more appealing when we remember the complicated nature of the Proca equation and the few existing papers on it. The DKP equation has already been used to study various phenomena in particle and nuclear physics, including deuteron–nucleus scattering [5], meson-nuclear interactions [6] and α-nucleus elastic scattering [7]. The equation has also been analyzed in connection with the Quantum Chromodynamics (QCD) [8], the covariant Hamiltonian formulation [9] and the causal approach [10, 11]. The successful predictions of the equation in such fields have motivated some authors to investigate the equation under some interactions [12–25]. Nevertheless, there are still important phenomenological interactions, which have not been solved within this framework. In our study, we intend to solve the equation with the Kratzer potential. This potential, because of having an ad hoc inverse square term, has been in some cases superior to the Coulomb interaction [26, 27]. In Sec. 1 we review the basic ingredients of the equation [28]. In Sec. 2 we review the equation in two spatial dimensions. In our last step, the eigenfunctions and the corresponding eigenvalues of the arising differential equation are reported via the Nikiforov–Uvarov (NU) technique [29, 30].

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1. THE FORMULATION OF DKP EQUATION

The first-order relativistic DKP equation for a free boson of mass m is (in natural units $\hbar = c = 1$) [1–3]

$$(i\beta^\mu \partial_\mu - m)\Psi = 0, \tag{1}$$

where the matrices β^μ $(\mu = 0, 1, 2, 3)$ satisfy the commutation relation

$$\beta^\mu \beta^\nu \beta^\lambda + \beta^\lambda \beta^\nu \beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\lambda\nu}\beta^\mu. \tag{2}$$

The engaged matrices have three irreducible representations; the one-dimensional representation that is trivial, the five-dimensional form that corresponds to spin-zero bosons and the ten-dimensional one, which is for a spin-one system. In the presence of an interaction, the equation reads [5]

$$(i\beta^\mu \partial_\mu - m - U)\Psi = 0. \tag{3}$$

For the spin-zero version of DKP equation, there are two scalar, two vector and two tensor terms. Within the spin-one formulation, there are two scalar, two vector, one pseudoscalar, two pseudovector and eight tensor terms. Because of noncausal effects, the tensor terms are avoided in applications. The interaction term U, in its most general form, is

$$U = S(r) + PS_P(r) + \beta^\mu V_\mu(r) + \beta^\mu P V_{p\mu}(r). \tag{4}$$

For an elastic interaction, U is normally written as [5]

$$U = S(r) + PS_P(r) + \beta^0 V(r) + \beta^0 P V_P(r), \tag{5}$$

where each term has a specific Lorentz character. In particular, two Lorentz vectors may be written as β^μ and $P\beta^\mu$ by assuming rotational invariance and parity conservation. The other term, i.e., $P = (\beta^\mu \beta_\mu - 2) = \mathrm{diag}\,(1,1,1,1,0,0,0,0,0,0)$, is a projection operator. In the five-dimensional representation, the β^μ matrices are the following 5×5 ones:

$$\beta^0 = \begin{pmatrix} \Theta & \widehat{0} \\ \widehat{0}^T & 0 \end{pmatrix}, \quad \beta^i = \begin{pmatrix} \tilde{0} & \rho^i \\ -(\rho^i)^T & 0 \end{pmatrix}, \quad i = 1,2,3 \tag{6}$$

with

$$\Theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

where $\widehat{0}$, $\tilde{0}$ and 0 are 2×3, 2×2 and 3×3 zero matrices, respectively.

Within the ten-dimensional representation,

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & 0 & I & 0 \\ \bar{0}^T & I & 0 & 0 \\ \bar{0}^T & 0 & 0 & 0 \end{pmatrix}, \quad \beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & 0 & 0 & -iS_i \\ -e_i^T & 0 & 0 & 0 \\ \bar{0}^T & -iS_i & 0 & 0 \end{pmatrix}, \tag{8}$$

where the S matrices are 3×3 ones and $(S_i)_{jk} = -i\varepsilon_{ijk}$, where ε_{ijk} is 1, -1, 0 for an even permutation, an odd permutation and repeated indices, respectively. The e_i matrices are 1×3, $(e_i)_{1j} = \delta_{ij}$, i.e., $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, and $e_3 = (0,0,1)$. I and 0 respectively represent unit and null 3×3 matrices and $\bar{0}$s are 1×3 ones [28].

2. SPIN-ONE DKP EQUATION IN $(1+2)$ DIMENSIONS

The DKP equation with the interaction $\beta^0 PV(r)$, which is the time-like component of a Lorentz vector, takes the form

$$(E\beta^0 - \beta^0 PV(r) + i\beta^1\partial_1 + i\beta^2\partial_2 - m)\,\Psi(x,y,t) = 0. \tag{9}$$

As usual, a solution of the form

$$\Psi(x,y,t) = \exp(-iE_{n,\lambda}t)\,\psi_{n,\lambda}(x,y) \tag{10}$$

removes the time-dependence of the wave function, and we write the stationary wave function as a ten-component spinor of the form

$$\psi^T_{n,\lambda}(x,y) = (\psi^{(1)}_{n,\lambda}, \psi^{(2)}_{n,\lambda}, \psi^{(3)}_{n,\lambda}, \psi^{(4)}_{n,\lambda}, \psi^{(5)}_{n,\lambda}, \psi^{(6)}_{n,\lambda}, \psi^{(7)}_{n,\lambda}, \psi^{(8)}_{n,\lambda}, \psi^{(9)}_{n,\lambda}, \psi^{(10)}_{n,\lambda})^T. \tag{11}$$

Substitution of the latter in Eq. (9) yields the coupled equations

$$i\partial_1\psi^{(5)}_{n,\lambda}(x,y) + i\partial_2\psi^{(6)}_{n,\lambda}(x,y) - m\psi^{(1)}_{n,\lambda}(x,y) = 0, \tag{12}$$

$$E_{n,\lambda}\psi^{(5)}_{n,\lambda}(x,y) + i\partial_2\psi^{(10)}_{n,\lambda}(x,y) - m\psi^{(2)}_{n,\lambda}(x,y) = 0, \tag{13}$$

$$E_{n,\lambda}\psi^{(6)}_{n,\lambda}(x,y) - i\partial_1\psi^{(10)}_{n,\lambda}(x,y) - m\psi^{(3)}_{n,\lambda}(x,y) = 0, \tag{14}$$

$$E_{n,\lambda}\psi^{(7)}_{n,\lambda}(x,y) + i\partial_1\psi^{(9)}_{n,\lambda}(x,y) - i\partial_2\psi^{(8)}_{n,\lambda}(x,y) - m\psi^{(4)}_{n,\lambda}(x,y) = 0, \tag{15}$$

$$E_{n,\lambda}\psi^{(2)}_{n,\lambda}(x,y) - V(\rho)\psi^{(2)}_{n,\lambda}(x,y) - i\partial_1\psi^{(1)}_{n,\lambda}(x,y) - m\psi^{(5)}_{n,\lambda}(x,y) = 0, \tag{16}$$

$$E_{n,\lambda}\psi^{(3)}_{n,\lambda}(x,y) - V(\rho)\psi^{(3)}_{n,\lambda}(x,y) - i\partial_2\psi^{(1)}_{n,\lambda}(x,y) - m\psi^{(6)}_{n,\lambda}(x,y) = 0, \tag{17}$$

$$E_{n,\lambda}\psi^{(4)}_{n,\lambda}(x,y) - V(\rho)\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(7)}_{n,\lambda}(x,y) = 0, \tag{18}$$

$$i\partial_2\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(8)}_{n,\lambda}(x,y) = 0, \tag{19}$$

$$-i\partial_1\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(9)}_{n,\lambda}(x,y) = 0, \tag{20}$$

$$i\partial_1\psi^{(3)}_{n,\lambda}(x,y) - i\partial_2\psi^{(2)}_{n,\lambda}(x,y) - m\psi^{(10)}_{n,\lambda}(x,y) = 0. \tag{21}$$

If we suppose $\psi^2_{n,\lambda}(x,y) = \psi^3_{n,\lambda}(x,y) = 0$, it is simply seen that

$$\psi^{(1)}_{n,\lambda}(x,y) = \psi^{(5)}_{n,\lambda}(x,y) = \psi^{(6)}_{n,\lambda}(x,y) = \psi^{(10)}_{n,\lambda}(x,y) = 0,$$

and

$$E_{n,\lambda}\psi^{(7)}_{n,\lambda}(x,y) + i\partial_1\psi^{(9)}_{n,\lambda}(x,y) - i\partial_2\psi^{(8)}_{n,\lambda}(x,y) - m\psi^{(4)}_{n,\lambda}(x,y) = 0, \tag{22a}$$

$$E_{n,\lambda}\psi^{(4)}_{n,\lambda}(x,y) - V(\rho)\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(7)}_{n,\lambda}(x,y) = 0, \tag{22b}$$

$$i\partial_2\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(8)}_{n,\lambda}(x,y) = 0, \tag{22c}$$

$$-i\partial_1\psi^{(4)}_{n,\lambda}(x,y) - m\psi^{(9)}_{n,\lambda}(x,y) = 0. \tag{22d}$$

Elimination of other components in favor of the fourth one yields

$$(E_{n,\lambda}(E_{n,\lambda} - V(\rho)) - m^2)\psi^{(4)}_{n,\lambda}(x,y) + \partial_1^2\psi^{(4)}_{n,\lambda}(x,y) + \partial_2^2\psi^{(4)}_{n,\lambda}(x,y) = 0. \tag{23}$$

Let us now rewrite Eq. (23) as

$$(\partial_1^2 + \partial_2^2)\psi_{n,\lambda}^{(4)}(x,y) = -(E_{n,m}(E_{n,m} - V(\rho)) - m^2)\psi_{n,\lambda}^{(4)}(x,y), \tag{24}$$

in which the first and the second terms, respectively, are

$$(\partial_1^2 + \partial_2^2)\psi_{n,\lambda}^{(4)}(x,y) = \nabla^2\psi_{n,\lambda}^{(4)}(x,y), \tag{25}$$

with

$$\nabla^2 = \left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right). \tag{26}$$

Applying the frequently used separation of variable

$$\psi_{n,\lambda}^{(4)}(\rho,\varphi) = R_n^{(4)}(\rho)Q_\lambda(\varphi), \tag{27}$$

we easily find

$$Q_\lambda(\varphi) = \mathrm{e}^{i\lambda\varphi}, \tag{28}$$

and the wave equation appears as

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{\lambda^2}{\rho^2}\right)R_n^{(4)}(\rho) + (E_{n,\lambda}^2 - E_{n,\lambda}V(\rho) - m^2)R_n^{(4)}(\rho) = 0. \tag{29}$$

To solve Eq. (30), we use the parametric version of the NU technique introduced in the Appendix.

3. SOLUTION OF THE DKP EQUATION FOR THE KRATZER POTENTIAL

We already mentioned that we consider the Kratzer potential $V(\rho) = \dfrac{a}{\rho} + \dfrac{b}{\rho^2}$, which is plotted in Fig. 1, for some values of constant parameters a and b. In Eq. (30), we arrive at the following second-order ordinary differential equation:

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{\lambda^2}{\rho^2}\right)R_n^{(4)}(\rho) + \left(E_{n,\lambda}^2 - E_{n,\lambda}\left(\frac{a}{\rho} + \frac{b}{\rho^2}\right) - m^2\right)R_n^{(4)}(\rho) = 0, \tag{30}$$

which we rewrite as

$$\frac{d^2R_n^{(4)}(\rho)}{d\rho^2} + \frac{1}{\rho}\frac{dR_n^{(4)}(\rho)}{d\rho} + \frac{1}{\rho^2}(-(\lambda^2 + E_{n,\lambda}b) + (E_{n,\lambda}^2 - m^2)\rho^2 - E_{n,\lambda}a\rho)R_n^{(4)}(\rho) = 0. \tag{31}$$

A comparison with Eq. (A. 1) of the Appendix reveals the correspondence

$$\begin{gathered}
\xi_1 = m^2 - E_{n,\lambda}^2, \quad \xi_2 = -E_{n,\lambda}a, \quad \xi_3 = \lambda^2 + E_{n,\lambda}b, \\
\alpha_1 = 1, \quad \alpha_2 = 0, \quad \alpha_3 = 0, \quad \alpha_4 = 0, \quad \alpha_5 = 0, \\
\alpha_6 = m^2 - E_{n,\lambda}^2, \quad \alpha_7 = E_{n,\lambda}a, \\
\alpha_8 = \lambda^2 + E_{n,\lambda}b, \quad \alpha_9 = m^2 - E_{n,\lambda}^2, \\
\alpha_{10} = 1 + 2\sqrt{\lambda^2 + E_{n,\lambda}b}, \quad \alpha_{11} = 2\sqrt{m^2 - E_{n,\lambda}^2}, \\
\alpha_{12} = \sqrt{\lambda^2 + E_{n,\lambda}^b}, \quad \alpha_{13} = -\sqrt{m^2 - E_{n,\lambda}^2}.
\end{gathered} \tag{32}$$

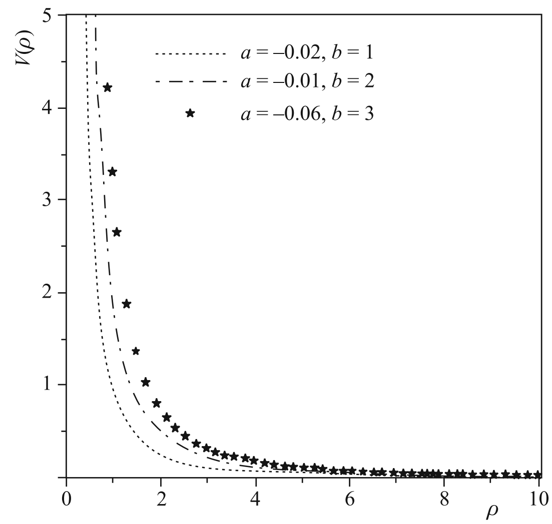

Fig. 1. Kratzer potential vs. ρ

Thus, from Eq. (A. 4), the energy relation is simply found:

$$\sqrt{m^2-E_{n,\lambda}^2}\left(2n+1+2\sqrt{\lambda^2+E_{n,\lambda}b}\right)+E_{n,\lambda}a=0, \tag{33}$$

and the wave function, from Eqs. (A. 5) and (A. 8), is written as

$$R_{n,\lambda}^{(4)}(\rho)=\rho^{\sqrt{\lambda^2+E_{n,\lambda}b}}\,\mathrm{e}^{-\sqrt{m^2-E_{n,\lambda}^2}\rho}L_n^{2\sqrt{\lambda^2+E_{n,\lambda}b}}\left(2\sqrt{m^2-E_{n,\lambda}^2}\rho\right). \tag{34}$$

We have plotted wave function vs. ρ in Fig. 2. The energy eigenvalues for different values of λ are reported in the table to give a better view of the obtained results.

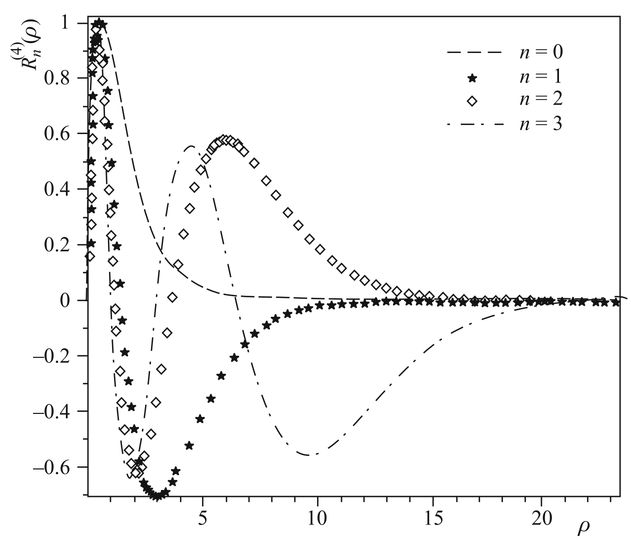

Fig. 2. Wave functions vs. ρ for $\lambda=0$, $a=-8$, $b=1$

Energy eigenvalues for $(m = 1,\ a = -0.1,\ b = 3)$

$\|n,\lambda\rangle$	$E_{n,\lambda}$	$\|n,\lambda\rangle$	$E_{n,\lambda}$	$\|n,\lambda\rangle$	$E_{n,\lambda}$	$\|n,\lambda\rangle$	$E_{n,\lambda}$
$\|0,0\rangle$	0.999749	$\|0,1\rangle$	0.999800	$\|0,2\rangle$	0.999874	$\|0,3\rangle$	0.999920
$\|1,0\rangle$	0.999880	$\|1,1\rangle$	0.999898	$\|1,2\rangle$	0.999927	$\|1,3\rangle$	0.999949
$\|2,0\rangle$	0.999930	$\|2,1\rangle$	0.999938	$\|2,2\rangle$	0.999953	$\|2,3\rangle$	0.999965
$\|3,0\rangle$	0.999954	$\|3,1\rangle$	0.999959	$\|3,2\rangle$	0.999967	$\|3,3\rangle$	0.999974
$\|4,0\rangle$	0.999968	$\|4,1\rangle$	0.999970	$\|4,2\rangle$	0.999976	$\|4,3\rangle$	0.999980
$\|5,0\rangle$	0.999976	$\|5,1\rangle$	0.999978	$\|5,2\rangle$	0.999981	$\|5,3\rangle$	0.999984
$\|6,0\rangle$	0.999982	$\|6,1\rangle$	0.999983	$\|6,2\rangle$	0.999985	$\|6,3\rangle$	0.999987
$\|7,0\rangle$	0.999985	$\|7,1\rangle$	0.999986	$\|7,2\rangle$	0.999988	$\|7,3\rangle$	0.999990
$\|8,0\rangle$	0.999988	$\|8,1\rangle$	0.999989	$\|8,2\rangle$	0.999990	$\|8,3\rangle$	0.999991

CONCLUSION

We studied the spin-one DKP equation with a Kratzer interaction in $(1+2)$-dimensional Minkowski space-time. We showed that the problem could be solved in an exact analytical manner via the powerful NU technique. The wave functions of the problem were reported in terms of the Laguerre polynomials and the corresponding eigenvalues were reported in the table. From the table, we see that with increasing the quantum numbers, the energy increases. Our obtained energy relation and the corresponding wave functions can be used to study relativistic spin-one bosonic systems such as photons, relativistic excitons and spin-one mesons.

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APPENDIX

The Nikiforov–Uvarov method enables us to solve second-order differential equations. We consider the following differential equation [29, 30]:

$$\left\{\frac{d^2}{ds^2} + \frac{\alpha_1 - \alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds} + \frac{1}{[s(1-\alpha_3 s)]^2}[-\xi_1 s^2 + \xi_2 s - \xi_3]\right\}\psi_n(s) = 0. \tag{A. 1}$$

According to the NU method, the eigenfunctions are [30]

$$\psi_n(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} P_n^{\left(\alpha_{10}-1,\,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \tag{A. 2}$$

where the Jacobi polynomial is

$$\begin{aligned} P_n^{(c,d)}(z) &= 2^{-n}\sum_{p=0}^{n}\binom{n+c}{P}\binom{n+d}{n-p}(1-z)^{n-p}(1+z)^p, \\ P_n^{(c,d)}(z) &= \frac{\Gamma(n+c+1)}{n!\Gamma(n+c+d+1)}\sum_{r=0}^{n}\binom{n}{r}\frac{\Gamma(n+c+d+r+1)}{\Gamma(r+c+1)}\left(\frac{z-1}{2}\right)^r, \end{aligned} \tag{A. 3}$$

where

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r+1)}.$$

And the eigenenergies satisfy

$$\alpha_2 n - (2n+1)\alpha_5 + (2n+1)\left(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\right) + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0, \quad \text{(A. 4)}$$

where

$$\begin{gathered} \alpha_4 = \frac{1}{2}(1-\alpha_1), \quad \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3), \quad \alpha_6 = \alpha_5^2 + \xi_1, \\ \alpha_7 = 2\alpha_4\alpha_5 - \xi_2, \quad \alpha_8 = \alpha_4^2 + \xi_3, \quad \alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6, \\ \alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}, \quad \alpha_{11} = \alpha_2 - 2\alpha_5 + 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}), \\ \alpha_{12} = \alpha_4 + \sqrt{\alpha_8}, \quad \alpha_{13} = \alpha_5 - (\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}). \end{gathered} \quad \text{(A. 5)}$$

Furthermore, in some cases we can use

$$\begin{gathered} \psi_n(s) = s^{\alpha_{12}^*}(1-\alpha_3 s)^{-\alpha_{12}^* - \frac{\alpha_{13}^*}{\alpha_3}} P_n^{\left(\alpha_{10}^*-1,\, \frac{\alpha_{11}^*}{\alpha_3} - \alpha_{10} - 1\right)}(1-2\alpha_3 s), \\ \alpha_{10}^* = \alpha_1 + 2\alpha_4 - 2\sqrt{\alpha_8}, \quad \alpha_{11}^* = \alpha_2 - 2\alpha_5 + 2\left(\sqrt{\alpha_9} - \alpha_3\sqrt{\alpha_8}\right), \\ \alpha_{12}^* = \alpha_4 - \sqrt{\alpha_8}, \quad \alpha_{13}^* = \alpha_5 - \left(\sqrt{\alpha_9} - \alpha_3\sqrt{\alpha_8}\right). \end{gathered} \quad \text{(A. 6)}$$

Also for this problem when

$$\lim_{\alpha_3 \to 0} P_n^{\left(\alpha_{10}-1,\, \frac{\alpha_{11}}{\alpha_3} - \alpha_{10} - 1\right)}(1-\alpha_3 s) = L_n^{\alpha_{10}-1}(\alpha_{11} s) \quad \text{(A. 7)}$$

and $\lim\limits_{\alpha_3 \to 0}(1-\alpha_3 s)^{-\alpha_{12} - \frac{\alpha_{13}}{\alpha_3}} = \mathrm{e}^{\alpha_{13} s}$.

Thus, the solution given in Eq. (A. 4) becomes

$$\psi_n(s) = s^{\alpha_{12}}\, \mathrm{e}^{\alpha_{13} s}\, L_n^{\alpha_{10}-1}(\alpha_{11} s). \quad \text{(A. 8)}$$

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