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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

NEW METHOD OF THE FUNCTIONAL RENORMALIZATION GROUP APPROACH FOR YANG–MILLS FIELDS

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We propose a new formulation of the functional renormalization group (FRG) approach based on the use of regulator functions as composite operators. In this case, one can provide (in contrast with standard approach) on-shell gauge invariance for the effective average action.

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INTRODUCTION

Functional renormalization group (FRG) approach has been proposed in [1, 2] in terms of effective average action. Now, it is one of the most popular and developed methods in Quantum Field Theory (QFT), which can be seen from the review papers on the FRG method [3–6, 8, 9]. Note that the special attention has been paid to the study of effective average action in gauge theories [10, 11] (see also [12–19] and a very clear and complete review [20]).

Despite the fact that many aspects of gauge theories in the FRG approach have been discussed with success [11, 13–15, 17, 19, 20], nevertheless, the problem of gauge dependence remains unsolved. The gauge dependence of effective average action in the standard formulation of FRG for Yang–Mills fields was analyzed in [21] and it was shown that even on-shell the effective average action depends on gauge regardless of the approximate schemes. We present here a possibility to solve the gauge dependence in FRG, which is based on an old idea of [22] about introduction of composite operators in gauge theories. Namely, we propose to consider the regulator functions being main ingredients of FRG as composite operators and show that, in this case, the effective average action does not depend on gauge on-shell.

We employ the standard condensed notation of DeWitt [23]. Derivatives with respect to sources and antifields are taken from the left, while those with respect to fields are taken from the right. Left derivatives with respect to fields are labeled by a subscript l. The Grassmann parity of a quantity F is denoted as $\varepsilon(F)$.

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A NEW APPROACH

Here we are going to describe an approach in spirit of the FRG, which is free of the gauge dependence problem for the Yang–Mills theory. This new approach is based on implementing regulator functions of FRG by means of composite fields.

Effective action for composite fields in QFT was introduced in [24]. Later on, effective action for composite fields in gauge theories was introduced and studied in papers [22,25,26]. The effective action depends on gauge, but this dependence has a very special form and there is a possibility to define a theory with composite fields in such a way that the effective action of these fields becomes gauge-independent on-shell.

Let us consider Lagrangian of the regulator functions $(R_{k,A})^{ab}_{\mu\nu}(x)$ and $(R_{k,\mathrm{gh}})^{ab}(x)$,

$$L^1_k(x) = \frac{1}{2}\, A^{a\mu}(x)(R_{k,A})^{ab}_{\mu\nu}(x)A^{b\nu}(x), \qquad L^2_k(x) = \bar{C}^a(x)(R_{k,\mathrm{gh}})^{ab}(x)C^b(x), \tag{1}$$

appearing in sectors of Yang–Mills $A^a_\mu(x)$ and ghost $C^a(x)$ and antighost $\bar{C}^a(x)$ fields in the standard FRG approach. Here k is a parameter, such that

$$\lim_{k\to 0}(R_{k,A})^{ab}_{\mu\nu}(x) = 0, \quad \lim_{k\to 0}(R_{k,\mathrm{gh}})^{ab}(x) = 0. \tag{2}$$

Now, we introduce external scalar sources $\Sigma_1(x)$ and $\Sigma_2(x)$ and construct the generating functional of Green's functions for Yang–Mills theories with composite fields

$$Z_k(J,K;\Sigma) = \int \mathcal{D}\Phi\, \exp\left\{\frac{i}{\hbar}\big[S_{\mathrm{FP}}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k(\Phi)\big]\right\}, \tag{3}$$

where $\Phi = \{\Phi^A = (A^{a\mu}, B^a, C^a, \bar{C}^a)\}$ is the set of all fields of configuration space for the given Yang–Mills theory, $S_{\mathrm{FP}}(\Phi)$ is the Faddeev–Popov action, $J_A = (j^a_\mu, \sigma^a, \bar{\eta}^a, \eta^a)$ are external sources to fields Φ^A ($J\Phi = J_A\Phi^A$), $\hat{s}$ is the operator of the BRST transformations, $K_A = (K^a_\mu, N^a, \bar{P}^a, P^a)$ are external sources to the BRST transformations $\hat{s}\Phi^A$ and $\Sigma L_k = \Sigma_1 L^1_k + \Sigma_2 L^2_k$.

The generating functional (3) may be regarded as a generalization of the generating functionals of the standard FRG approach, the difference is related to the new sources $\Sigma_1(x)$ and $\Sigma_2(x)$ and to the corresponding composite fields. Consider the FRG flow equations in the approach with composite operators, based on the new generating functional (3). Repeating usual manipulations with (3), we arrive at the new version of the FRG flow equation for the generating functional $Z_k = Z_k(J,K;\Sigma)$,

$$\partial_t Z_k = \frac{\hbar}{i}\left\{\frac{1}{2}\,\Sigma_1\,\partial_t(R_{k,A})^{ab}_{\mu\nu}\frac{\delta^2 Z_k}{\delta j^a_\mu\,\delta j^b_\nu} + \Sigma_2\,\partial_t(R_{k,\mathrm{gh}})^{ab}\frac{\delta^2 Z_k}{\delta\eta^a\,\delta\bar{\eta}^b}\right\}, \tag{4}$$

where we used the standard notation

$$\partial_t = k\,\frac{d}{dk}$$

and took into account that the dependence on k comes only from the corresponding dependence of the regulator functions in (1).

The effective average action with composite fields, $\Gamma_k = \Gamma_k(\Phi, K; F)$, can be introduced by means of the following double Legendre transformations (see [24] for the details in case of usual effective action):

$$\Gamma_k(\Phi, K; F) = W_k(J, K; \Sigma) - J_A\Phi^A - \Sigma_i\big[L_k^i(\Phi) + \hbar F^i\big], \tag{5}$$

where $W_k = -i\hbar \ln Z_k$ and

$$\Phi^A = \frac{\delta W_k}{\delta J_A}, \quad \hbar F^i = \frac{\delta W_k}{\delta \Sigma_i} - L_k^i\left(\frac{\delta W_k}{\delta J}\right), \quad i = 1, 2. \tag{6}$$

From (5) and (6) it follows that

$$\frac{\delta \Gamma_k}{\delta \Phi^A} = -J_A - \Sigma_i \frac{\delta L_k^i(\Phi)}{\delta \Phi^A}, \quad \frac{\delta \Gamma_k}{\delta F^i} = -\hbar\Sigma_i. \tag{7}$$

Let us introduce the full sets of fields $\mathcal{F}^{\mathcal{A}}$ and sources $\mathcal{J}_{\mathcal{A}}$ according to

$$\mathcal{F}^{\mathcal{A}} = (\Phi^A, \hbar F^i), \quad \mathcal{J}_{\mathcal{A}} = (J_A, \hbar\Sigma_i). \tag{8}$$

From the condition of solvability of Eqs. (7) with respect to the sources J and Σ, it follows that

$$\frac{\delta \mathcal{F}^{\mathcal{C}}(\mathcal{J})}{\delta \mathcal{J}_{\mathcal{B}}}\frac{\delta_l \mathcal{J}_{\mathcal{A}}(\mathcal{F})}{\delta \mathcal{F}^{\mathcal{C}}} = \delta_{\mathcal{A}}^{\mathcal{B}}. \tag{9}$$

One can express $\mathcal{J}_{\mathcal{A}}$ as a function of the fields in the form

$$\mathcal{J}_{\mathcal{A}} = \left(-\frac{\delta \Gamma_k}{\delta \Phi^A} - \frac{\delta \Gamma_k}{\delta F^i}\frac{\delta L_k^i(\Phi)}{\delta \Phi^A},\; -\frac{\delta \Gamma_k}{\delta F^i}\right) \tag{10}$$

and, therefore,

$$\frac{\delta_l \mathcal{J}_{\mathcal{B}}(\mathcal{F})}{\delta \mathcal{F}^{\mathcal{A}}} = -(G_k'')_{\mathcal{A}\mathcal{B}}, \quad \frac{\delta \mathcal{F}^{\mathcal{B}}(\mathcal{J})}{\delta \mathcal{J}_{\mathcal{A}}} = -(G_k''^{-1})^{\mathcal{A}\mathcal{B}}. \tag{11}$$

Now, we are in a position to derive the FRW equation for $\Gamma_k = \Gamma_k(\Phi, K; F)$ and see that it gains more complicated form due to the presence of composite fields. The expression for the generating functional of connected Green's functions can be obtained from (3), (5) and (6). The equation for Γ_k reads

$$\exp\left\{\frac{i}{\hbar}\left[\Gamma_k(\Phi, K; F) + \hbar\Sigma F\right]\right\} = \\ = \int \mathcal{D}\Phi' \exp\left\{\frac{i}{\hbar}\Big[S_{\rm FP}(\Phi') + J(\Phi' - \Phi) + K\hat{s}\Phi + \Sigma\big(L_k(\Phi') - L_k(\Phi)\big)\Big]\right\}.$$

Making shift of the variables of integration in the functional integral, $\Phi' - \Phi = \varphi$, and using (7), we obtain the equation for the effective action $\Gamma_k = \Gamma_k(\Phi; F)$,

$$\exp\left\{\frac{i}{\hbar}\left[\Gamma_k - \frac{\delta \Gamma_k}{\delta F}\,F\right]\right\} = \int \mathcal{D}\varphi \exp\Bigg\{\frac{i}{\hbar}\Bigg[S_{\rm FP}(\Phi + \varphi) - \frac{\delta \Gamma_k}{\delta \Phi}\,\varphi + \\ + K\hat{s}(\Phi + \varphi) - \frac{1}{\hbar}\frac{\delta \Gamma_k}{\delta F}\left(L_k(\Phi + \varphi) - L_k(\Phi) - \frac{\delta L_k(\Phi)}{\delta \Phi}\,\varphi\right)\Bigg]\Bigg\}. \tag{12}$$

The solution of this equation in the tree-level approximation has the form

$$\Gamma_k^{(0)}(\Phi, K; F) = S_{\mathrm{FP}}(\Phi) + K\hat{s}\Phi, \tag{13}$$

which does not depend on the fields F^i and parameter k. The next step is to define loop corrections to (13), so we assume that

$$\Gamma_k(\Phi, K; F) = S_{\mathrm{FP}}(\Phi) + \hbar\bar{\Gamma}_k(\Phi; F). \tag{14}$$

By taking into account the explicit structure of regulator functions, we obtain the equation which can be, for example, a basis for deriving the loop expansion of $\bar{\Gamma}_k = \bar{\Gamma}_k(\Phi; F)$,

$$\exp\left\{i\left[\bar{\Gamma}_k - \frac{\delta\bar{\Gamma}_k}{\delta F}\,F\right]\right\} = \int \mathcal{D}\varphi\,\exp\left\{\frac{i}{\hbar}\left[S_{\mathrm{FP}}(\Phi+\varphi) - \right.\right.$$
$$\left.\left. - S_{\mathrm{FP}}(\Phi) - \frac{\delta S_{\mathrm{FP}}(\Phi)}{\delta\Phi}\varphi - \hbar\,\frac{\delta\bar{\Gamma}_k}{\delta\Phi}\,\varphi - \frac{\delta\bar{\Gamma}_k}{\delta F}\,L_k(\varphi)\right]\right\}. \tag{15}$$

The FRG flow equation, in terms of the functional Γ_k, is cast into the form

$$\partial_t\Gamma_k = -i\left\{\frac{1}{2}\frac{\delta\Gamma_k}{\delta F^1}\,\partial_t(R_{k,A})^{ab}_{\mu\nu}\big(G_k''^{-1}\big)^{(a\mu)(b\nu)} + \frac{\delta\Gamma_k}{\delta F^2}\,\partial_t(R_{k,\mathrm{gh}})\big(G_k''^{-1}\big)^{ab}\right\} =$$
$$= -\frac{i}{2}\,\mathrm{Tr}\left\{\frac{\delta\Gamma_k}{\delta F^1}\,\partial_t(R_{k,A})(G_k''^{-1})\right\}_A + i\,\mathrm{Tr}\left\{\frac{\delta\Gamma_k}{\delta F^2}\,\partial_t(R_{k,\mathrm{gh}})(G_k''^{-1})\right\}_C, \tag{16}$$

when we used usual traces in the sectors of vector $A^{a\mu}$ and ghost C^a fields, while the Grassmann parity of quantum fields is taken into account explicitly. We conclude that the dependence of effective average action Γ_k on the cut-off parameter k disappears already on surface defined by the equations

$$\frac{\delta\Gamma_k}{\delta F^i} = 0, \quad i = 1, 2.$$

It differs from the standard formulation of effective average action, which depends essentially on k for any nonzero scale.

The BRST invariance of S_{FP} [27] leads to the generalized Slavnov–Taylor identity for the generating functional (3):

$$J_A\frac{\delta Z_k}{\delta K_A} + \frac{\hbar}{i}\left\{\Sigma_1(R_{k,A})^{ab}_{\mu\nu}\frac{\delta^2 Z_k}{\delta j^b_\nu\delta K^a_\mu} + \Sigma_2(R_{k,\mathrm{gh}})^{ab}\frac{\delta^2 Z_k}{\delta\eta^a\delta\bar{P}^a} - \right.$$
$$\left. - \Sigma_2(R_{k,\mathrm{gh}})^{ab}\frac{\delta^2 Z_k}{\delta\bar{\eta}^b\delta P^a}\right\} \equiv 0. \tag{17}$$

Let us explore the gauge dependence of generating functional of Green's functions in the presence of composite fields, (3). Under an infinitesimal variation of gauge function, $\chi^a \to \chi^a + \delta\chi^a$, the variation of $Z_k = Z_k(J, K; \Sigma)$ reads

$$\delta Z_k = \frac{i}{\hbar}\int \mathcal{D}\Phi\frac{\delta\,\delta\psi}{\delta\Phi^A}\frac{\delta(K\hat{s}\Phi)}{\delta K_A}\exp\left\{\frac{i}{\hbar}\big[S_{\mathrm{FP}}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k\big]\right\}, \tag{18}$$

where the Grassmann-odd functional $\delta\psi = \bar{C}^a \delta\chi^a$ has been introduced. Starting from the identity

$$\int \mathcal{D}\Phi \, \frac{\delta}{\delta\Phi^A} \left[\delta\psi \, \frac{\delta(K\hat{s}\Phi)}{\delta K_A} \, \exp\left\{ \frac{i}{\hbar} \left[S_{\mathrm{FP}}(\Phi) + J\Phi + K\hat{s}\Phi + \Sigma L_k \right] \right\} \right] = 0, \tag{19}$$

we can derive the final form of equation, describing the gauge dependence of the generating functional Z_k,

$$\delta Z_k = \frac{i}{\hbar} J_A \frac{\delta}{\delta K_A} \delta\psi \left(\frac{\hbar}{i} \frac{\delta}{\delta J} \right) Z_k + \left\{ \Sigma_1 (R_{k,A})^{ab}_{\mu\nu} \frac{\delta^2 Z_k}{\delta j^a_\mu \delta j^b_\nu} + \right.$$
$$\left. + \Sigma_2 (R_{k,\mathrm{gh}})^{ab} \frac{\delta^2 Z_k}{\delta \eta^a \delta \bar{P}^b} - \Sigma_2 (R_{k,\mathrm{gh}})^{ba} \, \frac{\delta^2 Z_k}{\delta \bar{\eta}^a \delta P^b} \right\} \delta\psi \left(\frac{\hbar}{i} \frac{\delta}{\delta J} \right) Z_k. \tag{20}$$

In particular, from (20) it follows that the vacuum functional in presence of sources K_A, $Z_k(K) = Z_k(0, K; 0)$, does not depend on gauge.

If we introduce the notations

$$L^i_{k,A}(\Phi) = \frac{\partial L^i_k(\Phi)}{\partial \Phi^A}, \quad i = 1, 2, \tag{21}$$

then the gauge dependence of the effective average action is described by the equation

$$\delta\Gamma_k = -\frac{\delta\Gamma_k}{\delta\Phi^A} \frac{\delta}{\delta K_A} \delta\psi(\hat{\Phi}) - \frac{1}{\hbar} \frac{\delta\Gamma_k}{\delta F^i} L^i_{k,A}(\Phi - \hat{\Phi}) \, \frac{\delta}{\delta K_A} \, \delta\psi(\hat{\Phi}). \tag{22}$$

Let us define the mass-shell of the quantum theory by the equations

$$\frac{\delta\Gamma_k}{\delta\Phi^A} = 0, \qquad \frac{\delta\Gamma_k}{\delta F^i} = 0. \tag{23}$$

Then, from (22) immediately follows the gauge independence of the effective action $\Gamma_k = \Gamma_k(\Phi, K; F)$ on-shell. Namely, when the relations (23) are satisfied, we have $\delta\Gamma_k = 0$. Moreover, all physical quantities calculated on the basis of the modified version of the effective average action do not depend on gauge and on the parameter k. The last feature is common for the renormalization group based on the abstract scale parameters (such as cut-off, k or μ), which require an additional identification of scale to be applied to one or another physical problem.

CONCLUSIONS

We have proposed the new formulation of the theory with cut-off-dependent regulator functions within the FRG approach. We have derived the FRG flow equation for generating functional of Green's functions and effective average action. It was found an interesting feature of the FRG flow for effective average action Γ_k consisting in the independence of this action on the cut-off parameter k already on surface defined by $\delta\Gamma_k/\delta F^i = 0$. It was shown that if the regulator functions are introduced by means of the special composite fields, the vacuum functional does not depend on the choice of gauge and, finally, the new theory, in contrast with the standard formulation, is free from the on-shell gauge dependence.

REFERENCES

1. *Wetterich C.* Average Action and the Renormalization Group Equations // Nucl. Phys. B. 1991. V. 352. P. 529–584.
2. *Wetterich C.* Exact Evolution Equation for the Effective Potential // Phys. Lett. B. 1993. V. 301. P. 90–94.
3. *Berges J., Tetradis N., Wetterich C.* Nonperturbative Renormalization Flow in Quantum Field Theory and Statistical Physics // Phys. Rep. 2002. V. 363. P. 223–386.
4. *Bagnuls C., Bervillier C.* Exact Renormalization Group Equations: An Introductory Review // Phys. Rep. 2001. V. 348. P. 91–150.
5. *Polonyi J.* Lectures on the Functional Renormalization Group Method // Centr. Eur. J. Phys. 2003. V. 1. P. 1–71.
6. *Pawlowski J. M.* Aspects of the Functional Renormalization Group // Ann. Phys. 2007. V. 322. P. 2831–2915.
7. *Igarashi Y., Itoh K., Sonoda H.* Realization of Symmetry in the ERG Approach to Quantum Field Theory // Prog. Theor. Phys. Suppl. 2010. V. 181. P. 1–166.
8. *Delamotte B.* An Introduction to the Nonperturbative Renormalization Group // Lect. Notes Phys. 2012. V. 852. P. 49–132.
9. *Rosten O. J.* Fundamentals of the Exact Renormalization Group // Phys. Rep. 2012. V. 511. P. 177–272.
10. *Reuter M., Wetterich C.* Average Action for the Higgs Model with Abelian Gauge Symmetry // Nucl. Phys. B. 1993. V. 391. P. 147–175.
11. *Reuter M., Wetterich C.* Effective Average Action for Gauge Theories and Exact Evolution Equations // Nucl. Phys. B. 1994. V. 417. P. 181–214.
12. *Becchi C.* On the Construction of Renormalized Gauge Theories Using Renormalization Group Techniques // Elementary Particle, Field Theory and Statistical Mechanics / Eds. M. Bonini, G. Marchesini, and E. Onofri. Parma Univ., 1993.
13. *Bonini M., D'Attanasio M., Marchesini G.* Ward Identities and Wilson Renormalization Group for QED // Nucl. Phys. B. 1994. V. 418. P. 81–112.
14. *Ellwanger U.* Gauge Invariance, the Quantum Action Principle, and the Renormalization Group // Phys. Lett. B. 1994. V. 335. P. 364–370.
15. *D'Attanasio M., Morris T. R.* Gauge Invariance, the Quantum Action Principle, and the Renormalization Group // Phys. Lett. B. 1996. V. 378. P. 213–221.
16. *Reuter M., Wetterich C.* Gluon Condensation in Nonperturbative Flow Equations // Phys. Rev. D. 1997. V. 56. P. 7893–7916.
17. *Litim D. F., Pawlowski J. M.* Flow Equations for Yang–Mills Theories in General Axial Gauges // Phys. Lett. B. 1998. V. 435. P. 181–188.
18. *Freire F., Litim D. F., Pawlowski J. M.* Gauge Invariance and Background Field Formalism in the Exact Renormalization Group // Phys. Lett. B. 2000. V. 495. P. 256–262.
19. *Igarashi Y., Itoh K., So H.* Regularized Quantum Master Equation in the Wilsonian Renormalization Group // JHEP. 2001. V. 0110. P. 032-1–032-17.
20. *Gies H.* Introduction to the Functional RG and Applications to Gauge Theories // Lect. Notes Phys. 2012. V. 852. P. 287–348.
21. *Lavrov P. M., Shapiro I. L.* On the Functional Renormalization Group Approach for Yang–Mills Fields // JHEP. 2013. V. 1306. P. 086-1–086-29.
22. *Lavrov P. M.* Effective Action for Composite Fields in Gauge Theories // Theor. Math. Phys. 1990. V. 82. P. 282–289.

23. *DeWitt B. S.* Dynamical Theory of Groups and Fields. New York: Gordon and Breach, 1965.
24. *Cornwell J. M., Jackiw R., Tomboulis E.* Effective Action for Composite Operators // Phys. Rev. D. 1974. V. 10. P. 2428–2445.
25. *Lavrov P. M., Odintsov S. D.* The Gauge Dependence of the Effective Action of Composite Fields in General Gauge Theories // Intern. J. Mod. Phys. A. 1989. V. 4. P. 5205–5212.
26. *Lavrov P. M., Odintsov S. D., Reshetnyak A. A.* Effective Action of Composite Fields for General Gauge Theories in BLT Covariant Formalism // J. Math. Phys. 1997. V. 38. P. 3466–3478.
27. *Becchi C., Rouet A., Stora R.* Renormalization of the Abelian Higgs–Kibble Model // Commun. Math. Phys. 1975. V. 42. P. 127–162;

    Tyutin I. V. Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism. Lebedev Inst. Preprint No. 39. 1975.