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POMERON IN THE $\mathcal{N}=4$ SYM AT LARGE COUPLING CONSTANT

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We show the result for the BFKL Pomeron intercept at $\mathcal{N}=4$ SYM in the form of the inverse coupling expansion $j_0 = 2-2\lambda^{-1/2}-\lambda^{-1}+1/4\lambda^{-3/2}+2(1+3\zeta_3)\lambda^{-2}+O(\lambda^{-5/2})$, which has been calculated in [1] with the use of the AdS/CFT correspondence.

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The investigation of the high-energy behavior of scattering amplitudes in the $\mathcal{N}=4$ Supersymmetric Yang–Mills (SYM) model [2, 3] is important for our understanding of the Regge processes in QCD. Indeed, this conformal model can be considered as a simplified version of QCD, in which the next-to-leading order (NLO) corrections [4] to the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [5] are comparatively simple and numerically small. On the other hand, due to the AdS/CFT correspondence [6, 7], in $\mathcal{N}=4$ SYM some physical quantities can be also computed at large couplings.

With the use of the BFKL equation in a diffusion approximation [2], strong coupling results [8, 9] for anomalous dimension (AD) γ of twist-2 Wilson operators and the Pomeron-graviton duality [10], the Pomeron intercept was calculated at the leading order in the inverse coupling constant (see Erratum [11] to the paper [12]). Similar results were obtained also in [13].

Due to the symmetry of the BFKL equation to the substitution $\gamma_{\rm BFKL} \to 1-\gamma_{\rm BFKL}$, its r.h.s. is an even function of ν (j is spin and the number of derivations in the Wilson operators)

$$\omega = \omega_0 + \sum_{m=1}^{\infty} (-1)^m D_m \nu^{2m}, \quad \gamma_{\rm BFKL} = \frac{1}{2} + i\nu, \quad \omega = j-1, \tag{1}$$

where ω_0 and D_m are functions of the 't Hooft coupling constant λ with known first two coefficients, which are $\sim \lambda$ and $\sim \lambda^2$.

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Due to the Möbius invariance and hermicity of the BFKL Hamiltonian in $\mathcal{N} = 4$ SYM, expansion (1) is valid also at large coupling constants. In the framework of the AdS/CFT correspondence, the BFKL Pomeron is equivalent to the Reggeized graviton [10]. In particular, in the strong coupling regime $\lambda \to \infty$

$$j_0 = 2 - \Delta, \tag{2}$$

where the leading contribution $\Delta = 2/\sqrt{\lambda}$ was calculated in [11,13]. Below we present the following several terms in the strong coupling expansion of the Pomeron intercept.

Due to the energy-momentum conservation, the universal AD of the stress tensor $T_{\mu\nu}$ should be zero, i.e.,

$$\gamma(j = 2) = 0. \tag{3}$$

It is important, that the AD γ does not coincide with $\gamma_{\rm BFKL}$ appearing in the BFKL equation. They are related as follows [4,18]:

$$\gamma = \gamma_{\rm BFKL} + \frac{\omega}{2} = \frac{j}{2} + i\nu, \tag{4}$$

where the additional contribution $\omega/2$ is responsible, in particular, for the cancellation of the singular terms $\sim 1/\gamma^3$ obtained from the NLO corrections to the eigenvalue of the BFKL kernel [4,18]. Using the above relations, one obtains $\nu(j = 2) = i$. As a result, from Eq. (1) for the Pomeron trajectory we derive the following representation for the correction Δ to the spin-2 graviton:

$$\Delta = \sum_{m=1}^{\infty} D_m. \tag{5}$$

According to (2) and (5), we have the following small-ν expansion for the eigenvalue of the BFKL kernel:

$$j - 2 = \sum_{m=1}^{\infty} D_m \left(\left(-\nu^2\right)^m - 1 \right), \tag{6}$$

where ν^2 is related to γ according to Eq. (4)

$$\nu^2 = -\left(\frac{j}{2} - \gamma\right)^2. \tag{7}$$

On the other hand, due to the AdS/CFT correspondence, the string energies E in dimensionless units are related to the AD γ of the twist-two operators as follows [7]:

$$E^2 = (j + \Gamma)^2 - 4, \quad \Gamma = -2\gamma, \tag{8}$$

and therefore we can obtain from (7) the relation between the parameter ν for the principal series of unitary representations of the Möbius group and the string energy E: $\nu^2 = -(E^2/4 + 1)$. This expression for ν^2 can be inserted in the r.h.s. of Eq. (6) leading to the following expression for the Regge trajectory of the graviton in the anti-de Sitter space:

$$j - 2 = \sum_{m=1}^{\infty} D_m \left[\left(\frac{E^2}{4} + 1 \right)^m - 1 \right]. \tag{9}$$

We assume, that Eq. (9) is valid also at large j and large λ in the region $1 \ll j \ll \sqrt{\lambda}$, where the strong coupling calculations of energies were performed [14, 15]. These energies can be presented in the form [1]

$$\frac{E^2}{4} = \sqrt{\lambda}\frac{S}{2}\left[h_0(\lambda) + h_1(\lambda)\frac{S}{\sqrt{\lambda}} + h_2(\lambda)\frac{S^2}{\lambda}\right] + O\left(S^{7/2}\right), \tag{10}$$

where

$$h_i(\lambda) = a_{i0} + \frac{a_{i1}}{\sqrt{\lambda}} + \frac{a_{i2}}{\lambda} + \frac{a_{i3}}{\sqrt{\lambda^3}} + \frac{a_{i2}}{\lambda^2}. \tag{11}$$

The contribution $\sim \sqrt{S}$ can be extracted directly from the Basso result [16] taking $J_{\rm an} = 2$ according to [17]:

$$a_{00} = 1, \quad a_{01} = -\frac{1}{2}, \quad a_{02} = a_{03} = \frac{15}{8}, \quad a_{04} = \frac{135}{128}. \tag{12}$$

The coefficients a_{10} and a_{20} come from considerations of the classical part of the spinning folded string corresponding to the twist-2 operators (see, for example, [15])

$$a_{10} = \frac{3}{4}, \quad a_{20} = -\frac{3}{16}. \tag{13}$$

The one-loop coefficient a_{11} is found recently in the paper [17], considering different asymptotical regimes with taking into account the Basso result [16] (ζ_3 is the Euler ζ-function)

$$a_{11} = \frac{3}{16}\left(1 - \zeta_3\right). \tag{14}$$

Comparing the l.h.s. and r.h.s. of (9) at large j values gives us the coefficients D_m and Δ. So, at $\lambda \to \infty$, the correction Δ for the Pomeron intercept $j_0 = 2 - \Delta$ has the form

$$\Delta = \frac{2}{\lambda^{1/2}}\left[1 + \frac{1}{2\lambda^{1/2}} - \frac{1}{8\lambda} - \left(1 + 3\zeta_3\right)\frac{1}{\lambda^{3/2}} + \right. \\ \left. + \left(2a_{12} - \frac{145}{128} - \frac{9}{2}\zeta_3\right)\frac{1}{\lambda^2} + O\left(\frac{1}{\lambda^{5/2}}\right)\right]. \tag{15}$$

The fourth corrections in (15) contain unknown coefficient a_{12}, which will be obtained after the evaluation of the spinning folded string on the two-loop level. Some estimations were given in Sec. 6 of [1].

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[1] Here, we put $S = j - 2$, which, in particular, is related to the use of the angular momentum $J_{\rm an} = 2$ in calculations of [14, 15].

REFERENCES

1. *Kotikov A. V., Lipatov L. N.* Pomeron in the $N = 4$ Supersymmetric Gauge Model at Strong Couplings // Nucl. Phys. B. 2013. V. 874. P. 889; Pomeron in the $N = 4$ SYM at Strong Couplings. Preprint arXiv:1401.0639 [hep-th]. 5 p.
2. *Kotikov A. V., Lipatov L. N.* NLO Corrections to the BFKL Equation in QCD and in Supersymmetric Gauge Theories // Nucl. Phys. B. 2000. V. 582. P. 19.
3. *Kotikov A. V., Lipatov L. N.* DGLAP and BFKL Equations in the $N = 4$ Supersymmetric Gauge Theory // Nucl. Phys. B. 2003. V. 661. P. 19; DGLAP and BFKL Evolution Equations in the $N = 4$ Supersymmetric Gauge Theory. Preprint hep-ph/0112346. 22 p.
4. *Fadin V. F., Lipatov L. N.* BFKL Pomeron in the Next-to-Leading Approximation // Phys. Lett. B. 1998. V. 429. P. 127;
   Camici G., Ciafaloni M. Energy scale(s) and Next-to-Leading BFKL Equation // Ibid. V. 430. P. 349.
5. *Lipatov L. N.* Reggeization of the Vector Meson and the Vacuum Singularity in Non-Abelian Gauge Theories // Sov. J. Nucl. Phys. 1976. V. 23. P. 338;
   Fadin V. F., Kuraev E. A., Lipatov L. N. On the Pomeranchuk Singularity in Asymptotically Free Theories // Phys. Lett. B. 1975. V. 60. P. 50; Multi-Reggeon Processes in the Yang–Mills Theory // Sov. Phys. JETP. 1976. V. 44. P. 443; The Pomeranchuk Singularity in Non-Abelian Gauge Theories // Sov. Phys. JETP. 1977. V. 45. P. 199;
   Balitsky I. I., Lipatov L. N. The Pomeranchuk Singularity in Quantum Chromodynamics // Sov. J. Nucl. Phys. 1978. V. 28. P. 822.
6. *Maldacena J.* The Large N Limit of Superconformal Field Theories and Supergravity // Adv. Theor. Math. Phys. 1998. V. 2. P. 231.
7. *Gubser S. S., Klebanov I. R., Polyakov A. M.* Gauge Theory Correlators from Noncritical String Theory // Phys. Lett. B. 1998. V. 428. P. 105;
   Witten E. Anti-de Sitter Space, Thermal Phase Transition, and Confinement in Gauge Theories // Adv. Theor. Math. Phys. 1998. V. 2. P. 253.
8. *Gubser S. S., Klebanov I. R., Polyakov A. M.* A Semiclassical Limit of the Gauge/String Correspondence // Nucl. Phys. B. 2002. V. 636. P. 99.
9. *Frolov S., Tseytlin A. A.* Semiclassical Quantization of Rotating Superstring in $AdS(5) \times S(5)$ // JHEP. 2002. V. 0206. P. 007;
   Roiban R., Tirziu A., Tseytlin A. A. Two-Loop World-Sheet Corrections in $AdS_5 \times S^5$ Superstring // JHEP. 2007. V. 0707. P. 056.
10. *Polchinski J., Strassler M. J.* Hard Scattering and Gauge/String Duality // Phys. Rev. Lett. 2002. V. 88. P. 031601; Deep Inelastic Scattering and Gauge/String Duality // JHEP. 2003. V. 0305. P. 012.
11. *Kotikov A. V. et al.* Three-Loop Universal Anomalous Dimension of the Wilson Operators in $N = 4$ SUSY Yang–Mills Model // Phys. Lett. B. 2006. V. 632. P. 754 (Erratum to Ref. [12]).
12. *Kotikov A. V. et al.* Three-Loop Universal Anomalous Dimension of the Wilson Operators in $N = 4$ SUSY Yang–Mills Model // Phys. Lett. B. 2004. V. 595. P. 521.
13. *Brower R. C. et al.* The Pomeron and Gauge/String Duality // JHEP. 2007. V. 0712. P. 005.
14. *Gromov N. et al.* Quantum Folded String and Integrability: From Finite Size Effects to Konishi Dimension // JHEP. 2011. V. 1108. P. 046.
15. *Roiban R., Tseytlin A. A.* Semiclassical String Computation of Strong-Coupling Corrections to Dimensions of Operators in Konishi Multiplet // Nucl. Phys. B. 2011. V. 848. P. 251.
16. *Basso B.* An Exact Slope for AdS/CFT. Preprint arXiv:1109.3154 [hep-th]. 6 p.; Scaling Dimensions at Small Spin in $N = 4$ SYM Theory. Preprint arXiv:1205.0054 [hep-th]. 25 p.
17. *Gromov N., Valatka S.* Deeper Look into Short Strings // JHEP. 2012. V. 1203. P. 058.
18. *Salam G. P.* An Introduction to Leading and Next-to-Leading BFKL // Acta Phys. Polon. B. 1999. V. 30. P. 3679.