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# APPROXIMATE SOLUTIONS OF DIRAC EQUATION FOR TIETZ AND GENERAL MANNING–ROSEN POTENTIALS USING SUSYQM

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In this paper, we consider the relativistic Dirac equation with Tietz and general Manning–Rosen potentials. By using appropriate approximation, we obtained the approximate analytical solutions of the Dirac equation for the combined potential via the supersymmetric quantum mechanics (SUSYQM). Within the framework of spin and pseudospin symmetry limits, we obtained the relativistic energy eigenvalues and the corresponding components of the wave functions for Tietz and Manning–Rosen potentials using the SUSYQM. We have also reported some numerical results and figures to show the effect of the tensor interactions.

В статье рассматривается релятивистское уравнение Дирака с обобщенным потенциалом Маннинга–Розена и потенциалом Тиетца. Используя приближение, авторы получают приближительные аналитические решения уравнения Дирака для комбинированного потенциала на базе суперсимметричной квантовой механики (SUSYQM). В рамках спиновой и псевдоспиновой симметрий находятся собственные значения релятивистских энергий и соответствующие компоненты волновых функций для потенциалов Тиетца и Маннинга–Розена на основе SUSYQM. Представлены также некоторые численные результаты и рисунки, иллюстрирующие эффект тензорных взаимодействий.

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## INTRODUCTION

The problem of finding the exact or approximate solutions of the Dirac equation for a number of special potentials has been of great interest in recent years, and many authors, by using different methods, have studied the bound states solution of the Dirac equation under the condition of spin and pseudospin symmetries [1]. The concept of pseudospin symmetry observed many years ago in the nuclei has been used to explain quasi-degeneracy in heavy nuclei between single-nucleon doublets with quantum number $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$, where $n$, $l$ and $j$ are the single-nucleon radials, orbital and

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total angular momentum quantum number, respectively [2]. Ginocchio et al. explain that the pseudospin-orbital angular momentum defined as $\tilde{l} = l + 1$ is nothing but the orbital angular momentum of the lower component of the Dirac spinor [3]. Also, the tensor interaction term was introduced into the Dirac equation with the replacement $\mathbf{p} = \mathbf{p} - iM\omega\hat{\beta}\cdot\hat{r}U(r)$ and a spin-orbit coupling is added to the Dirac Hamiltonian [4]. Within the Dirac theory, pseudospin and spin symmetries are used to study features of deformed nuclei, superdeformation and effective shell model [5, 6]. It has been shown that the exact pseudopsin symmetry occurs in the Dirac equation when $\dfrac{d\Sigma(r)}{dr} = 0$, i.e., $\Sigma(r) = V(r) + S(r) = C_{\rm ps} = \text{const}$ [7], where $V(r)$, $S(r)$ are the repulsive and attractive vector and scalar potentials, respectively. Likewise, the exact spin symmetry occurs in the Dirac theory when $\dfrac{d\Delta(r)}{dr} = 0$, where $\Delta(r) = V(r) - S(r) = C_s = \text{const}$ [5, 6]. Recently, there has been a lot of interest in searching for the analytical solutions of the Dirac equation with physically motivated potential model under spin and pseudodspin symmetries [8–12]. In this paper, we shall attempt to solve the Dirac equation with combined Tietz and geneal Manning–Rosen potential using the supersymmetric quantum mechanics (SUSYQM) [13].

## 1. DIRAC EQUATION WITH A TENSOR COUPLING

Dirac equation with a tensor potential $U(r)$ in the relativistic unit ($\hbar = c = 1$) is written as [14–18]

$$[\boldsymbol{\alpha}\cdot\mathbf{p} + \beta(M + S(r) - i\beta\boldsymbol{\alpha}\cdot\hat{\mathbf{r}}\,U(r)]\;\psi(r) = [E - V(r)]\;\psi(r), \tag{1}$$

where $E$ is the relativistic energy of the system; $\mathbf{p} = -i\boldsymbol{\nabla}$ is the three-dimensional momentum operator; and $M$ is the mass of the fermionic particle. $\boldsymbol{\alpha}$, $\beta$ are the $4\times 4$ Dirac matrices given as

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma}_i \\ \boldsymbol{\sigma}_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{2}$$

where $I$ is a $2\times 2$ unit matrix and $\boldsymbol{\sigma}_i$ are the Pauli three-vector matrices defined as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

The eigenvalues of the spin-orbit coupling operator are $\kappa = (j+1/2) > 0$, $\kappa = -(j+1/2) < 0$ for unaligned $j = l - 1/2$ and aligned spin $j = l + 1/2$ cases, respectively. The set $\left(H, K, J^2, J_z\right)$ forms a complete set of conserved quantities. Thus, we can write the spinors as [16–18]

$$\psi_{n\kappa}(r) = \frac{1}{r}\begin{pmatrix} F_{n\kappa}(r)\,Y^{l}_{jm}(\theta,\varphi) \\ iG_{n\kappa}(r)\,Y^{\tilde{l}}_{jm}(\theta,\varphi) \end{pmatrix}, \tag{4}$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ represent the upper and lower components of the Dirac spinors; $Y^{l}_{jm}(\theta,\varphi)$, $Y^{\tilde{l}}_{jm}(\theta,\varphi)$ are the spin and pseudospin spherical harmonics; and $m$ is the projection on the $z$ axis. With other known identities [16–17]

$$\begin{gathered}(\boldsymbol{\sigma}\cdot\mathbf{A})\,(\boldsymbol{\sigma}\cdot\mathbf{B}) = \mathbf{A}\cdot\mathbf{B} + i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B}),\\ \boldsymbol{\sigma}\cdot\mathbf{p} = \boldsymbol{\sigma}\cdot\hat{r}\left(\hat{r}\cdot\mathbf{p} + i\frac{\boldsymbol{\sigma}\cdot\mathbf{L}}{r}\right)\end{gathered} \tag{5}$$

as well as

$$\begin{aligned}(\boldsymbol{\sigma}\cdot\mathbf{L})\,Y_{jm}^{\tilde{l}}(\theta,\varphi)=(\kappa-1)Y_{jm}^{\tilde{l}}(\theta,\varphi),\quad(\boldsymbol{\sigma}\cdot\mathbf{L})\,Y_{jm}^{l}(\theta,\varphi)=-(\kappa-1)Y_{jm}^{l}(\theta,\varphi),\\(\boldsymbol{\sigma}\cdot\hat{r})\,Y_{jm}^{l}(\theta,\varphi)=-Y_{jm}^{\tilde{l}}(\theta,\varphi),\quad(\boldsymbol{\sigma}\cdot\hat{r})\,Y_{jm}^{\tilde{l}}(\theta,\varphi)=-Y_{jm}^{l}(\theta,\varphi),\end{aligned}\tag{6}$$

we obtain the coupled equations [16–20],

$$\left(\frac{d}{dr}+\frac{\kappa}{r}-U(r)\right)F_{n\kappa}(r)=(M+E_{n\kappa}-\Delta(r))\,G_{n\kappa}(r),\tag{7}$$

$$\left(\frac{d}{dr}-\frac{\kappa}{r}+U(r)\right)G_{n\kappa}(r)=(M-E_{n\kappa}+\Sigma(r))\,F_{n\kappa}(r),\tag{8}$$

where

$$\Delta(r)=V(r)-S(r),\tag{9}$$

$$\Sigma(r)=V(r)+S(r).\tag{10}$$

Eliminating $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ in favor of each other in Eqs. (7) and (8), we obtain the second-order Schrödinger-like equations

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{\kappa(\kappa+1)}{r^2}+\dfrac{2\kappa U(r)}{r}-\dfrac{dU(r)}{dr}-U^2(r)-(M+E_{n\kappa}-\Delta(r))\times\\\times(M-E_{n\kappa}+\Sigma(r))+\dfrac{\dfrac{d\Delta(r)}{dr}\left(\dfrac{d}{dr}+\dfrac{\kappa}{r}-U(r)\right)}{(M+E_{n\kappa}-\Delta(r))}\end{array}\right\}F_{n\kappa}(r)=0,\tag{11}$$

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{\kappa(\kappa-1)}{r^2}+\dfrac{2\kappa U(r)}{r}+\dfrac{dU(r)}{dr}-U^2(r)-(M+E_{n\kappa}-\Delta(r))\times\\\times(M-E_{n\kappa}+\Sigma(r))+\dfrac{\dfrac{d\Sigma(r)}{dr}\left(\dfrac{d}{dr}-\dfrac{\kappa}{r}+U(r)\right)}{(M-E_{n\kappa}+\Sigma(r))}\end{array}\right\}G_{n\kappa}(r)=0,\tag{12}$$

with $\kappa(\kappa-1)=\tilde{l}(\tilde{l}+1)$, $\kappa(\kappa+1)=l(l+1)$.

**1.1. Pseudospin Symmetry Limit.** In the pseudospin symmetry limit, $d\Sigma(r)/dr=0$ or $\Sigma(r)=C_{\rm ps}=\text{const}$ [2, 3]. Here, we consider the Tietz and general Manning–Rosen potentials as [19]

$$\Delta(r)=V_0^{\rm ps}\left(\frac{\sinh\beta(r-r_0)}{\sinh\beta r}\right)^2+A^{\rm ps}+B^{\rm ps}\coth(\beta r)-C^{\rm ps}\,\mathrm{cosech}^2(\beta r),\tag{13}$$

besides the Coulomb-like tensor interaction [21]

$$U(r)=-\frac{H_c}{r}.\tag{14}$$

Substituting Eqs. (13) and (14) into Eq. (12) yields

$$\left\{\frac{d^2}{dr^2}-\frac{\eta_\kappa(\eta_\kappa-1)}{r^2}+(M-E^{\rm ps}_{n\kappa}+C_{\rm ps})\times\right.$$
$$\left.\times\begin{pmatrix}V_0^{\rm ps}\cosh^2(\beta r_0)+V_0^{\rm ps}\sinh^2(\beta r_0)-2V_0^{\rm ps}\sinh(\beta r_0)\cosh(\beta r_0)\coth(\beta r_0)+\\+V_0^{\rm ps}\sinh^2(\beta r_0)\,{\rm cosech}^2\,(\beta r)\end{pmatrix}\right\}G^{\rm ps}_{n\kappa}+$$
$$+\left\{\beta_{\rm ps}A^{\rm ps}+\beta_{\rm ps}B^{\rm ps}\coth(\beta r)-\beta_{\rm ps}C^{\rm ps}\,{\rm cosech}^2\,(\beta r)\right\}G^{\rm ps}_{n\kappa}=$$
$$=(M+E^{\rm ps}_{n\kappa})\,(M-E^{\rm ps}_{n\kappa}+C_{\rm ps})\,G^{\rm ps}_{n\kappa}, \quad (15)$$

where

$$\left(H_c^2+2\kappa H_c-H_c+\kappa(\kappa-1)\right)=$$
$$=(\kappa+H_c)\,(\kappa+H_c-1)=\eta_\kappa(\eta_\kappa-1)\to\eta_\kappa=(\kappa+H_c)\,. \quad (16)$$

Equation (15) cannot be solved exactly because of the centrifugal term. Thus, we introduce the approximation for the centrifugal term as [22]

$$\frac{1}{r^2}\approx\beta^2\,{\rm cosech}^2\,(\beta r). \quad (17)$$

Substituting Eq. (17) into Eq. (15), we obtain

$$\left\{\frac{d^2}{dr^2}-\beta^2\eta_\kappa(\eta_\kappa-1)\,{\rm cosech}^2\,(\beta r)+\beta_{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0)+\beta_{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0)+\right.$$
$$+\beta_{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0)\,{\rm cosech}^2\,(\beta r)-2\beta_{\rm ps}V_0^{\rm ps}\sinh\,(\beta r_0)\cosh\,(\beta r_0)\coth\,(\beta r)+$$
$$\left.+\beta_{\rm ps}A^{\rm ps}+\beta_{\rm ps}B^{\rm ps}\coth\,(\beta r)-\beta_{\rm ps}C^{\rm ps}\,{\rm cosech}^2\,(\beta r)\right\}G^{\rm ps}_{n\kappa}(r)=$$
$$=(M+E_{n\kappa})(M-E_{n\kappa}+C_{\rm ps})G^{\rm ps}_{n\kappa}, \quad (18)$$

or more explicitly, we write Eq. (18) as

$$-\frac{d^2G^{\rm ps}_{n\kappa}}{dr^2}+V_{\rm eff}(r)G^{\rm ps}_{n\kappa}=\tilde{E}^{\rm ps}_{n\kappa}G^{\rm ps}_{n\kappa}, \quad (19)$$

where

$$V_{\rm eff}(r)=\tilde{V}_{1\rm ps}\,{\rm cosech}^2\,(\beta r)+\tilde{V}_{2\rm ps}\coth\,(\beta r), \quad (20)$$

$$\tilde{E}^{\rm ps}_{n\kappa}=-M^2-C_{\rm ps}M+(E^{\rm ps}_{n\kappa})^2-C_{\rm ps}E^{\rm ps}_{n\kappa}+MV_0^{\rm ps}\cosh^2(\beta r_0)-$$
$$-E^{\rm ps}_{n\kappa}V_0^{\rm ps}\cosh^2(\beta r_0)+C_{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0)+MV_0^{\rm ps}\sinh^2(\beta r_0)-$$
$$-E^{\rm ps}_{n\kappa}V_0^{\rm ps}\sinh^2(\beta r_0)+C_{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0)+MA^{\rm ps}-E^{\rm ps}_{n\kappa}A^{\rm ps}+C_{\rm ps}A^{\rm ps}, \quad (21)$$

$$\tilde{V}_{1\mathrm{ps}} = \beta^2\eta_\kappa(\eta_\kappa - 1) + MC^{\mathrm{ps}} - E^{\mathrm{ps}}_{n\kappa}C^{\mathrm{ps}} + C_{\mathrm{ps}}C^{\mathrm{ps}} - MV_0^{\mathrm{ps}}\sinh^2(\beta r_0) + \\ + V_0^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}\sinh^2(\beta r_0) - C_{\mathrm{ps}}V_0^{\mathrm{ps}}\sinh^2(\beta r_0), \quad (22)$$

$$\tilde{V}_{2\mathrm{ps}} = 2MV_0^{\mathrm{ps}}\sinh(\beta r_0)\cosh(\beta r_0) - 2E^{\mathrm{ps}}_{n\kappa}V_0^{\mathrm{ps}}\sinh(\beta r_0)\cosh(\beta r_0) + \\ + 2C_{\mathrm{ps}}V_0^{\mathrm{ps}}\sinh(\beta r_0)\cosh(\beta r_0) - MB^{\mathrm{ps}} + E^{\mathrm{ps}}_{n\kappa}B^{\mathrm{ps}} - C_{\mathrm{ps}}B^{\mathrm{ps}}. \quad (23)$$

Considering the Schrödinger-like equation (19), one may introduce the following operators [23–25]:

$$\hat{A} = \frac{d}{dr} - W(r), \quad \hat{A}^\dagger = -\frac{d}{dr} - W(r), \quad (24)$$

where $W(r)$ is the superpotential partner. In the SUSYQM formalism [23–25], the ground state wave function for the lower component is given as

$$G^{\mathrm{ps}}_{0,\kappa}(r) = \exp\left(-\int W(r)\,dr\right). \quad (25)$$

This ground state is equivalent to the following nonlinear Riccati equation:

$$W^2 - W' = V_{\mathrm{eff}}(r) - \tilde{E}^{\mathrm{ps}}_{0,\kappa}, \quad (26)$$

for which we proposed a superpotential of the form

$$W^{\mathrm{ps}} = g^{\mathrm{ps}} - f^{\mathrm{ps}}\coth(\beta r). \quad (27)$$

Now by using this superpotential and the expression for the effective potential, the Riccati equation (26) turns into exact parameter of our study as

$$(f^{\mathrm{ps}})^2 + (g)^2 + (f)^2\,\mathrm{cosech}^2(\beta r) + 2f^{\mathrm{ps}}g^{\mathrm{ps}}\coth(\beta r) + \beta f^{\mathrm{ps}}\,\mathrm{cosech}^2(\beta r) = \\ = \tilde{V}_{1\mathrm{ps}}\,\mathrm{cosech}^2(\beta r) + \tilde{V}_{2\mathrm{ps}}\coth(\beta r) - \tilde{E}^{\mathrm{ps}}_{0,\kappa}. \quad (28)$$

Or solving Eq. (28) completely yields

$$\tilde{E}^{\mathrm{ps}}_{0,\kappa} = -\left((f^{\mathrm{ps}})^2 + (g^{\mathrm{ps}})^2\right), \quad (29)$$

$$f^{\mathrm{ps}} = \frac{\beta - \sqrt{\beta^2 + 4\tilde{V}_{1\mathrm{ps}}}}{2}, \quad (30)$$

$$g^{\mathrm{ps}} = -\frac{\tilde{V}_{2\mathrm{ps}}}{2f^{\mathrm{ps}}}. \quad (31)$$

Using Eq. (24), we can construct the partner Hamiltonian as

$$V_{\mathrm{eff}+}(r) = W^2 + \frac{dW}{dr} = f^{\mathrm{ps}}(f^{\mathrm{ps}} + \beta)\,\mathrm{cosech}^2(\beta r) + \tilde{V}_{2\mathrm{ps}}\coth(\beta r) + (f^{\mathrm{ps}})^2 + \frac{\tilde{V}^2_{2\mathrm{ps}}}{4(f^{\mathrm{ps}})^2}, \quad (32)$$

$$V_{\mathrm{eff}-}(r) = \phi^2 - \frac{d\phi}{dr} = f^{\mathrm{ps}}(f^{\mathrm{ps}} - \beta)\,\mathrm{cosech}^2(\beta r) + \tilde{V}_{2\mathrm{ps}}\coth(\beta r) + (f^{\mathrm{ps}})^2 + \frac{\tilde{V}^2_{2\mathrm{ps}}}{4(f^{\mathrm{ps}})^2}. \quad (33)$$

Thus, it is not difficult to show that $V_+(r)$ and $V_-(r)$ are shape-invariant, i.e,

$$V_+(r,\rho_0) = V_-(r,\rho_i) + R(\rho_i), \tag{34}$$

where $\rho_0 = f^{\rm ps}$ and $\rho_i$ is a function of $\rho_0$, i.e., $\rho_1 = f(\rho_0) = \rho_0 + \beta$. Therefore, $\rho_n = f(\rho_0) = \rho_0 + n\beta$. Consequently, the shape invariance holds via the mapping $f^{\rm ps} \to f^{\rm ps} + \beta$. Thus, from Eq. (34), we obtained

$$\begin{aligned}
R(\rho_1) &= \left(\rho_0^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_0^2}\right) - \left(\rho_1^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_1^2}\right),\\
R(\rho_2) &= \left(\rho_1^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_1^2}\right) - \left(\rho_2^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_2^2}\right),\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
R(\rho_n) &= \left(\rho_{n-1}^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_{n-1}^2}\right) - \left(\rho_n^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_n^2}\right),
\end{aligned} \tag{35}$$

$$\tilde{E}_{0,\kappa}^{-} = 0. \tag{36}$$

Consequently, the energy eigenvalues can be obtained from Eq. (35) as

$$\tilde{E}_{n\kappa}^{\rm ps-} = \sum_{k=1}^{n} R(\rho_k) = \left(\rho_0^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_0^2}\right) - \left(\rho_n^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_n^2}\right), \tag{37}$$

but

$$\tilde{E}_{n\kappa}^{\rm ps} = \tilde{E}_{n\kappa}^{\rm ps-} + \tilde{E}_{0,\kappa}^{\rm ps} = -\left(\rho_n^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_n^2}\right). \tag{38}$$

With the aid of Eqs. (29)–(31), (35) and (37), we obtain the energy eigenvalues for the Tietz and general Manning–Rosen potentials for the pseudospin symmetry for any spin-orbit quantum number as

$$\begin{aligned}
&-M^2 - C_{\rm ps}M + (E_{n\kappa}^{\rm ps})^2 - C_{\rm ps}E_{n\kappa}^{\rm ps} + MV_0^{\rm ps}\cosh^2(\beta r_0) - E_{n\kappa}^{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0) +\\
&\quad + C_{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0) + MV_0^{\rm ps}\sinh^2(\beta r_0) - E_{n\kappa}^{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0) +\\
&\quad + C_{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0) + MA^{\rm ps} - E_{n\kappa}^{\rm ps}A^{\rm ps} + C_{\rm ps}A^{\rm ps} + \left(\rho_n^2 + \frac{\tilde{V}_{2\rm ps}^2}{4\rho_n^2}\right) = 0,
\end{aligned} \tag{39}$$

where

$$\rho_n = \beta\left(n + \sigma^{\rm ps}\right), \tag{40}$$

$$\sigma^{\rm ps} = \frac{1}{2}\left(1 \pm \sqrt{1 + \frac{4\tilde{V}_{1\rm ps}}{\beta^2}}\right). \tag{41}$$

For completeness of description of the potential model under investigation, let us find the corresponding wave functions for the pseudospin symmetry limit. By using the transformation, $z = \coth(\beta r)$, we obtain lower component of the wave function as

$$G_{n\kappa}(r) = N_{n\kappa}\left(\frac{1}{2}+\frac{\coth(\beta r)}{2}\right)^{\sqrt{\gamma^{\rm ps}+\varepsilon_0^{\rm ps}}}\left(\frac{1}{2}-\frac{\coth(\beta r)}{2}\right)^{\sqrt{-\gamma^{\rm ps}+\varepsilon_0^{\rm ps}}}\times \\ \times\, {}_1F_2\left(-n; n+1+2\sqrt{\gamma^{\rm ps}+\varepsilon_0^{\rm ps}}+2\sqrt{-\gamma^{\rm ps}+\varepsilon_0^{\rm ps}}; \right. \\ \left. 2\sqrt{\gamma^{\rm ps}+\varepsilon_0^{\rm ps}}+1; \frac{1}{2}+\frac{\coth(\beta r)}{2}\right), \quad (42)$$

where

$$\gamma^{\rm ps} = -2MV_0^{\rm ps}\sinh(\beta r_0)\cosh(\beta r_0) + 2E_{n\kappa}^{\rm ps}V_0^{\rm ps}\sinh(\beta r_0)\cosh(\beta r_0) - \\ - 2C_{\rm ps}V_0^{\rm ps}\sinh(\beta r_0)\cosh(\beta r_0) + MB^{\rm ps} - E_{n\kappa}^{\rm ps}B^{\rm ps} + C_{\rm ps}B^{\rm ps}, \quad (43)$$

$$\varepsilon_0^{\rm ps} = M^2 + C_{\rm ps}M - (E_{n\kappa}^{\rm ps})^2 + C_{\rm ps}E_{n\kappa}^{\rm ps} - MV_0^{\rm ps}\cosh^2(\beta r_0) + \\ + E_{n\kappa}^{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0) - C_{\rm ps}V_0^{\rm ps}\cosh^2(\beta r_0) - MV_0^{\rm ps}\sinh^2(\beta r_0) + \\ + E_{n\kappa}^{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0) - C_{\rm ps}V_0^{\rm ps}\sinh^2(\beta r_0) - MA^{\rm ps} + E_{n\kappa}^{\rm ps}A^{\rm ps} - C_{\rm ps}A^{\rm ps}. \quad (44)$$

**1.2. Spin Symmetry Limit.** In the spin symmetry limit $d\Delta(r)/dr = 0$ or $\Delta(r) = C_s =$ const [2, 3]. As in the previous subsection, we consider the sum of the potentials as Tietz and general Manning–Rosen potentials and Coulomb-like tensor interaction as

$$\Sigma(r) = V_0^s\left(\frac{\sinh\beta(r-r_0)}{\sinh\beta r}\right)^2 + A^s + B^s\coth(\beta r) - C^s\,\mathrm{cosech}^2(\beta r), \quad (45)$$

$$U(r) = -\frac{H_c}{r}. \quad (46)$$

Substituting Eqs. (45) and (46) into Eq. (11) gives

$$\left\{\frac{d^2}{dr^2} - \frac{\Lambda_\kappa(\Lambda_\kappa-1)}{r^2}\right\}F_{n\kappa}(r) - (M+E_{n\kappa}-C_s)\times \\ \times\left(V_0^s\left(\frac{\sinh\beta(r-r_0)}{\sinh\beta r}\right)^2 + A^s + B^s\coth(\beta r) - C^s\,\mathrm{cosech}^2(\beta r)\right)F_{n\kappa}(r) = \\ = (M+E_{n\kappa}-C_s)(M-E_{n\kappa})F_{n\kappa}(r). \quad (47)$$

Using approximations (17) in Eq. (47) yields

$$-\frac{d^2F_{n\kappa}}{dr^2} + V_{\rm eff}(r)F_{n\kappa} = \tilde{E}_{n\kappa}^s F_{n\kappa}, \quad (48)$$

where

$$V_{\text{eff}}(r) = \tilde{V}_{1s}\,\text{cosech}^2\,(\beta r) + \tilde{V}_{2s}\coth(\beta r), \tag{49}$$

$$\begin{aligned}\tilde{E}^s_{n\kappa} = &-(M + E_{n\kappa} - C_s)(M - E_{n\kappa}) - MV^s_0\cosh^2(\beta r_0) - \\ &- E^s_{n\kappa}V^s_0\cosh^2(\beta r_0) + C_sV^s_0\cosh^2(\beta r_0) - MV^s_0\sinh^2(\beta r_0) - \\ &- E^s_{n\kappa}V^s_0\sinh^2(\beta r_0) + C_sV^s_0\sinh^2(\beta r_0) - MA^s - E^s_{n\kappa}A^s + C_sA^s,\end{aligned} \tag{50}$$

$$\begin{aligned}\tilde{V}_{1s} = &\beta^2\Lambda_\kappa(\Lambda_\kappa - 1) + MV^s_0\sinh^2(\beta r_0) + E^s_{n\kappa}V^s_0\sinh^2(\beta r_0) - \\ &- C_sV^s_0\sinh^2(\beta r_0) - MC^s - E^s_{n\kappa}C^s + C^sC_s,\end{aligned} \tag{51}$$

$$\begin{aligned}\tilde{V}_{2s} = &MB^s + E^s_{n\kappa}B^s - C_sB^s - 2MV^s_0\sinh(\beta r_0)\cosh(\beta r_0) - \\ &- 2E^s_{n\kappa}V^s_0\sinh(\beta r_0)\cosh(\beta r_0) + 2C_sV^s_0\sinh(\beta r_0)\cosh(\beta r_0),\end{aligned} \tag{52}$$

$$\begin{aligned}&\left(H_c^2 + 2\kappa H_c + H_c + \kappa(\kappa+1)\right) = (\kappa + H_c)(\kappa + H_c + 1) = \\ &= \Lambda_\kappa(\Lambda_\kappa - 1) \to \Lambda_\kappa = (\kappa + H_c + 1)\,.\end{aligned} \tag{53}$$

Following the same procedures as in the previous subsection, we find the energy eigenvalues equation and the corresponding wave function of the Dirac particles for the Tietz and general Manning potentials in the presence of Coulomb-like tensor interactions for the spin symmetry limits as follows:

$$\begin{aligned}&-(M + E_{n\kappa} - C_s)(M - E_{n\kappa}) - MV^s_0\cosh^2(\beta r_0) - E^s_{n\kappa}V^s_0\cosh^2(\beta r_0) + \\ &+ C_sV^s_0\cosh^2(\beta r_0) - MV^s_0\sinh^2(\beta r_0) - E^s_{n\kappa}V^s_0\sinh^2(\beta r_0) + C_sV^s_0\sinh^2(\beta r_0) - \\ &- MA^s - E^s_{n\kappa}A^s + C_sA^s + \left(\rho_n^2 + \frac{\tilde{V}_{2s}^2}{4\rho_n^2}\right) = 0,\end{aligned} \tag{54}$$

where $\rho_n = \beta\,(n + \sigma^s)$ and $\sigma^{\text{ps}} = \frac{1}{2}\left(1 \pm \sqrt{1 + \frac{4\tilde{V}_{1s}}{\beta^2}}\right)$. The upper component of the wave function is obtained as follows:

$$\begin{aligned}F_{n\kappa}(r) = &N_{n\kappa}\left(\frac{1}{2} + \frac{\coth(\beta r)}{2}\right)^{\sqrt{\gamma^s + \varepsilon_0^{\text{ps}}}}\left(\frac{1}{2} - \frac{\coth(\beta r)}{2}\right)^{\sqrt{-\gamma^s + \varepsilon_0^{\text{ps}}}} \times \\ &\times {}_2F_1\left(-n; n + 1 + 2\sqrt{\gamma^s + \varepsilon^s_0} + 2\sqrt{-\gamma^s + \varepsilon^s_0}; 2\sqrt{\gamma^s + \varepsilon^s_0} + 1; \frac{1}{2} + \frac{\coth(\beta r)}{2}\right),\end{aligned} \tag{55}$$

where

$$\gamma^s = -2MV_0^{\text{ps}} \sinh(\beta r_0)\cosh(\beta r_0) + \\ + 2E_{n\kappa}^{\text{ps}} V_0^{\text{ps}} \sinh(\beta r_0)\cosh(\beta r_0) - 2C_{\text{ps}} V_0^{\text{ps}} \sinh(\beta r_0)\cosh(\beta r_0) + \\ + MB^{\text{ps}} - E_{n\kappa}^{\text{ps}} B^{\text{ps}} + C_{\text{ps}} B^{\text{ps}}, \quad (56)$$

$$\varepsilon_0^s = M^2 + C_{\text{ps}} M - (E_{n\kappa}^{\text{ps}})^2 + C_{\text{ps}} E_{n\kappa}^{\text{ps}} - MV_0^{\text{ps}} \cosh^2(\beta r_0) + \\ + E_{n\kappa}^{\text{ps}} V_0^{\text{ps}} \cosh^2(\beta r_0) - C_{\text{ps}} V_0^{\text{ps}} \cosh^2(\beta r_0) - MV_0^{\text{ps}} \sinh^2(\beta r_0) + \\ + E_{n\kappa}^{\text{ps}} V_0^{\text{ps}} \sinh^2(\beta r_0) - C_{\text{ps}} V_0^{\text{ps}} \sinh^2(\beta r_0) - MA^{\text{ps}} + E_{n\kappa}^{\text{ps}} A^{\text{ps}} - C_{\text{ps}} A^{\text{ps}}, \quad (57)$$

and the other component can be simply found via

$$G_{n\kappa}(r) = \frac{1}{M + E_{n\kappa} - C_s}\left(\frac{d}{dr} + \frac{\kappa}{r} - U(r)\right) F_{n\kappa}(r). \quad (58)$$

## 2. NUMERICAL RESULTS

We have calculated the energy eigenvalues in the absence ($H = 0$) and the presence ($H = 1$) of the Coulomb-like tensor potential for various values of the quantum numbers *n* and $\kappa$. The results are presented in Tables 1 and 2 for the Tietz and general Manning–Rosen potentials under the condition of the pseudospin and spin symmetries, respectively, and we

*Table* 1. **Energies in the pseudospin symmetry limit for** $M = 1$**,** $C_{\text{ps}} = -5$**,** $V_0 = -1.2$**,** $\beta = 0.01$**,** $r_0 = 0.2$**,** $A^{\text{ps}} = 0.5$**,** $B^{\text{ps}} = 0.8$**,** $C^{\text{ps}} = -1$

| $\tilde{\ell}$ | $n$, $k < 0$ | $(\ell, j)$ | $E_{n\kappa}^{\text{ps}}$, fm$^{-1}$ ($H = 0$) | $E_{n\kappa}^{\text{ps}}$, fm$^{-1}$ ($H = 1$) | $n - 1$, $\kappa > 0$ | $(\ell + 2, j + 1)$ | $E_{n\kappa}^{\text{ps}}$, fm$^{-1}$ ($H = 0$) | $E_{n\kappa}^{\text{ps}}$, fm$^{-1}$ ($H = 1$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1, –1 | $1S_{1/2}$ | –2.839045187 | –2.838905765 | 0, 2 | $0d_{3/2}$ | –2.839045187 | –2.839324161 |
| 2 | 1, –2 | $1P_{3/2}$ | –2.839324161 | –2.839045187 | 0, 3 | $0f_{5/2}$ | –2.839324161 | –2.839742946 |
| 3 | 1, –3 | $1d_{5/2}$ | –2.839742946 | –2.839324161 | 0, 4 | $0g_{7/2}$ | –2.839742946 | –2.84030193 |
| 4 | 1, –4 | $1f_{7/2}$ | –2.84030193 | –2.839742946 | 0, 5 | $0h_{9/2}$ | –2.840301930 | –2.841001635 |
| 1 | 2, –1 | $2S_{1/2}$ | –2.824238247 | –2.824103718 | 1, 2 | $1d_{3/2}$ | –2.824238247 | –2.824507427 |
| 2 | 2, –2 | $2P_{3/2}$ | –2.824507427 | –2.824238247 | 1, 3 | $1f_{5/2}$ | –2.824507427 | –2.824911498 |
| 3 | 2, –3 | $2d_{5/2}$ | –2.824911498 | –2.824507427 | 1, 4 | $1g_{7/2}$ | –2.824911498 | –2.825450823 |
| 4 | 2, –4 | $2f_{7/2}$ | –2.825450823 | –2.824911498 | 1, 5 | $1h_{9/2}$ | –2.825450823 | –2.826125889 |
| 1 | 3, –1 | $3S_{1/2}$ | –2.809857312 | –2.809727420 | 2, 2 | $2d_{3/2}$ | –2.809857312 | –2.810117210 |
| 2 | 3, –2 | $3P_{3/2}$ | –2.810117210 | –2.809857312 | 2, 3 | $2f_{5/2}$ | –2.81011721 | –2.810507339 |
| 3 | 3, –3 | $3d_{5/2}$ | –2.810507339 | –2.810117210 | 2, 4 | $2g_{7/2}$ | –2.810507339 | –2.81102804 |
| 4 | 3, –4 | $3f_{7/2}$ | –2.81102804 | –2.810507339 | 2, 5 | $2h_{9/2}$ | –2.811028040 | –2.811679766 |

*Table* 2. **Energies in the spin symmetry limit for** $M = 1$**,** $C_s = 5$**,** $V_0 = 1.2$**,** $\beta = 0.01$**,** $r_0 = 0.2$**,** $A^s = -0.5$**,** $B^s = -0.8$**,** $C^s = 1$

| $\ell$ | $n$, $k < 0$ | $(\ell, j)$ | $E^s_{n\kappa}$, fm$^{-1}$ $(H = 0)$ | $E^s_{n\kappa}$, fm$^{-1}$ $(H = 1)$ | $n$, $\kappa > 0$ | $(\ell, j)$ | $E^s_{n\kappa}$, fm$^{-1}$ $(H = 0)$ | $E^s_{n\kappa}$, fm$^{-1}$ $(H = 1)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0, –2 | $0P_{3/2}$ | 2.854297362 | 2.854152761 | 0, 1 | $0P_{1/2}$ | 2.854297362 | 2.854586705 |
| 2 | 0, –3 | $0d_{5/2}$ | 2.854586705 | 2.854297362 | 0, 2 | $0d_{3/2}$ | 2.854586705 | 2.855021067 |
| 3 | 0, –4 | $0f_{7/2}$ | 2.855021067 | 2.854586705 | 0, 3 | $0f_{5/2}$ | 2.855021067 | 2.855600869 |
| 4 | 0, –5 | $0g_{9/2}$ | 2.855600869 | 2.855021067 | 0, 4 | $0g_{7/2}$ | 2.855600869 | 2.85632667 |
| 1 | 1, –2 | $1P_{3/2}$ | 2.839045187 | 2.838905765 | 1, 1 | $1P_{1/2}$ | 2.839045187 | 2.839324161 |
| 2 | 1, –3 | $1d_{5/2}$ | 2.839324161 | 2.839045187 | 1, 2 | $1d_{3/2}$ | 2.839324161 | 2.839742946 |
| 3 | 1, –4 | $1f_{7/2}$ | 2.839742946 | 2.839324161 | 1, 3 | $1f_{5/2}$ | 2.839742946 | 2.84030193 |
| 4 | 1, –5 | $1g_{9/2}$ | 2.84030193 | 2.839742946 | 1, 4 | $10g_{7/2}$ | 2.84030193 | 2.841001635 |
| 1 | 2, –2 | $2P_{3/2}$ | 2.824238247 | 2.824103718 | 2, 1 | $2P_{1/2}$ | 2.824238247 | 2.824507427 |
| 2 | 2, –3 | $2d_{5/2}$ | 2.824507427 | 2.824238247 | 2, 2 | $2d_{3/2}$ | 2.824507427 | 2.824911498 |
| 3 | 2, –4 | $2f_{7/2}$ | 2.824911498 | 2.824507427 | 2, 3 | $2f_{5/2}$ | 2.824911498 | 2.825450823 |
| 4 | 2, –5 | $2g_{9/2}$ | 2.825450823 | 2.824911498 | 2, 4 | $2g_{7/2}$ | 2.825450823 | 2.826125889 |

can clearly see that there is the degeneracy between the bound states and, in the presence of the tensor interaction, these degeneracies are changed or removed. Also Tables 3–6 show behavior of the energy eigenvalues versus difference values of the $V_0^{\text{ps}}$, $V_0^s$ and $C_{\text{ps}}$, $C_s$ for both

*Table* 3. **Energies in the pseudospin symmetry limit for** $M = 1$**,** $C_{\text{ps}} = -5$**,** $H = 1.2$**,** $\beta = 0.01$**,** $r_0 = 0.2$**,** $A^{\text{ps}} = 0.5$**,** $B^{\text{ps}} = 0.8$**,** $C^{\text{ps}} = -1$

| $V_0$ | $E^{\text{ps}}_{n\kappa}$, fm$^{-1}$ | | | | | |
|---|---|---|---|---|---|---|
| | $1S_{1/2}$ | $1P_{3/2}$ | $1d_{5/2}$ | $1f_{7/2}$ | $2S_{1/2}$ | $2P_{3/2}$ |
| –1.3 | –2.938036055 | –2.938188076 | –2.938492284 | –2.938949009 | –2.922620024 | –2.922766226 |
| –1.1 | –2.739653307 | –2.739782049 | –2.740039638 | –2.740426279 | –2.725397958 | –2.725522525 |
| –0.9 | –2.540865672 | –2.540977298 | –2.541200619 | –2.541535777 | –2.527544433 | –2.527652908 |
| –0.7 | –2.341796511 | –2.341895031 | –2.342092120 | –2.342387878 | –2.329247240 | –2.329343289 |
| –0.5 | –2.142522420 | –2.142610586 | –2.142786956 | –2.143051603 | –2.130624600 | –2.130710771 |
| –0.3 | –1.943093759 | –1.943173543 | –1.943333138 | –1.943572600 | -1.931755039 | –1.931833173 |
| –0.1 | –1.743545163 | –1.743618022 | –1.743763760 | –1.743982422 | –1.732692985 | –1.732764453 |
| 0.1 | –1.543901330 | –1.543968372 | –1.544102472 | –1.544303667 | –1.533477487 | –1.533543339 |
| 0.3 | –1.344180404 | –1.344242491 | –1.344366680 | –1.344552999 | –1.334137376 | –1.334198431 |
| 0.5 | –1.144396051 | –1.144453868 | –1.144569514 | –1.144743014 | –1.134694456 | –1.134751368 |
| 0.7 | –0.944558787 | –0.944612886 | –0.944721094 | –0.944883431 | –0.935165561 | –0.935218859 |
| 0.9 | –0.744676852 | –0.744727684 | –0.744829357 | –0.744981888 | –0.735563920 | –0.735614037 |
| 1.1 | –0.544756806 | –0.544804745 | –0.544900631 | –0.545044479 | –0.535900091 | –0.535947387 |
| 1.3 | –0.344803945 | –0.344849305 | –0.344940031 | –0.345076136 | –0.336182616 | –0.336227393 |

*Table* 4. **Energies in the spin symmetry limit for** $M = 1$, $C_s = 5$, $H = 1$, $\beta = 0.01$, $r_0 = 0.2$, $A^s = -0.5$, $B^s = -0.8$, $C^s = 1$

| $V_0$ | $E^s_{n\kappa}$, fm$^{-1}$ | | | | | |
|---|---|---|---|---|---|---|
| | $1f_{7/2}$ | $1d_{5/2}$ | $1P_{3/2}$ | $2f_{7/2}$ | $2d_{5/2}$ | $2P_{3/2}$ |
| –1.3 | 0.344940031 | 0.344849305 | 0.344803945 | 0.336316954 | 0.336227393 | 0.336182616 |
| –1.1 | 0.544900631 | 0.544804745 | 0.544756806 | 0.536041986 | 0.535947387 | 0.535900091 |
| –0.9 | 0.744829357 | 0.744727684 | 0.744676852 | 0.735714279 | 0.735614037 | 0.735563920 |
| –0.7 | 0.944721094 | 0.944612886 | 0.944558787 | 0.935325464 | 0.935218859 | 0.935165561 |
| –0.5 | 1.144569514 | 1.144453868 | 1.144396051 | 1.134865204 | 1.134751368 | 1.134694456 |
| –0.3 | 1.34436668 | 1.344242491 | 1.344180404 | 1.334320556 | 1.334198431 | 1.334137376 |
| –0.1 | 1.544102472 | 1.543968372 | 1.543901330 | 1.533675059 | 1.533543339 | 1.533477487 |
| 0.1 | 1.743763760 | 1.743618022 | 1.743545163 | 1.732907411 | 1.732764453 | 1.732692985 |
| 0.3 | 1.943333138 | 1.943173543 | 1.943093759 | 1.931989467 | 1.931833173 | 1.931755039 |
| 0.5 | 2.142786956 | 2.142610586 | 2.142522420 | 2.130883148 | 2.130710771 | 2.130624600 |
| 0.7 | 2.342092120 | 2.341895031 | 2.341796511 | 2.329535434 | 2.329343289 | 2.329247240 |
| 0.9 | 2.541200619 | 2.540977298 | 2.540865672 | 2.527869924 | 2.527652908 | 2.527544433 |
| 1.1 | 2.740039638 | 2.739782049 | 2.739653307 | 2.725771758 | 2.725522525 | 2.725397958 |
| 1.3 | 2.938492284 | 2.938188076 | 2.938036055 | 2.923058782 | 2.922766226 | 2.922620024 |

*Table* 5. **Energies in the pseudospin symmetry limit** $M = 1$, $H = 1$, $V_0 = -1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^{\rm ps} = 0.5$, $B^{\rm ps} = 0.8$, $C^{\rm ps} = -1$

| $C_{\rm ps}$ | $E^{\rm ps}_{n\kappa}$, fm$^{-1}$ | | | |
|---|---|---|---|---|
| | $1S_{1/2}$ | $1P_{3/2}$ | $1d_{5/2}$ | $1f_{7/2}$ |
| –6 | –2.844969952 | –2.84504602 | –2.845198181 | –2.845426483 |
| –5.5 | –2.842616995 | –2.842715463 | –2.842912449 | –2.843208053 |
| –5 | –2.838905765 | –2.839045187 | –2.839324161 | –2.839742946 |
| –4.5 | –2.831710714 | –2.831948096 | –2.832423446 | –2.833137937 |
| –4 | –2.806615503 | –2.807332449 | –2.808781743 | –2.81099585 |

*Table* 6. **Energies in the spin symmetry limit** $M = 1$, $H = 1$, $V_0 = 1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^s = -0.5$, $B^s = -0.8$, $C^s = 1$

| $C_s$ | $E^s_{n\kappa}$, fm$^{-1}$ | | | |
|---|---|---|---|---|
| | $1g_{9/2}$ | $1f_{7/2}$ | $1d_{5/2}$ | $1P_{3/2}$ |
| 4 | 2.810995850 | 2.808781743 | 2.807332449 | 2.806615503 |
| 4.5 | 2.833137937 | 2.832423446 | 2.831948096 | 2.831710714 |
| 5 | 2.839742946 | 2.839324161 | 2.839045187 | 2.838905765 |
| 5.5 | 2.843208053 | 2.842912449 | 2.842715463 | 2.842616995 |
| 6 | 2.845426483 | 2.845198181 | 2.845046020 | 2.844969952 |

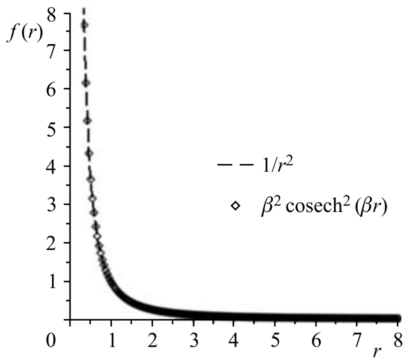


Fig. 1. $1/r^2$ and its approximations for $\beta = 0.01$

symmetries limit. In Fig. 2, we obtain the effects of the tensor interaction on the bound states in view of the pseudospin and spin symmetry limits for the Tietz and general Manning–Rosen potentials. Figure 2 shows that the magnitude of the energy difference between the degenerate states increases as $H$ increases. In Fig. 3, we show the effects of the potential parameters $A^{\mathrm{ps}}$, $A^s$ on the bound states under the conditions of the pseudospin and spin symmetry limits for $H = 1$ for the Tietz and general Manning–Rosen potentials. In Fig. 3, we can see that, although bound states obtained in view of spin symmetry become more bounded with increasing $A^s$, they become less bounded in the pseudospin symmetry limit with increasing $A^{\mathrm{ps}}$. Figures 4 and 5 show the magnitude of the energy versus the potential parameters $B^{\mathrm{ps}}$, $B^s$ and $C^{\mathrm{ps}}$, $C^s$ under the conditions of the pseudospin and spin symmetry limits for $H = 1$ for the Tietz and general Manning–Rosen potentials.

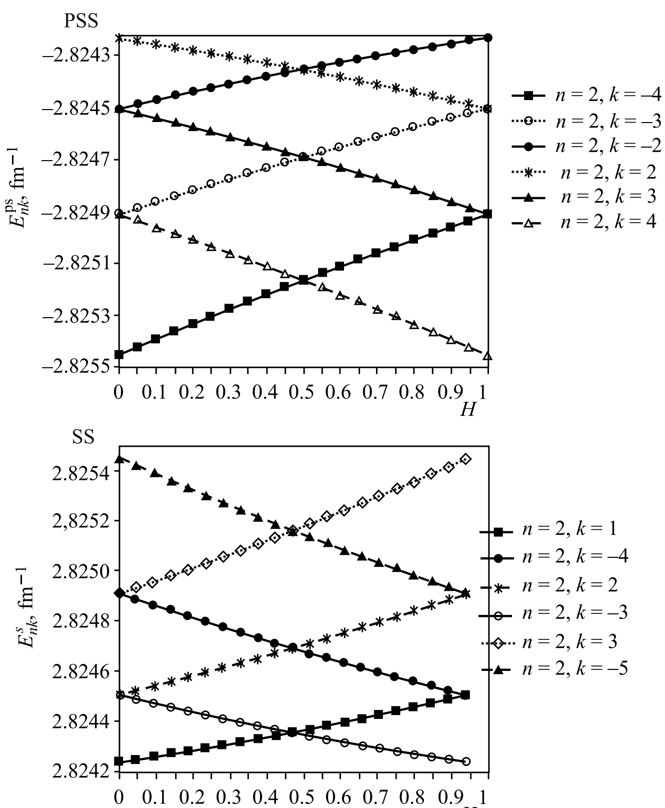


Fig. 2. Type 1: Energy vs. $H$ for pseudospin and spin symmetries limit. PSS: $M = 1$, $C_{\mathrm{ps}} = -5$, $V_0 = -1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^{\mathrm{ps}} = 0.5$, $B^{\mathrm{ps}} = 0.8$, $C^{\mathrm{ps}} = -1$. SS: $M = 1$, $C_s = 5$, $V_0 = 1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^s = -0.5$, $B^s = -0.8$, $C^s = 1$

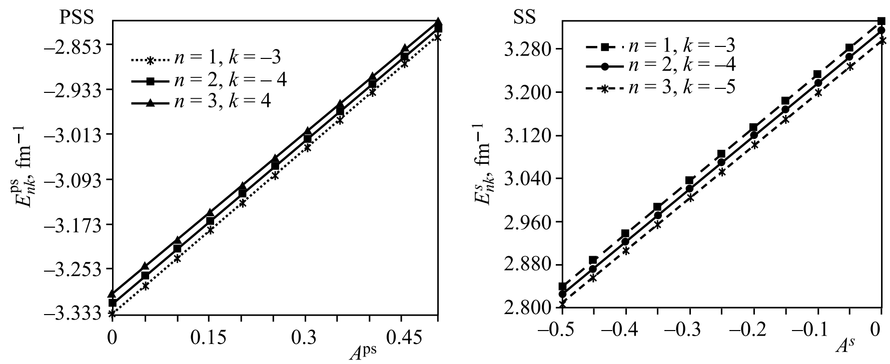


Fig. 3. PSS: Energy vs. $A^{\mathrm{ps}}$ for pseudospin symmetry limit for $M = 1$, $C_{\mathrm{ps}} = -5$, $H = 1$, $V_0 = -1.2$, $\beta = 0.01$, $r_0 = 0.2$, $B^{\mathrm{ps}} = 0.8$, $C^{\mathrm{ps}} = -1$. SS: Energy vs. $A^{\mathrm{ps}}$ for spin symmetry limit for $M = 1$, $C_s = 5$, $H = 1$, $V_0 = 1.2$, $\beta = 0.01$, $r_0 = 0.2$, $B^s = -0.8$, $C^s = 1$

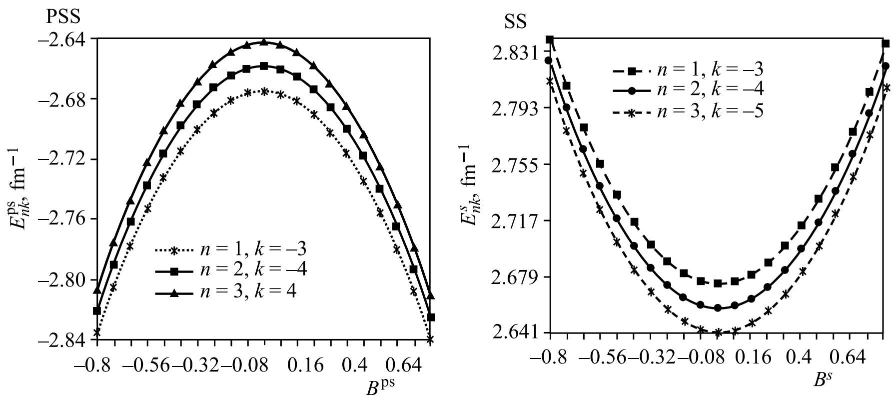


Fig. 4. PSS: Energy vs. $B^{\mathrm{ps}}$ for pseudospin symmetry limit for $M = 1$, $C_{\mathrm{ps}} = -5$, $H = 1$, $V_0 = -1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^{\mathrm{ps}} = 0.5$, $C^{\mathrm{ps}} = -1$. SS: Energy vs. $B^s$ for spin symmetry limit for $M = 1$, $C_s = 5$, $H = 1$, $V_0 = 1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^s = -0.5$, $C^s = 1$

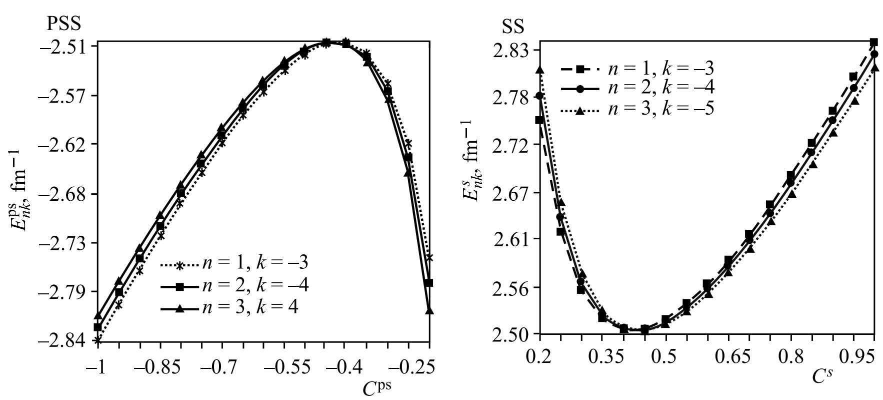


Fig. 5. PSS: Energy vs. $C^{\mathrm{ps}}$ for pseudospin symmetry limit for $M = 1$, $C_{\mathrm{ps}} = -5$, $H = 1$, $V_0 = -1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^{\mathrm{ps}} = 0.5$, $B^{\mathrm{ps}} = 0.8$. SS: Energy vs. $C^s$ for spin symmetry limit for $M = 1$, $C_s = 5$, $H = 1$, $V_0 = 1.2$, $\beta = 0.01$, $r_0 = 0.2$, $A^s = -0.5$, $B^s = -0.8$

## CONCLUSIONS

In this paper, we have obtained approximate solutions of the Dirac equation with combined Tietz and general Manning–Rosen potentials in addition to Coulomb-like tensor term using the SUSYQM technique. The results of our work find many applications in both nuclear and hadron physics and therefore provide a more general solution compared to other previous work [25].

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