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3D SUPERSYMMETRIC NEW MASSIVE GRAVITY WITH AUXILIARY FIELDS

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New Massive Gravity is three-dimensional higher-derivative modification of General Relativity. The number of derivatives can be lowered down by introducing the auxiliary fields. We discuss the construction of the low-derivative linearized supersymmetric New Massive Gravity.

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INTRODUCTION

Einsteins theory of General Relativity (GR) to a great extent explains the dynamics of our Universe, but still it faces many problems. For instance, the predictions of GR do not match with the observed data of the galaxy rotation nor the measured value of the cosmological constant. Problems exist in the ultraviolet regime as well, where the quantum effects of gravity are significant. It is not known how to quantize GR. In this respect, physicists have been trying to modify GR. One idea, which could improve on some problems, is to give a mass to the graviton. The idea of giving a mass to the graviton was introduced by Fierz and Pauli (FP). They proposed adding an explicit mass term to the Einstein–Hilbert action:

$$S = \int d^4x \left\{ h^{\mu\nu} G^{lin}_{\mu\nu}(h) + m^2 (h^{\mu\nu} h_{\mu\nu} - h^2 \right\}, \qquad (1)$$

where $G^{lin}_{\mu\nu}(h)$ is the linearized Einstein tensor. The equations of motion, derived from this action, are FP equations. The massless limit of FP theory does not lead to linearized GR and this is known as the van Dam–Veltman–Zakharov discontinuity. Hence, FP theory and GR give different predictions for the Newtonian potential and the light-bending, for instance. Since adding the explicit mass term can generate new problems, one can think about other ways of introducing the massive gravitons. One idea is to add higher-derivative terms to the Einstein–Hilbert action and it will be discussed in the next section.

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1. NEW MASSIVE GRAVITY

As was mentioned in Introduction, there is another way of introducing massive gravitons: by adding higher-order derivative terms to the Einstein–Hilbert action. The higher-derivative terms will lead to the propagation of some additional degrees of freedom. Already in the early 1970s, such an action with higher-derivative terms was considered by Stelle, at first as an attempt to improve the renormalizability properties of general relativity:

$$S = \int d^D x \sqrt{-g} \left\{ R + \frac{1}{m^2} (aR^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + bR^{\mu\nu} R_{\mu\nu} + cR^2) \right\}, \tag{2}$$

where a, b, and c are some generic parameters, and $R_{\mu\nu\rho\sigma}$ is Riemann tensor, $R_{\mu\nu}$ is Ricci tensor, and R is Ricci scalar. The Einstein–Hilbert term in the action above plays a role of the mass term, and 4th-order derivative terms are kinetic terms. In the limit $m \to \infty$, it reduces to GR. This action contains a massive graviton, a massless graviton, and a scalar, in general. By tuning the parameters, one can eliminate the scalar, but it turns out that one of the massless or massive gravitons is always a ghost. The way out of this problem is to go to three dimensions. Since the massless graviton does not propagate in three dimensions, we can change the overall sign of the action in order to obtain the ghost free theory. This three-dimensional higher-derivative model is called New Massive Gravity (NMG) and was proposed by Bergshoeff, Hohm, and Townsend. The action of NMG is given by [1]

$$S = \frac{M_P}{2} \int d^3 x \sqrt{-g} \left\{ -R + \frac{1}{m^2} (R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2) \right\}, \tag{3}$$

where M_P is the Planck mass. The special tuning of parameters in front of higher-derivative terms is needed to eliminate the scalar mode. NMG is a ghost-free theory. The NMG action describes two massive degrees of freedom of helicities $+2$ and -2. One nice property of the NMG model is the diffeomorphism invariance. Using the auxiliary fields, one can lower the number of derivatives in the NMG action. At the linearized level, the action reduces to massless Einstein–Hilbert term plus the massive spin-2 Fierz–Pauli terms. It is known that starting from the 3D spin-2 FP theory and applying the «boosting up derivative» procedure [1], we can obtain the linearized (higher-derivative) NMG. This can be applied for gravitini as well. Namely, starting from the massive spin-3/2 FP equations and applying the «boosting up derivative» procedure, we obtain the higher-derivative NMG-like theory (for more details, we refer to Appendix B in [2]). We want to construct $\mathcal{N} = 1$ supersymmetrization of linearized low-derivative version of NMG.

2. SUPERSYMMETRIC NMG

Although the supersymmetrized NMG was constructed in [3], we want to construct the supersymmetric low-derivative NMG version. This requires introduction of the auxiliary fields. We consider only the linearized NMG here. As was mentioned in the previous section, linearized NMG decomposes into the massless spin-2 (Einstein–Hilbert) and massive spin-2 (Fierz–Pauli) theory. Hence, we need to construct the massive spin-2 and massless spin-2 multiplets in 3D for the supersymmetric extension of linearized NMG. The massless multiplet is known and the massive one is recently constructed in [2]. The massive spin-2 multiplet can be obtained by the Kaluza–Klein (KK) reduction of the massless spin-2 (off-shell) $\mathcal{N}=1$ multiplet in 4D and a truncation to the first massive mode, see [2].

We will start from the action for the linearized NMG auxiliary field version:

$$S=\int d^3x\left\{-h^{\mu\nu}G^{\rm lin}_{\mu\nu}(h)+2q^{\mu\nu}G^{\rm lin}_{\mu\nu}(h)-m^2(q^{\mu\nu}q_{\mu\nu}-q^2)\right\}, \tag{4}$$

where $q_{\mu\nu}$ is a symmetric auxiliary field and $q=\eta^{\mu\nu}q_{\mu\nu}$. The action can be diagonalized, by imposing redefinitions $h_{\mu\nu}=A_{\mu\nu}+B_{\mu\nu}$ and $q_{\mu\nu}=B_{\mu\nu}$, into the following one:

$$S=\int d^3x\left\{-A^{\mu\nu}G^{\rm lin}_{\mu\nu}(A)+B^{\mu\nu}G^{\rm lin}_{\mu\nu}(B)-m^2(B^{\mu\nu}B_{\mu\nu}-B^2)\right\}. \tag{5}$$

We can identify now the massless mode $A_{\mu\nu}$ given by the Einstein–Hilbert term and the massive mode $B_{\mu\nu}$ given by the Fierz–Pauli terms. Hence, we can supersymmetrize the action above in terms of the massless multiplet $(A_{\mu\nu},\lambda_\mu,S)$ and the massive multiplet $(B_{\mu\nu},\psi_\mu,\chi_\mu,M,N,P,A_\mu)$, as it is explained below.

The off-shell massless spin-2 multiplet in 3D is very well known: it consists of a massless spin-2 $A_{\mu\nu}$, massless gravitino λ_μ, and an auxiliary scalar S. The off-shell supersymmetry rules are

$$\delta A_{\mu\nu}=\bar\epsilon\gamma_{(\mu}\lambda_{\nu)},\quad \delta\lambda_\mu=-\frac14\gamma^{\rho\sigma}\partial_\rho A_{\sigma\mu}\epsilon+\frac12 S\gamma_\mu\epsilon,\quad \delta S=\frac14\bar\epsilon\gamma^{\mu\nu}\partial_\mu\lambda_\nu, \tag{6}$$

and they leave the following action invariant:

$$S_{m=0}=\int d^3x\left\{-A^{\mu\nu}G^{\rm lin}_{\mu\nu}(A)+4\bar\lambda_\mu\gamma^{\mu\nu\rho}\partial_\nu\lambda_\rho+8S^2\right\}.$$

The supersummetry rules of the 3D $\mathcal{N}=1$ off-shell massive spin-2 multiplet obtained in [2] are

$$\begin{aligned}
\delta B_{\mu\nu}&=\bar\epsilon\gamma_{(\mu}\psi_{\nu)}+\frac1m\bar\epsilon\partial_{(\mu}\chi_{\nu)},\\
\delta\psi_\mu&=-\frac14\gamma^{\rho\lambda}\partial_\rho B_{\lambda\mu}\epsilon+\frac1{12}\gamma_\mu(M+P)\epsilon+\frac1{12m}\partial_\mu(N+\gamma^\rho A_\rho)\epsilon,\\
\delta\chi_\mu&=\frac14 m\gamma^\rho B_{\rho\mu}\epsilon+\frac14 A_\mu\epsilon-\frac1{12}\gamma_\mu(N+\gamma^\rho A_\rho)\epsilon-\frac1{12m}\partial_\mu(M-2P)\epsilon,\\
\delta M&=\bar\epsilon\gamma^{\rho\lambda}\partial_\rho\psi_\lambda-m\bar\epsilon\gamma^\rho\chi_\rho,\\
\delta N&=-\bar\epsilon\gamma^{\rho\lambda}\partial_\rho\chi_\lambda+m\bar\epsilon\gamma^\rho\psi_\rho,\\
\delta P&=\frac12\bar\epsilon\gamma^{\rho\lambda}\partial_\rho\psi_\lambda+m\bar\epsilon\gamma^\rho\chi_\rho,\\
\delta A_\mu&=\frac32\bar\epsilon\gamma_\mu{}^{\rho\lambda}\partial_\rho\chi_\lambda-\bar\epsilon\gamma_\mu\gamma^{\rho\lambda}\partial_\rho\chi_\lambda-\frac12 m\bar\epsilon\gamma_\mu{}^\rho\psi_\rho+m\bar\epsilon\psi_\mu,
\end{aligned} \tag{7}$$

and the action invariant under (7) is

$$\begin{aligned}
S_{m\neq0}=\int d^3x\Big\{&B^{\mu\nu}G^{\rm lin}_{\mu\nu}(B)-m^2\big(B^{\mu\nu}B_{\mu\nu}-B^2\big)-\\
&-4\bar\psi_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho-4\bar\chi_\mu\gamma^{\mu\nu\rho}\partial_\nu\chi_\rho+8m\bar\psi_\mu\gamma^{\mu\nu}\chi_\nu-\frac23M^2-\frac23N^2+\frac23P^2+\frac23A_\mu A^\mu\Big\}.
\end{aligned}$$

By combining the action for the massive multiplet and the action for the massless multiplet, we obtain the action for the supersymmetric NMG:

$$S_{\rm SNMG} = \int d^3x \Big\{ -h^{\mu\nu} G^{\rm lin}_{\mu\nu}(h) + 2q^{\mu\nu} G^{\rm lin}_{\mu\nu}(h) - m^2 \big(q^{\mu\nu} q_{\mu\nu} - q^2\big) + 8S^2 + \\ + 4\bar\rho_\mu \gamma^{\mu\nu\rho} \partial_\nu \rho_\rho - 8\bar\psi_\mu \gamma^{\mu\nu\rho} \partial_\nu \rho_\rho - 4\bar\chi_\mu \gamma^{\mu\nu\rho} \partial_\nu \chi_\rho + 8m\bar\psi_\mu \gamma^{\mu\nu} \chi_\nu - \\ - \frac{2}{3}M^2 - \frac{2}{3}N^2 + \frac{2}{3}P^2 + \frac{2}{3}A_\mu A^\mu \Big\}.$$

and the supersymmetry rules are the following:

$$\begin{aligned} \delta h_{\mu\nu} &= \bar\epsilon \gamma_{(\mu} \rho_{\nu)}, \\ \delta \rho_\mu &= -\frac{1}{4}\gamma^{\rho\sigma}\big(\partial_\rho h_{\mu\sigma}\big)\epsilon + \frac{1}{2} S \gamma_\mu \epsilon + \frac{1}{12}\gamma_\mu (M+P)\epsilon, \\ \delta S &= \frac{1}{4}\bar\epsilon \gamma^{\mu\nu} \partial_\mu \rho_\nu - \frac{1}{4}\bar\epsilon\gamma^{\mu\nu}\partial_\mu \psi_\nu, \end{aligned} \qquad (8)$$

together with transformation rules (7). We can distinguish between two types of auxiliary fields in the final action. One type of auxiliary fields upon their elimination from the action leads to the new action with higher derivatives. Those auxiliary fields we call nontrivial ones. The nontrivial auxiliary fields are $q_{\mu\nu}$, $\psi_{\mu\nu}$, and $\chi_{\mu\nu}$. Elimination of other auxiliary fields, M, N, P, S, and A_μ from the action does not lead to the higher-derivative action, since the equations of motion for these fields are zero. This kind of auxiliary fields we call trivial ones. They are needed only to close the algebra off-shell.

CONCLUSIONS

We constructed the linearized low-derivative $\mathcal{N} = 1$ SNMG. For this purpose, we needed the massless spin-2 3D off-shell multiplet, which was known, and we derived the 3D off-shell massive spin-2 multiplet from the 4D off-shell massless spin-2 multiplet by performing the KK reduction. We investigated the structure of the auxiliary fields needed for the closure of the algebra. Extension to the nonlinearized level is possible, but the answer is not elegant. Maybe the superspace technique can help with addressing this problem.

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