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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ELECTROWEAK MODEL AT INFINITE ENERGY

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The infinite energy limit of the Electroweak Model with the contracted gauge group is regarded at the level of classical gauge fields. Masses of all particles of the Electroweak Model disappear under contraction, so the Lagrangian of the limit model includes massless neutral Z bosons, massless u quarks, neutrinos and photons as well as weak and electromagnetic interactions. The weak interactions become long-range and are mediated by neutral currents. The limit model represents the development of the early Universe from the Big Bang up to the end of the first second.

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INTRODUCTION

The modern theory of electroweak processes is the Electroweak Model, which is in good agreement with the experimental data, including the latest ones from LHC. This model is a gauge theory based on the gauge group $SU(2) \times U(1)$, which is the direct product of two simple groups. The operation of group contraction [1] transforms a simple or semisimple group to a nonsemisimple one. For better understanding of a complicated physical system it is useful to investigate its limits for the limiting values of its physical parameters. In this paper we discuss the modified Electroweak Model with the contracted gauge group $SU(2;j) \times U(1)$ at the level of classical gauge fields. It was shown [2,3] that the contraction parameter depends on the energy s in the center-of-mass system, so the contracted gauge group corresponds to the zero energy limit of the Electroweak Model. In this paper we discuss another contraction scheme, which describes the infinite energy limit of the Electroweak Model.

1. TWO POSSIBLE CONTRACTIONS OF THE ELECTROWEAK MODEL

We consider the Electroweak Model where the contracted gauge group $SU(2;j) \times U(1)$ acts in the boson, lepton and quark sectors. The contracted group $SU(2;j)$ and its fundamental representation space $C_2(j)$ can be obtained by the *consistent rescaling* of the group $SU(2)$ and the space C_2:

$$z'(j) = \begin{pmatrix} jz_1' \\ z_2' \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} jz_1 \\ z_2 \end{pmatrix} = u(j)z(j),$$
$$\det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1, \qquad (1)$$

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when the contraction parameter tends to zero $j \to 0$ or is equal to the nilpotent unit $j = \iota$, $\iota^2 = 0$. The contracted group $SU(2;\iota)$ is isomorphic to Euclide group $E(2)$ and the space $C_2(\iota)$ is the fiber space with the one-dimensional base $\{z_2\}$ and the one-dimensional fiber $\{z_1\}$. The actions of the unitary group $U(1)$ and the electromagnetic subgroup $U(1)_{\rm em}$ in the space $C_2(j)$ are given by the same matrices as in C_2.

The space $C_2(j)$ can be obtained from C_2 by the substitution $z_1 \to jz_1$, which induces the substitution of Lie algebra generators: $T_1 \to jT_1$, $T_2 \to jT_2$, $T_3 \to T_3$. As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely: $A^1_\mu \to jA^1_\mu$, $A^2_\mu \to jA^2_\mu$, $A^3_\mu \to A^3_\mu$, $B_\mu \to B_\mu$. For the standard gauge boson fields these substitutions are as follows:

$$W^\pm_\mu \to jW^\pm_\mu, \quad Z_\mu \to Z_\mu, \quad A_\mu \to A_\mu. \tag{2}$$

The lepton fields $L_l = \left(\begin{array}{c} \nu_l \\ e_l \end{array}\right)$ and the quark fields $Q_l = \left(\begin{array}{c} u_l \\ d_l \end{array}\right)$ are $SU(2)$-doublets, i.e., vectors in the space C_2, so their components are transformed similarly to the components of z, namely:

$$\nu_l \to j\nu_l, \quad e_l \to e_l, \quad u_l \to ju_l, \quad d_l \to d_l. \tag{3}$$

The right lepton and quark fields are $SU(2)$-singlets, i.e., scalars, and therefore are not transformed. In this scheme the contraction parameter is connected with the energy s in center-of-mass system and is evaluated through the fundametal parameters of the Electroweak Model:

$$j^2(s) = \frac{g}{m_W}\sqrt{s}, \tag{4}$$

where m_W is W-boson mass and g is constant [2, 3].

It follows from the last equation (4) that the contraction $j \to 0$ corresponds to zero energy limit of the Electroweak Model. In this limit the first components of the lepton and quark doublets become infinitely small in comparison with their second components. On the contrary, when energy increases the first components of the doublets become greater than their second ones. In the infinite energy limit the second components of the lepton and quark doublets will be infinitely small as compared with their second components. To describe this limit, we introduce a new contraction parameter ϵ and *new consistent rescaling* of the group $SU(2)$ and the space C_2 as follows:

$$\begin{gathered} z'(\epsilon) = \left(\begin{array}{c} z'_1 \\ \epsilon z'_2 \end{array}\right) = \left(\begin{array}{cc} \alpha & \epsilon\beta \\ -\epsilon\bar\beta & \bar\alpha \end{array}\right)\left(\begin{array}{c} z_1 \\ \epsilon z_2 \end{array}\right) = u(\epsilon)z(\epsilon), \\ \det u(\epsilon) = |\alpha|^2 + \epsilon^2|\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1. \end{gathered} \tag{5}$$

Both contracted groups $SU(2;j)$ (1) and $SU(2;\epsilon)$ (5) are the same and are isomorphic to Euclide group $E(2)$, but the space $C_2(\epsilon)$ is split in the limit $\epsilon \to 0$ into the one-dimensional base $\{z_1\}$ and the one-dimensional fiber $\{z_2\}$. From the mathematical point of view it is not important if the first or the second Cartesian axis forms the base of fibering and in this sense constructions (1) and (5) are equivalent. But the doublet components are interpreted as certain physical fields; therefore, the fundamental representations (1) and (5) of the same contracted unitary group lead to different limit cases of the Electroweak Model, namely, its zero energy and infinite energy limits.

In the second contraction scheme (5) all gauge bosons are transformed according to the rules (2) with the natural substitution of j by ϵ. Instead of (3) the lepton and quark fields are transformed now as follows:

$$e_l \to \epsilon e_l, \quad d_l \to \epsilon d_l, \quad \nu_l \to \nu_l, \quad u_l \to u_l. \tag{6}$$

The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, which is used to generate mass of vector bosons and other elementary particles of the model. In this mechanism one of Lagrangian ground states $\phi^{\mathrm{vac}} = \begin{pmatrix} 0 \\ v \end{pmatrix}$ is taken as vacuum of the model and then small field excitations $v + \chi(x)$ with respect to this vacuum are regarded. So Higgs boson field χ and the constant v are multiplied by ϵ. As far as masses of all particles are proportionate to v, we obtain the following transformation rule for contraction (5):

$$\chi \to \epsilon\chi, \quad v \to \epsilon v, \quad m_p \to \epsilon m_p, \tag{7}$$

where $p = \chi, W, Z, e, u, d$.

2. HIGH-ENERGY LAGRANGIAN OF ELECTROWEAK MODEL

After transformations (2), (6), (7) the boson Lagrangian of the Electroweak Model can be represented in the form

$$L_B(\epsilon) = -\frac{1}{4}\mathcal{Z}_{\mu\nu}^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \epsilon^2 L_{B,2} + \epsilon^3 g W_\mu^+ W_\mu^- \chi + \epsilon^4 L_{B,4},$$

$$L_{B,4} = m_W^2 W_\mu^+ W_\mu^- - \frac{1}{2}m_\chi^2\chi^2 - \lambda v \chi^3 - \frac{\lambda}{4}\chi^4 + \frac{g^2}{4}\left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+\right)^2 + \frac{g^2}{4} W_\mu^+ W_\nu^- \chi^2,$$

$$\begin{aligned} L_{B,2} = {} & \frac{1}{2}\left(\partial_\mu\chi\right)^2 + \frac{1}{2}m_Z^2\left(Z_\mu\right)^2 - \frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}_{\mu\nu}^- + \frac{g m_z}{2\cos\theta_W}\left(Z_\mu\right)^2\chi + \frac{g^2}{8\cos^2\theta_W}\left(Z_\mu\right)^2\chi^2 - \\ & - 2ig\left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+\right)\left(\mathcal{F}_{\mu\nu}\sin\theta_W + \mathcal{Z}_{\mu\nu}\cos\theta_W\right) - \\ & - \frac{i}{2}e\left[A_\mu\left(\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+\right) + \frac{i}{2}eA_\nu\left(\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+\right)\right] - \\ & - \frac{i}{2}g\cos\theta_W\left[Z_\mu\left(\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+\right) - Z_\nu\left(\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+\right)\right] - \\ & - \frac{e^2}{4}\Big\{\left[(W_\mu^+)^2 + (W_\mu^-)^2\right](A_\nu)^2 - 2(W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-)A_\mu A_\nu + \left[(W_\nu^+)^2 + (W_\nu^-)^2\right](A_\mu)^2\Big\} - \\ & - \frac{g^2}{4}\cos\theta_W\Big\{\left[(W_\mu^+)^2 + (W_\mu^-)^2\right](Z_\nu)^2 - 2(W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-)Z_\mu Z_\nu + \\ & + \left[(W_\nu^+)^2 + (W_\nu^-)^2\right](Z_\mu)^2\Big\} - eg\cos\theta_W\Big[W_\mu^+ W_\mu^- A_\nu Z_\nu + W_\nu^+ W_\nu^- A_\mu Z_\mu - \\ & - \frac{1}{2}(W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-)(A_\mu Z_\nu + A_\nu Z_\mu)\Big]. \end{aligned} \tag{8}$$

In terms of electron and neutrino fields the lepton Lagrangian takes the form

$$L_L(\epsilon) = L_{L,0} + \epsilon^2 L_{L,2} = \nu_l^\dagger i\tilde{\tau}_\mu \partial_\mu \nu_l + e_r^\dagger i\tau_\mu \partial_\mu e_r + g' \sin\theta_w e_r^\dagger \tau_\mu Z_\mu e_r - \\ - g' \cos\theta_w e_r^\dagger \tau_\mu A_\mu e_r + \frac{g}{2\cos\theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \epsilon^2 \Big\{ e_l^\dagger i\tilde{\tau}_\mu \partial_\mu e_l - m_e(e_r^\dagger e_l + e_l^\dagger e_r) + \\ + \frac{g\cos 2\theta_w}{2\cos\theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \frac{g}{\sqrt{2}} \left(\nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l \right) \Big\}. \quad (9)$$

In terms of u- and d-quarks fields the quark Lagrangian can be written as

$$L_Q(\epsilon) = L_{Q,0} - \epsilon\, m_u(u_r^\dagger u_l + u_l^\dagger u_r) + \epsilon^2 L_{Q,2},$$

$$L_{Q,0} = d_r^\dagger i\tau_\mu \partial_\mu d_r + u_l^\dagger i\tilde{\tau}_\mu \partial_\mu u_l + u_r^\dagger i\tau_\mu \partial_\mu u_r - \frac{1}{3} g' \cos\theta_w d_r^\dagger \tau_\mu A_\mu d_r + \\ + \frac{1}{3} g' \sin\theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \frac{g}{\cos\theta_w} \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + \\ + \frac{2}{3} g' \cos\theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin\theta_w u_r^\dagger \tau_\mu Z_\mu u_r, \quad (10)$$

$$L_{Q,2} = d_l^\dagger i\tilde{\tau}_\mu \partial_\mu d_l - m_d(d_r^\dagger d_l + d_l^\dagger d_r) - \frac{e}{3} d_l^\dagger \tilde{\tau}_\mu A_\mu d_l - \\ - \frac{g}{\cos\theta_w} \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w \right) d_l^\dagger \tilde{\tau}_\mu Z_\mu d_l + \frac{g}{\sqrt{2}} \left[u_l^\dagger \tilde{\tau}_\mu W_\mu^+ d_l + d_l^\dagger \tilde{\tau}_\mu W_\mu^- u_l \right].$$

The complete Lagrangian of the modified model is given by the sum $L(\epsilon) = L_B(\epsilon) + L_L(\epsilon) + L_Q(\epsilon)$ and for the infinite energy (for $\epsilon = 0$) is equal to

$$L_\infty = -\frac{1}{4}\mathcal{Z}_{\mu\nu}^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \nu_l^\dagger i\tilde{\tau}_\mu \partial_\mu \nu_l + u_l^\dagger i\tilde{\tau}_\mu \partial_\mu u_l + e_r^\dagger i\tau_\mu \partial_\mu e_r + d_r^\dagger i\tau_\mu \partial_\mu d_r + \\ + u_r^\dagger i\tau_\mu \partial_\mu u_r + L_\infty^{\text{int}}(A_\mu, Z_\mu),$$

$$L_\infty^{\text{int}}(A_\mu, Z_\mu) = \frac{g}{2\cos\theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \frac{g}{\cos\theta_w} \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + \\ + g' \sin\theta_w e_r^\dagger \tau_\mu Z_\mu e_r - g' \cos\theta_w e_r^\dagger \tau_\mu A_\mu e_r - \frac{1}{3} g' \cos\theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin\theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \\ + \frac{2}{3} g' \cos\theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin\theta_w u_r^\dagger \tau_\mu Z_\mu u_r. \quad (11)$$

The limit model includes only massless particles: neutral massless Z bosons Z_μ and photons A_μ, massless right electrons e_r and neutrinos ν_l, and massless left and right quarks u_l, u_r, d_r. The electroweak interactions become long-range because they are mediated by the massless neutral Z bosons and photons. There are no interactions between particles of different kind; for example, neutrinos interact only with each other by neutral currents. Similar higher energies can exist in the early Universe after inflation and reheating in the first stages of the Hot Big Bang [4, 5]. The electroweak phase transition and neutrino decoupling which take place during the first second after the Big Bang [6] are apparently in correspondence with the infinity energy limit of the Electroweak Model (11). The mass term of u quark in the complete Lagrangian is proportional to ϵ, whereas the mass terms of electron and d quark are multiplied by ϵ^2, so u quark first restores its mass in the evolution of the Universe.

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