

## MASSIVE SPIN-2 IN FRADKIN–VASILIEV FORMALISM

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Here, using massive spin-2 case as an example, we discuss the possibility to extend Fradkin–Vasiliev formalism, initially developed for investigation of massless higher-spin fields interactions, to the interactions involving both massless and/or massive or partially massless fields.

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### INTRODUCTION

Today we have rather complete understanding of cubic interaction vertices for massless higher-spin fields (see, e.g., [1] and references therein). As for their explicit construction, it turns out that one of the very effective way is the Fradkin–Vasiliev formalism [2,3]. Two main ingredients of this formalism are gauge invariance and frame-like description of massless higher-spin fields [4–6]. But frame-like gauge-invariant description exists for massive (and partially massless) fields as well [7,8]. So it seems natural to try to extend this formalism to the case of interactions involving both massless and/or massive or partially massless fields. Here we discuss such a possibility using massive spin-2 case as an example.

First of all, we illustrate the formalism with the simplest example possible — massless spin-2 self-interaction. In particular, we show how usual low derivative gravity can be reproduced in such a formalism. For the main part of our discussion we use partially massless spin-2 field, because being much simpler, it conveniently illustrates both similarities as well as differences with the massless case. Namely, we consider two explicit examples: self-interaction and interaction with massless graviton. At last, we briefly present analogous results for the general massive case leaving detailed discussion for the forthcoming publication.

### 1. MASSLESS CASE

In this section, we briefly recall main basic points of the frame-like description for massless higher-spin particles and Fradkin–Vasiliev formalism. Then, we illustrate the construction of cubic vertices using massless spin-2 case as an example.

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**1.1. Frame-like Formalism.** For the description of massless spin- $s$  particle in the frame-like formalism one uses a set of fields

$$\Phi_{\mu}^{a_1 \dots a_{s-1}}, \Phi_{\mu}^{a_1 \dots a_{s-1}, b_1}, \dots, \Phi_{\mu}^{a_1 \dots a_{s-1}, b_1 \dots b_{s-1}},$$

where the field  $\Phi^{a_1 \dots a_{s-1}, b_1 \dots b_k}$  is completely traceless, symmetric on  $a$  and  $b$  indices and satisfies the constraint  $\Phi^{(a_1 \dots a_{s-1}, b_1) b_2 \dots b_k} = 0$ . The physical field is the lowest one  $\Phi^{a_1 \dots a_{s-1}}$ , while all others are auxiliary, which by solving their algebraic equations, can be expressed as higher derivatives of the physical one  $\Phi^{a_1 \dots a_{s-1}, b_1 \dots b_k} \sim \partial^k \Phi^{a_1 \dots a_{s-1}}$ .

In spite of being auxiliary, each field (and not only physical one) plays the role of gauge field having its own gauge transformation

$$\delta \Phi_{\mu}^{a_1 \dots a_{s-1}, b_1 \dots b_k} \sim D_{\mu} \xi^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots,$$

where dots stand for the terms linear in fields without derivatives. Similarly, the main gauge parameter is the lowest one  $\xi^{a_1 \dots a_{s-1}}$ , while all others are equivalent to its higher derivatives.

For each field one can construct gauge-invariant object (we will call them curvatures):

$$\mathcal{R}_{\mu\nu}^{a_1 \dots a_{s-1}, b_1 \dots b_k} \sim D_{[\mu} \Phi_{\nu]}^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots,$$

where again dots stand for the terms linear in fields without derivatives. Moreover, for the nonzero cosmological constant the free Lagrangian can be rewritten in explicitly gauge-invariant form

$$\mathcal{L}_0 \sim \sum \mathcal{R} \wedge \mathcal{R},$$

where each term is separately gauge-invariant, while relative coefficients are determined by the requirement that all higher derivative terms must be absent.

**1.2. Nontrivial Cubic Vertices.** For the three massless fields with spins  $s_1 \geq s_2 \geq s_3$ , there exist a number of cubic vertices with  $N$  derivatives [9]:

$$s_1 + s_2 - s_3 \leq N \leq s_1 + s_2 + s_3,$$

where  $N$  must be even (odd) if  $s_1 + s_2 + s_3$  is even (odd). Vertices with the maximum number of derivatives  $N = s_1 + s_2 + s_3$  are trivially gauge-invariant and can be easily constructed using gauge-invariant curvatures for all three fields. Here and in what follows, we will call vertex nontrivial if its gauge invariance requires corrections to gauge transformations. Recently Vasiliev has shown [1] that all nontrivial vertices having up to  $N = s_1 + s_2 + s_3 - 2$  derivatives can be constructed as combinations of the so-called non-Abelian and Abelian ones.

All non-Abelian vertices have the general form

$$\mathcal{R} \wedge \Phi \wedge \Phi$$

and results from quadratic deformations of curvatures

$$\mathcal{R} \Rightarrow \hat{\mathcal{R}} = \mathcal{R} \oplus \Phi \wedge \Phi$$

determined by the requirement that deformed curvatures transform covariantly

$$\delta \hat{\mathcal{R}} \sim \mathcal{R} \xi.$$

At the same time, Abelian vertices have the general form

$$\mathcal{R} \wedge \mathcal{R} \wedge \Phi$$

and their gauge invariance heavily relies on Bianchi identities.

**1.3. Massless Spin-2 in  $AdS$ .** In the frame formalism the free Lagrangian for massless spin-2 field in  $AdS_d$  space has the form

$$\mathcal{L}_0 = \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \omega_\mu{}^{ac} \omega_\nu{}^{bc} - \frac{1}{2} \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \omega_\mu{}^{ab} D_\nu h_\alpha{}^c - \frac{(d-2)\kappa}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} h_\mu{}^a h_\nu{}^b, \quad (1)$$

where  $\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} = e^\mu{}_a e^\nu{}_b - e^\mu{}_b e^\nu{}_a$ ,  $\kappa = \frac{2\Lambda}{(d-1)(d-2)}$ .

This Lagrangian is invariant under the following local gauge transformations:

$$\delta_0 \omega_\mu{}^{ab} = D_\mu \hat{\eta}^{ab} + \kappa e_\mu{}^{[a} \hat{\xi}^{b]}, \quad \delta_0 h_\mu{}^a = D_\mu \hat{\xi}^a + \hat{\eta}_\mu{}^a. \quad (2)$$

Moreover, one can easily construct two gauge-invariant objects (linearized curvature and torsion):

$$\begin{aligned} R_{\mu\nu}{}^{ab} &= D_{[\mu} \omega_{\nu]}{}^{ab} + \kappa e_{[\mu}{}^{[a} h_{\nu]}{}^{b]}, \\ T_{\mu\nu}{}^a &= D_{[\mu} h_{\nu]}{}^a - \omega_{[\mu, \nu]}{}^a. \end{aligned} \quad (3)$$

In this, the free Lagrangian can be rewritten as

$$\mathcal{L}_0 = a_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd}, \quad a_0 = -\frac{1}{32(d-3)\kappa}. \quad (4)$$

**1.4. Cubic Vertices for Massless Spin-2.** Now let us illustrate the general procedure on the simple case — massless spin-2 self-interaction. As a first step one has to construct consistent quadratic deformations for curvatures. In this particular case this unambiguously gives the following result:

$$\begin{aligned} \Delta R_{\mu\nu}{}^{ab} &= b_0 [\omega_{[\mu}{}^{ca} \omega_{\nu]}{}^{bc} + \kappa h_{[\mu}{}^a h_{\nu]}{}^b], \\ \Delta T_{\mu\nu}{}^a &= b_0 \omega_{[\mu}{}^{ab} h_{\nu]}{}^b. \end{aligned} \quad (5)$$

Such a deformation results in the following transformations for deformed curvatures:

$$\begin{aligned} \delta \hat{R}_{\mu\nu}{}^{ab} &= b_0 R_{\mu\nu}{}^{c[a} \hat{\eta}^{b]c} + \kappa b_0 T_{\mu\nu}{}^{[a} \hat{\xi}^{b]}, \\ \delta \hat{T}_{\mu\nu}{}^a &= -b_0 \hat{\eta}^{ab} T_{\mu\nu}{}^b + b_0 R_{\mu\nu}{}^{ab} \hat{\xi}^b. \end{aligned} \quad (6)$$

Now let us consider the interacting Lagrangian in the form:

$$\mathcal{L} = a_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd} + c_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd} h_\gamma{}^e, \quad (7)$$

where the first term is just the free Lagrangian with the curvature replaced by the deformed one, while the second term is the only possible Abelian vertex in this case. Both terms are separately gauge-invariant, so we obtain two independent vertices having up to four derivatives. Note, however, that four derivative terms from these two vertices turn out to be equivalent on-shell (which in this particular case means just zero-torsion condition):

$$c_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} D_\mu \omega_\nu{}^{ab} D_\alpha \omega_\beta{}^{cd} h_\gamma{}^e \approx 3c_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} D_\mu \omega_\nu{}^{ab} \omega_\alpha{}^{ce} \omega_\beta{}^{de} + \dots$$

Thus, we may adjust coefficients so that all four derivative terms vanish on-shell leaving us with the two-derivative vertex.

## 2. PARTIALLY MASSLESS CASE

In this section, we apply the same procedure to construct nontrivial cubic vertices for the partially massless spin-2 fields.

**2.1. Partially Massless Spin-2.** Recall that partially massless fields correspond to exotic representation of the de Sitter group (see, e.g., [10, 11]). In  $d = 4$  the partially massless spin-2 has helicities  $\pm 2, \pm 1$  so the frame-like gauge-invariant description requires two pairs of fields  $(\Omega_\mu^{ab}, f_\mu^a)$  and  $(B^{ab}, B_\mu)$ . Free Lagrangian for partially massless spin-2 has the form [7]:

$$\mathcal{L}_0 = \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{ac} \Omega_\nu^{bc} - \frac{1}{2} \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \Omega_\mu^{ab} D_\nu f_\alpha^c + \frac{1}{2} B_{ab}^2 - \\ - \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} B^{ab} D_\mu B_\nu + m [ \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \omega_\mu^{ab} B_\nu + e^\mu{}_a B^{ab} f_\mu^b ], \quad (8)$$

where  $m^2 = (d-2)\kappa$ . This Lagrangian is invariant under the following gauge transformations:

$$\delta_0 \Omega_\mu^{ab} = D_\mu \eta^{ab}, \quad \delta_0 f_\mu^a = D_\mu \xi^a + \eta_\mu^a + \frac{2m}{(d-2)} e_\mu^a \xi, \\ \delta_0 B^{ab} = -m \eta^{ab}, \quad \delta_0 B_\mu = D_\mu \xi + \frac{m}{2} \xi_\mu. \quad (9)$$

Correspondingly, we have four gauge-invariant objects (curvatures):

$$\mathcal{F}_{\mu\nu}^{ab} = D_{[\mu} \Omega_{\nu]}^{ab} - \frac{m}{(d-2)} e_{[\mu}^{[a} B_{\nu]}^b], \\ \mathcal{T}_{\mu\nu}^a = D_{[\mu} f_{\nu]}^a - \Omega_{[\mu, \nu]}^a + \frac{2m}{(d-2)} e_{[\mu}^a B_{\nu]}, \\ \mathcal{B}_\mu^{ab} = D_\mu B^{ab} + m \Omega_\mu^{ab}, \\ \mathcal{B}_{\mu\nu} = D_{[\mu} B_{\nu]} - B_{\mu\nu} - \frac{m}{2} f_{[\mu, \nu]}. \quad (10)$$

Moreover, the free Lagrangian can be rewritten in terms of these curvatures as follows:

$$\mathcal{L}_0 = a_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}^{ab} \mathcal{F}_{\alpha\beta}^{cd} + a_2 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_\mu^{ac} \mathcal{B}_\nu^{bc} + a_3 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{B}_\mu^{ab} \mathcal{T}_{\nu\alpha}^c, \quad (11)$$

where

$$a_1 = -\frac{(d-2)}{32(d-3)m^2}, \quad a_2 = -\frac{1}{m^2}, \quad a_3 = -\frac{1}{4m}.$$

**2.2. Self-Interaction.** As our first example we consider self-interactions for partially massless spin-2. Similarly to the massless case, first of all, we have to construct the most general quadratic deformations for all four curvatures such that deformed curvatures transform covariantly. The solution turns out to be unique (up to possible field redefinitions related to the presence of zero form  $B^{ab}$ ) and produces the following transformations

for deformed curvatures:

$$\begin{aligned}
 \delta \hat{\mathcal{F}}_{\mu\nu}{}^{ab} &= 2d_1 \mathcal{F}_{\mu\nu}{}^{c[a\eta^b]c} + \frac{d_2}{2} \mathcal{B}_{[\mu}{}^{ab} \xi_{\nu]} + d_2 \mathcal{F}_{\mu\nu}{}^{ab} \xi, \\
 \delta \hat{\mathcal{T}}_{\mu\nu}{}^a &= -2d_1 \eta^{ab} \mathcal{T}_{\mu\nu}{}^b + 2d_1 \mathcal{F}_{\mu\nu}{}^{ab} \xi^b, \\
 \delta \hat{\mathcal{B}}_{\mu}{}^{ab} &= -d_1 \eta^{c[a} \mathcal{B}_{\mu}{}^{b]c} + d_2 \mathcal{B}_{\mu}{}^{ab} \xi, \\
 \delta \hat{\mathcal{B}}_{\mu\nu} &= -d_1 \mathcal{B}_{[\mu,\nu]}{}^a \xi^a,
 \end{aligned} \tag{12}$$

where  $d_2 = -\frac{4md_1}{(d-2)}$ .

Now we consider the following interacting Lagrangian:

$$\begin{aligned}
 \mathcal{L} = a_1 \left\{ \begin{matrix} \mu\nu\alpha\beta \\ abcd \end{matrix} \right\} \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd} + a_2 \left\{ \begin{matrix} \mu\nu \\ ab \end{matrix} \right\} \hat{\mathcal{B}}_{\mu}{}^{ac} \hat{\mathcal{B}}_{\nu}{}^{bc} + a_3 \left\{ \begin{matrix} \mu\nu\alpha \\ abc \end{matrix} \right\} \hat{\mathcal{B}}_{\mu}{}^{ab} \hat{\mathcal{T}}_{\nu\alpha}{}^c + \\
 + a_4 \left\{ \begin{matrix} \mu\nu\alpha\beta\gamma \\ abcde \end{matrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} f_{\gamma}{}^e + a_5 \left\{ \begin{matrix} \mu\nu\alpha \\ abc \end{matrix} \right\} \mathcal{B}_{\mu}{}^{ad} \mathcal{B}_{\nu}{}^{bd} f_{\alpha}{}^c + \\
 + a_6 \left\{ \begin{matrix} \mu\nu\alpha\beta \\ abcd \end{matrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{B}_{\alpha}{}^{cd} B_{\beta}, \tag{13}
 \end{aligned}$$

where the first line is just the free Lagrangian with curvatures replaced by the deformed ones, while the second line contains three Abelian vertices. Contrary to the massless case, the first and second lines are not separately gauge-invariant. In this, gauge invariance gives three equations on these three parameters  $a_4, a_5, a_6$ , hence we obtain one independent vertex containing terms with up to four derivatives.

Note, that in  $d = 4$  one of the Abelian vertices

$$a_4 \left\{ \begin{matrix} \mu\nu\alpha\beta\gamma \\ abcde \end{matrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} f_{\gamma}{}^e$$

is absent, but equations on remaining parameters  $a_5, a_6$  still have a solution. Moreover, all higher derivative terms (both with physical as well as with Stueckelberg fields) vanish on-shell, leaving us with a rather well-known two-derivative vertex that exists in  $d = 4$  and only in  $d = 4$  (e.g., [12, 13]).

**2.3. Gravitational Interaction.** Our second example — gravitational interaction for partially massless spin-2. In this case, we have to consider quadratic deformations both for gravitational curvatures as well as for the partially massless ones. The most general (up to possible field redefinitions) quadratic deformation for gravitational curvature has two free parameters and leads to

$$\delta \hat{R}_{\mu\nu}{}^{ab} \approx 2b_1 \mathcal{F}_{\mu\nu}{}^{c[a\eta^b]c} - \left( \frac{2mb_1}{(d-2)} + \frac{b_2}{2} \right) \mathcal{B}_{[\mu,\nu]}{}^{[a\xi^b]} - \frac{b_2}{2} \mathcal{B}_{[\mu}{}^{ab} \xi_{\nu]} + b_2 \mathcal{F}_{\mu\nu}{}^{ab} \xi. \tag{14}$$

At the same time, deformations for partially massless curvatures correspond to standard minimal substitution rules and give

$$\begin{aligned}
 \delta \hat{\mathcal{F}}_{\mu\nu}{}^{ab} &\approx -b_0 [\mathcal{F}_{\mu\nu}{}^{c[a\hat{\eta}^b]c} + R_{\mu\nu}{}^{c[a\eta^b]c} - \frac{m}{(d-2)} \mathcal{B}_{[\mu,\nu]}{}^{[a\hat{\xi}^b]}], \\
 \delta \hat{\mathcal{T}}_{\mu\nu}{}^a &\approx -b_0 [\mathcal{F}_{\mu\nu}{}^{ab} \hat{\xi}^b + R_{\mu\nu}{}^{ab} \xi^b], \\
 \delta \hat{\mathcal{B}}_{\mu}{}^{ab} &\approx -b_0 \mathcal{B}_{\mu}{}^{c[a\hat{\eta}^b]c}, \\
 \delta \hat{\mathcal{B}}_{\mu\nu} &\approx b_0 [\mathcal{B}_{[\mu,\nu]}{}^a \hat{\xi}^a + \frac{m}{2} \mathcal{F}_{\mu\nu}{}^a \hat{\xi}^a].
 \end{aligned} \tag{15}$$

Note, that at this stage deformation parameters  $b_0$  and  $b_1, b_2$  are independent. The reason is that covariance of the deformed curvatures guarantees that equations of the theory we are trying to construct will be gauge-invariant, but it does not guarantee that these equations will be Lagrangian. But when we put these curvatures into the Lagrangian and require it to be invariant, we have to expect that these parameters would be related.

Now we construct the interacting Lagrangian

$$\begin{aligned} \mathcal{L} = & a_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd} + a_2 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \hat{\mathcal{B}}_{\mu}{}^{ac} \hat{\mathcal{B}}_{\nu}{}^{bc} + \\ & + a_3 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \hat{\mathcal{B}}_{\mu}{}^{ab} \hat{\mathcal{T}}_{\nu\alpha}{}^c + a_0 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd} + \\ & + a_4 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} h_{\gamma}{}^e + a_5 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{B}_{\mu}{}^{ad} \mathcal{B}_{\nu}{}^{bd} h_{\alpha}{}^c + \\ & + a_6 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta\gamma \\ abcde \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd} f_{\gamma}{}^e + a_7 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} R_{\mu\nu}{}^{ab} \mathcal{B}_{\alpha}{}^{cd} B_{\beta} \end{aligned} \quad (16)$$

as the sum of free Lagrangian for partially massless and massless spin-2 fields with the deformed curvatures supplemented with four possible Abelian vertices. Gauge invariance requires (as expected)

$$b_1 = -\frac{b_0}{2}, \quad b_2 = 2mb_0$$

and also gives two equations on parameters  $a_{4,5,6,7}$ , hence we obtain three independent vertices.

In  $d = 4$  the terms with  $a_4$  and  $a_6$  are absent leaving us with one vertex only. Moreover, all higher derivative terms vanish on-shell, and we obtain two-derivative vertex known previously from metric-like formalism [12, 13].

### 3. MASSIVE CASE

Here we give a brief description of the results for the massive case leaving detailed discussion for the forthcoming publication.

**3.1. Massive Spin-2 Kinematics.** In the massless limit, massive spin-2 breaks into the massless spin-2, spin-1 and spin-0 ones, hence gauge-invariant frame-like description requires three pairs of fields:  $(\Omega_{\mu}{}^{ab}, f_{\mu}{}^a)$ ,  $(B^{ab}, B_{\mu})$  and  $(\pi^a, \varphi)$  [7]. Scalar field does not have its own gauge transformations, so the gauge symmetries are the same as in the partially massless case:  $(\eta^{ab}, \xi^a, \xi)$ .

Correspondingly, we have six gauge-invariant objects

$$\mathcal{F}_{\mu\nu}{}^{ab}, \quad \mathcal{T}_{\mu\nu}{}^a, \quad \mathcal{B}_{\mu}{}^{ab}, \quad \mathcal{B}_{\mu\nu}, \quad \Pi_{\mu}{}^a, \quad \Phi_{\mu},$$

and the free gauge-invariant Lagrangian can be expressed as

$$\begin{aligned} \mathcal{L}_0 = & a_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} + a_2 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_{\mu}{}^{ac} \mathcal{B}_{\nu}{}^{bc} + a_3 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{B}_{\mu}{}^{ab} \mathcal{T}_{\nu\alpha}{}^c + \\ & + a_4 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \Pi_{\alpha}{}^c + a_5 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \Pi_{\mu}{}^a \Pi_{\nu}{}^b + a_6 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_{\mu}{}^{ab} \Phi_{\nu}. \end{aligned} \quad (17)$$

**3.2. Cubic Vertices.** As in the partially massless case, we have considered two examples — self-interaction and interaction with massless graviton. Qualitative results are the following.

*Self-Interaction*

- There are three independent cubic vertices.
- By adjusting coefficients one can obtain one particular solution that has no more than two derivatives on-shell.
- General solution is singular in the partially massless limit and only one particular solution admits smooth limit.

*Gravitational Interaction*

- We obtained three independent solutions, two having four derivatives and one with no more than two.
- All three admit nonsingular partially massless limit.

REFERENCES

1. *Vasiliev M. A.* Cubic Vertices for Symmetric Higher-Spin Gauge Fields in  $(A)dS_d$  // Nucl. Phys. B. 2012. V. 862. P. 341.
2. *Fradkin E. S., Vasiliev M. A.* On the Gravitational Interaction of Massless Higher-Spin Fields // Phys. Lett. B. 1987. V. 189. P. 89.
3. *Fradkin E. S., Vasiliev M. A.* Cubic Interaction in Extended Theories of Massless Higher-Spin Fields // Nucl. Phys. B. 1987. V. 291. P. 141.
4. *Vasiliev M. A.* Gauge Form of Description of Massless Fields with Arbitrary Spin // Sov. J. Nucl. Phys. 1980. V. 32. P. 439.
5. *Lopatin V. E., Vasiliev M. A.* Free Massless Bosonic Fields of Arbitrary Spin in D-Dimensional de Sitter Space // Mod. Phys. Lett. A. 1988. V. 3. P. 257.
6. *Vasiliev M. A.* Free Massless Fermionic Fields of Arbitrary Spin in D-Dimensional de Sitter Space // Nucl. Phys. B. 1988. V. 301. P. 26.
7. *Zinoviev Yu. M.* Frame-like Gauge-Invariant Formulation for Massive High Spin Particles // Nucl. Phys. B. 2009. V. 808. P. 185.
8. *Ponomarev D. S., Vasiliev M. A.* Frame-like Action and Unfolded Formulation for Massive Higher-Spin Fields // Nucl. Phys. B. 2010. V. 839. P. 466.
9. *Metsaev R. R.* Cubic Interaction Vertices of Massive and Massless Higher-Spin Fields // Nucl. Phys. B. 2006. V. 759. P. 147.
10. *Zinoviev Yu. M.* On Massive High Spin Particles in  $(A)dS$ . arXiv:hep-th/0108192.
11. *Skvortsov E. D., Vasiliev M. A.* Geometric Formulation for Partially Massless Fields // Nucl. Phys. B. 2006. V. 756. P. 117.
12. *Zinoviev Yu. M.* On Massive Spin-2 Interactions // Nucl. Phys. B. 2007. V. 770. P. 83.
13. *Zinoviev Yu. M.* All Spin-2 Cubic Vertices with Two Derivatives // Nucl. Phys. B. 2013. V. 872. P. 21.