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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

RENORMDYNAMICS, UNIFIED FIELD THEORIES AND UNIVERSAL DISTRIBUTIONS OF THE MULTIPARTICLE PRODUCTION PROCESSES

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Short introduction to renormdynamics (in and beyond critical dimensions) is given. The fixed points of the pion–(rho-meson)–nucleon and unified field fine structure coupling constants are predicted. Universal distributions for the higher energy multiparticle production processes are constructed.

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Renormdynamics unifies different renormgroups in one society

1. RENORMDYNAMICS

Quantum Field Theory (QFT) and Fractal Calculus (FC) provide universal language of fundamental physics (see, e.g., [7]). In QFT existence of a given theory means that we can control its behavior at some scales (short or large distances) by renormalization theory [1, 2]. If the theory exists, then we want to solve it, which means to determine what happens on other (large or short) scales. This is the problem (and content) of Renormdynamics. The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one).

1.1. p-Adic Convergence of Perturbation Theory Series. Perturbation theory series (PTS) have the following qualitative form:

$$f(g) = f_0 + f_1 g + \ldots + f_n g^n + \ldots,$$
$$f(x) = \sum_{n\geqslant 0} P(n) n! x^n = P(\delta)\Gamma(1+\delta)\frac{1}{1-x}, \quad f_n = n!P(n), \quad \delta = x\frac{d}{dx}. \tag{1}$$

So, we reduce the previous series to the standard geometric progression series. This series is convergent for $|x| < 1$ or for $|x|_p = p^{-k} < 1$, $x = p^k a/b$, $k \geqslant 1$, $p = 2, 3, 5, \ldots, 29, \ldots, 137, \ldots$

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With an appropriate nomalization of the expansion parameter, the coefficients of the series are rational numbers and if experimental data indicates for some prime value for g, e.g., in QED

$$g = \frac{e^2}{4\pi} = \frac{1}{137.0\ldots}, \tag{2}$$

then we can take the corresponding prime number and consider p-adic convergence of the series. In the Yukawa theory of strong interactions (see, e.g., [1]), we take $g = 13$,

$$f(g) = \sum f_n p^n, \quad f_n = n!P(n), \quad p = 13, \quad |f|_p \leqslant \sum |f_n|_p p^{-n} < \frac{1}{1 - p^{-1}}. \tag{3}$$

So, the series is convergent. If the limit is rational number, we consider it as an observable value of the corresponding physical quantity. In MSSM (see [5]) coupling constants unify at $\alpha_u^{-1} = 26.3 \pm 1.9 \pm 1$. So, $23.4 < \alpha_u^{-1} < 29.2$.

Question: How many primes are in this interval? 24, 25, 26, 27, 28, 29.

Only one! Proposal: Take the value $\alpha_u^{-1} = 29.0\ldots$ which will be two orders of magnitude more precise than prediction and find the consequences for the SM scale observables.

1.2. The Goldberger–Treiman Relation and the Pion–Nucleon Coupling Constant. The Goldberger–Treiman Relation (GTR) [3] plays an important role in theoretical hadronic and nuclear physics. GTR relates the meson–nucleon coupling constants to the axial-vector coupling constant in β-decay:

$$g_{\pi N} f_\pi = g_A m_N, \tag{4}$$

where m_N is the nucleon mass; g_A is the axial-vector coupling constant in nucleon β-decay at vanishing momentum transfer; f_π is the π decay constant, and $g_{\pi N}$ is the π–N coupling constant. Since the days when the Goldberger–Treiman relation was discovered, the value of g_A has increased considerably. Also, f_π decreased a little, on account of radiative corrections. The main source of uncertainty is $g_{\pi N}$. If we take

$$\alpha_{\pi N} = \frac{g_{\pi N}^2}{4\pi} = 13 \Rightarrow g_{\pi N} = 12.78 \tag{5}$$

experimental values for f_π from pion decay and nucleon mass

$$f_\pi = \frac{130}{\sqrt{2}} = 91.9 \text{ MeV}, \quad m_N = 940 \text{ MeV}, \tag{6}$$

from (4), we find

$$g_A = \frac{f_\pi g_{\pi N}}{m_N} = \frac{91.9 \cdot \sqrt{52\pi}}{940} = 1.2496 \simeq 1.25 = \frac{5}{4}. \tag{7}$$

2. RENORMDYNAMICS OF QCD

QCD is the theory of the strong interactions with one mass parameter for each quark species and the value of the QCD coupling constant at some energy or momentum scale in some renormalization scheme. This last free parameter of the theory can be fixed by Λ_{QCD}, the energy scale used as the typical boundary condition for the integration of the Renormdynamic (RD) equation for the strong coupling constant. This is the parameter which expresses the scale of strong interactions, the only parameter in the limit of massless quarks. While the evolution of the coupling with the momentum scale is determined by the quantum corrections induced by the renormalization of the bare coupling and can be computed in perturbation theory, the strength itself of the interaction, given at any scale by the value of the renormalized coupling at this scale, or equivalently by $\Lambda_{\rm QCD}$, is one of the above-mentioned parameters of the theory and has to be taken from experiment. The RD equations play an important role in our understanding of Quantum Chromodynamics and the strong interactions. The beta function and the quarks mass anomalous dimension are among the most prominent objects for QCD RD equations. The MS-scheme [4] belongs to the class of massless schemes where the β function does not depend on masses of the theory and the first two coefficients of the β function are scheme-independent. The RD equation for the coupling constant is

$$\dot a = \beta(a) = \beta_2 a^2 + \beta_3 a^3 + \beta_4 a^4 + \beta_5 a^5 + O(a^6), \int\limits_{a_0}^{a} \frac{da}{\beta(a)} = t - t_0 = \ln \frac{\mu^2}{\mu_0^2}, \tag{8}$$

μ is the 't Hooft unit of mass, the renormalization point in the MS-scheme. To calculate the β function, we need to calculate the renormalization constant Z of the coupling constant, $a_b = Za$, where a_b is the bare (unrenormalized) charge. The expression of the β function can be obtained in the following way:

$$\begin{gathered} 0 = \frac{d(a_b \mu^{2\varepsilon})}{dt} = \mu^{2\varepsilon}\left(\varepsilon Z a + \frac{\partial (Za)}{\partial a}\frac{da}{dt}\right) \Rightarrow \frac{da}{dt} = \beta(a,\varepsilon) = \frac{-\varepsilon Z a}{\frac{\partial (Za)}{\partial a}} = -\varepsilon a + \beta(a), \\ \beta(a) = a\frac{d}{da}(aZ_1), \quad \beta(a,\varepsilon) = \frac{D-4}{2}a + \beta(a), \end{gathered} \tag{9}$$

where Z_1 is the residue of the first pole in ε expansion

$$Z(a,\varepsilon) = 1 + Z_1 \varepsilon^{-1} + \ldots + Z_n \varepsilon^{-n} + \ldots \tag{10}$$

For quark anomalous dimension, RD equation is

$$\begin{gathered} \dot b = \gamma(a) = \gamma_1 a + \gamma_2 a^2 + \gamma_3 a^3 + \gamma_4 a^4 + O(a^5), \\ b(t) = b_0 + \int\limits_{t_0}^{t} dt \gamma(a(t)) = b_0 + \int\limits_{a_0}^{a} da \gamma(a)/\beta(a). \end{gathered} \tag{11}$$

To calculate the quark mass anomalous dimension $\gamma(a)$, we need to calculate the renormalization constant Z_m of the quark mass $m_b = Z_m m,\ m_b$ is the bare quark mass. Then we find the function $\gamma(a)$ in the following way:

$$\begin{aligned} 0 = \dot m_b = \dot Z_m m + Z_m \dot m = Z_m m((\ln Z_m)^{\cdot} + (\ln m)^{\cdot}) \Rightarrow \gamma(a) = -\frac{d\ln Z_m}{dt} = \\ = \dot b = -\frac{d\ln Z_m}{da}\frac{da}{dt} = -\frac{d\ln Z_m}{da}(-\varepsilon a + \beta(a)) = a\frac{dZ_{m1}}{da},\ b = -\ln Z_m = \ln\frac{m}{m_b}, \end{aligned} \tag{12}$$

where Z_{m1} is the coefficient of the first pole in the ε-expansion of the Z_m in MS-scheme

$$Z_m(\varepsilon, a) = 1 + Z_{m1}\varepsilon^{-1} + Z_{m2}\varepsilon^{-2} + \ldots \tag{13}$$

2.1. Reparametrization and Solution of the RD Equation. RD equation

$$\dot{a} = \beta_1 a + \beta_2 a^2 + \ldots \tag{14}$$

can be reparametrized:

$$a(t) = f(A(t)) = A + f_2 A^2 + \ldots + f_n A^n + \ldots, \quad \dot{A} = b_1 A + b_2 A^2 + \ldots,$$

$$\begin{aligned} \dot{a} = \dot{A} f'(A) = (b_1 A + b_2 A^2 + \ldots)(1 + 2 f_2 A + \ldots + n f_n A^{n-1} + \ldots) = \\ = \beta_1 (A + f_2 A^2 + \ldots + f_n A^n + \ldots) + \beta_2 (A^2 + 2 f_2 A^3 + \ldots) + \ldots = \\ = \beta_1 A + (\beta_2 + \beta_1 f_2) A^2 + (\beta_3 + 2\beta_2 f_2 + \beta_1 f_3) A^3 + \\ + \ldots + (\beta_n + (n-1)\beta_{n-1} f_2 + \ldots + \beta_1 f_n) A^n + \ldots, \end{aligned} \tag{15}$$

$$\begin{aligned} & b_1 = \beta_1,\ b_2 = \beta_2 + f_2 \beta_1 - 2 f_2 b_1 = \beta_2 - f_2 \beta_1, \\ & b_3 = \beta_3 + 2 f_2 \beta_2 + f_3 \beta_1 - 2 f_2 b_2 - 3 f_3 b_1 = \beta_3 + 2(f_2^2 - f_3)\beta_1, \\ & b_4 = \beta_4 + 3 f_2 \beta_3 + f_2^2 \beta_2 + 2 f_3 \beta_2 - 3 f_4 b_1 - 3 f_3 b_2 - 2 f_2 b_3, \ldots, \\ & b_n = \beta_n + \ldots + \beta_1 f_n - 2 f_2 b_{n-1} - \ldots - n f_n b_1, \ldots \end{aligned}$$

so, by reparametrization, beyond the critical dimension ($\beta_1 \neq 0$) we can change any coefficient but β_1. We can fix any higher coefficient with zero value. In the critical dimension of space-time, $\beta_1 = 0$, and we can change by reparametrization any coefficient but β_2 and β_3. From the relations (15), in the critical dimenshion ($\beta_1 = 0$), we find that we can define the minimal form of the RD equation

$$\dot{A} = \beta_2 A^2 + \beta_3 A^3. \tag{16}$$

We can solve (16) as implicit function,

$$u^{\beta_3/\beta_2}\, \mathrm{e}^{-u} = c\, \mathrm{e}^{\beta_2 t}, \quad u = \frac{1}{A} + \frac{\beta_3}{\beta_2}, \tag{17}$$

then, as in the noncritical case, explicit solution for a will be given by reparametrization representation (15) [9]. If we know somehow the coefficients β_n, e.g., for first several exact and for other asymptotic values (see, e.g., [6]) then we can construct reparametrization function (15) and find the dynamics of the running coupling constant. At any given scale by reparametrization $a = f(A)$ we can define new expansion parameter A as appropriate prime number. For example, if beyond critical dimension we determine all $b_n = 0, n \geqslant 3$, demand that $A = p$, then

$$0 = \dot{A} = b_1 A + b_2 A^2 \Rightarrow b_2 = -\frac{b_1}{p} = \beta_2 - f_2 \beta_1 \Rightarrow f_2 = \frac{\beta_2 + \beta_1/p}{\beta_1}. \tag{18}$$

Statement: The reparametrization series for a is p-adically convergent, when β_n is rational number. The scale at which we have a fixed point is reparametrization-invariant, universal. Indeed,

$$0 = \dot{a} = f'(A)\dot{A} = 0. \tag{19}$$

So, when we calculate the scale, e.g., the hadronization scale in the case of QCD by lattice gauge theory methods, we can improve precision by comparing results of different definitions.

Let us take the anomalous dimension of some quantity

$$\gamma(a) = \gamma_1 a + \gamma_2 a^2 + \gamma_3 a^3 + \ldots, \quad a = f(A) = A + f_2 A^2 + f_3 A^3 + \ldots,$$

$$\gamma(a) = \gamma_1(A + f_2A^2 + f_3A^3 + \ldots) + \gamma_2(A^2 + 2f_2A^3 + \ldots) + \gamma_3(A^3 + \ldots) + \ldots = \\ = \Gamma_1 A + \Gamma_2 A^2 + \Gamma_3 A^3 + \ldots, \quad (20)$$

$$\Gamma_1 = \gamma_1, \quad \Gamma_2 = \gamma_2 + \gamma_1 f_2, \quad \Gamma_3 = \gamma_3 + 2\gamma_2 f_2 + \gamma_1 f_3, \ldots$$

When $\gamma_1 \neq 0$, we can take $\Gamma_n = 0,\ n \geqslant 2$. So, we get the exact value for the anomalous dimension

$$\gamma(A) = \gamma_1 A = \gamma_1 f^{-1}(a) = \gamma_1 \left(a + \frac{\gamma_2}{\gamma_1 a^2} + \frac{\gamma_3}{\gamma_1 a^3} + \ldots : \right). \quad (21)$$

2.2. QCD, Parton Model, Valence Quarks and $\alpha_s = 2$. While it has been well established in the perturbative regime at high energies, QCD still lacks a comprehensive solution at low and intermediate energies, even 40 years after its invention. In order to deal with the wealth of non-perturbative phenomena, various approaches are followed with limited validity and applicability. This is especially also true for lattice QCD, various functional methods, or chiral perturbation theory, to name only a few. In neither one of these approaches the full dynamical content of QCD can yet be included. Basically, the difficulties are associated with a relativistically covariant treatment of confinement and the spontaneous breaking of chiral symmetry, the latter being a well-established property of QCD at low and intermediate energies. As a result, most hadron reactions, like resonance excitations, strong and electroweak decays, etc., are nowadays only amenable to models of QCD. Most famous is the constituent-quark model (CQM), which essentially relies on a limited number of effective degrees of freedom with the aim of encoding the essential features of low- and intermediate-energy QCD. The CQM has a long history, and it has made important contributions to the understanding of many hadron properties, think only of the fact that the systematization of hadrons in the standard particle-data base follows the CQM or valence-quark picture.

It was noted [10] that parton densities given by the following solution:

$$M_2(Q^2) = \frac{3}{25} + \frac{2}{3}\omega^{32/81} + \frac{16}{75}\omega^{50/81}, \quad \bar{M}_2(Q^2) = M_2^s(Q^2) = \frac{3}{25} - \frac{1}{3}\omega^{32/81} + \frac{16}{75}\omega^{50/81},$$

$$M_2^G(Q^2) = \frac{16}{25}(1 - \omega^{50/81}), \quad \omega = \frac{\alpha_s(Q^2)}{\alpha_s(m^2)}, \quad Q^2 \in (5, 20)\ \mathrm{GeV}^2, \quad (22)$$

$$b = 11 - \frac{2}{3}n_f = 9,\ \alpha_s(Q^2) \simeq 0.2$$

of the Altarelli–Parisi equation

$$\dot{M} = AM, \quad M^T = (M_2, \bar{M}_2, M_2^s, M_2^G),$$

$$M_2 = \int_0^1 dx\, x(u(x) + d(x)), \quad \bar{M}_2 = \int_0^1 dx\, x(\bar{u}(x) + \bar{d}(x)),$$

$$M_2^s = \int_0^1 dx\, x(s(x) + \bar{s}(x)),\ M_2^G = \int_0^1 dx\, xG(x),$$

$$A = -a(Q^2)\begin{pmatrix} 32/9 & 0 & 0 & -2/3 \\ 0 & 32/9 & 0 & -2/3 \\ 0 & 0 & 32/9 & -2/3 \\ -32/9 & -32/9 & -32/9 & 2 \end{pmatrix}, \tag{23}$$

$$a = \left(\frac{g}{4\pi}\right)^2, \quad \dot{M} = Q^2\frac{dM}{dQ^2}$$

with the following valence quark initial condition at a scale m:

$$\bar{M}_2(m^2) = M_2^s(m^2) = M_2^G(m^2) = 0, \quad M_2(m^2) = 1 \tag{24}$$

and

$$\alpha_s(m^2) = 2, \tag{25}$$

give the experimental values

$$M_2 = 0.44, \quad \bar{M}_2 = M_2^s = 0.04, \quad M_2^G = 0.48. \tag{26}$$

So, for valence quark model (VQCD), $\alpha_s(m^2) = 2$. We have seen that for $\pi\rho N$ model $\alpha_{\pi\rho N} = 3$ and for πN model $\alpha_{\pi N} = 13$. It is nice that $\alpha_s^2 + \alpha_{\pi\rho N}^2 = \alpha_{\pi N}$. Note that to $\alpha_s = 2$ corresponds

$$g = \sqrt{4\pi\alpha_s} = 5.013 = 5 + . \tag{27}$$

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